This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

**Usage guidelines**

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

**About Google Book Search**

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

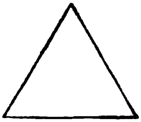
43. A polygon of nine sides is called a nonagon.

44. A polygon of ten sides is called a decagon.

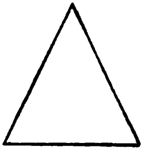
45. A polygon of twelve sides is called a dodocagon.

46. There are several kinds of triangles.

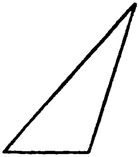
*First.* An equilateral triangle, which has its three sides all equal.



*Second.* An isosceles triangle, which has two of its sides equal.

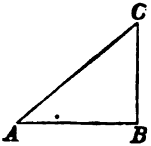


*Third.* A scalene triangle, which has its three sides all unequal.



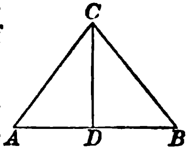
*Fourth.* A right angled triangle, which has one right angle.

In the right angled triangle *ABC*, the side *AC*, opposite the right angle, is called the hypothenuse.



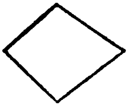
47. The base of a triangle is the side on which it stands. Thus, *AB* is the base of the triangle *ACB*.

The altitude of a triangle is a line drawn from the angle opposite the base and perpendicular to the base. Thus, *CD* is the altitude of the triangle *ACB*.

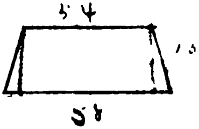


**48.** There are three kinds of quadrilaterals.

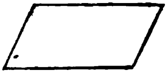
**1.** The *trapezium*, which has none of its sides parallel.



**2.** The *trapezoid*, which has only two of its sides parallel.

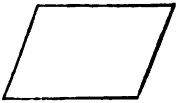


**3.** The *parallelogram*, which has its opposite sides parallel.

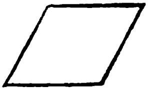


**49.** There are four kinds of parallelograms:

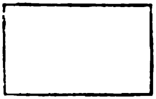
**1.** The *rhomboid*, which has no right angle.



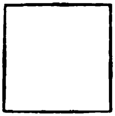
**2.** The *rhombus*, or *lozenge*, which is an equilateral rhomboid.



**3.** The *rectangle*, which is an equiangular parallelogram.



**4.** The *square*, which is both equilateral and equiangular.



Educ T 148.63.320

3 2044 097 045 009

Harvard College Library

FROM

Geo. P. Bradford,

Cambridge.

5 Feb. 1887.

Educ T 148.63.320

3 2044 097 045 009

Harvard College Library

FROM

Geo. P. Bradford,
Cambridge.
5 Feb, 1887.

Educ T 148.63.320

2044 097 045 009

Harvard College Library

FROM

Geo. P. Bradford,

Cambridge.

5 Feb. 1887.

# ELEMENTS OF GEOMETRY

AND

# TRIGONOMETRY,

WITH

## APPLICATIONS IN MENSURATION.

BY CHARLES DAVIES, LL. D.

AUTHOR OF FIRST LESSONS IN ARITHMETIC, ELEMENTARY ALGEBRA,
PRACTICAL MATHEMATICS FOR PRACTICAL MEN, ELEMENTS OF
SURVEYING, ELEMENTS OF DESCRIPTIVE GEOMETRY,
SHADES, SHADOWS, AND PERSPECTIVE, ANALYTICAL GEOMETRY, DIFFERENTIAL
AND INTEGRAL CALCULUS.

NEW YORK:
A. S. BARNES & BURR, 51 & 53 JOHN STREET.
SOLD BY BOOKSELLERS, GENERALLY, THROUGHOUT THE UNITED STATES.
1863.

Educ T 148.63.320

~~Math 5078.2.5~~

HARVARD COLLEGE
FEB 5 1887
LIBRARY.

Geo. P. Bradford,
Cambridge.

Entered according to Act of Congress, in the year Eighteen Hundred and Fifty-one,
By CHARLES DAVIES,
In the Clerk's Office of the District Court of the United States for the Southern District of New York.

Stereotyped by
RICHARD C. VALENTINE,
New York

# PREFACE.

Those who are conversant with the preparation of elementary text-books, have experienced the difficulty of adapting them to the various wants which they are intended to supply.

The institutions of education are of all grades, from the college to the district school, and although there is a wide difference between the extremes, the level, in passing from one grade to the other, is scarcely broken.

Each of these classes of seminaries requires text-books adapted to its own peculiar wants; and if each held its proper place in its own class, the task of supplying suitable works would not be difficult.

An indifferent college is generally inferior, in the system and scope of its instruction, to the academy or high school; while the district school is often found to be superior to its neighboring academy.

The Geometry of Legendre, embracing a complete course of Geometrical science, is all that is desired in the colleges and higher seminaries; while the Practical Mathematics for Practical Men, recently published, is designed to meet the wants of those schools which are strictly elementary and practical in their systems of instruction.

But still a large class of seminaries remained unsupplied with a suitable text-book on Elementary Geometry and Trigonometry: viz., those where the pupils are carried beyond the acquisition of facts and mere practical knowledge, but have not time to go through with a full course of mathematical studies.

It is for such, that the following work is designed. It has been the aim of the author to present the striking and important truths of Geometry in a form more simple and concise than could be adopted in a complete treatise, and yet to preserve the exactness of rigorous reasoning.

In this system of Geometry nothing has been taken for granted, and nothing passed over without being fully demonstrated.

The Trigonometry, including the applications to the measurements of heights and distances, has been written upon the same plan and for the same objects: it embraces all the important theorems and all the striking examples.

In order, however, to render the applications of Geometry to the mensuration of surfaces and solids complete in itself, a few rules have been given which are not demonstrated. This forms an exception to the general plan of the work, but being added in the form of an appendix, it does not materially break its unity.

That the work may be useful in advancing the interests of education, is the hope and ardent wish of the author.

FISHKILL LANDING,

*May*, 1851

# CONTENTS.

## BOOK I.

Page.
Definitions and Remarks, - - - - - - 9—16
Axioms, - - - - - - - - 16
Properties of Polygons, - - - - - - - 17—37

## BOOK II.

Of the Circle, - - - - - - - - - - 38
Problems relating to the First and Second Books, - - 53—68

## BOOK III.

Ratios and Proportions, - - - - - - - - 69—8

## BOOK IV.

Measurement of Areas and Proportions of Figures, - - 82—10
Problems relating to the Fourth Book, - - - - 109—113
Appendix—Regular Polygons, - - - - 113—115

## BOOK V.

Of Planes and their Angles, - - - - - - 116—125

## BOOK VI.

Of Solids, - - - - - - - - - 126—162
Appendix, - - - - - - - - - 163—164

## TRIGONOMETRY.

Page.
Of Logarithms, - - - - - - - - - - 165—176
Of Scales, - - - - - - - - - - 176—181
Definitions, and Explanation of Tables, - - - - - 181—189
Theorems, - - - - - - - - - - 189—192
Examples, - - - - - - - - - - 193—201
Application to Heights and Distances, - - - - - 202—210

## APPLICATIONS OF GEOMETRY.

Mensuration of Surfaces, - - - - - - - 211
General Principles, - - - - - - - 211—213
Contents of Figures, - - - - - - - 213—239

Mensuration of Solids, - - - - - - - - 239
General Principles, - - - - - - - 239—240
Solidities of Figures, - - - - - - - 240—247

Mensuration of the Round Bodies, - - - - - 248
To find the Surface of a Cylinder, - - - - - 248—249
To find the Solidity of a Cylinder, - - - - - 249—250
To find the Surface of a Cone, - - - - - - 250—251
To find the Solidity of a Cone, - - - - - - 251—252
To find the Surface of the Frustum of a Cone, - - 253
To find the Solidity of the Frustum of a Cone, - - 254
To find the Surface of a Sphere, - - - - - - 255
To find the Surface of a Spherical Zone, - - - 255—256
To find the Solidity of a Sphere, - - - - - - 256—257
To find the Solidity of a Spherical Segment, - - - 258
To find the Solidity of a Spheroid, - - - - - 259—260
To find the Surface of a Cylindrical Ring, - - 260—261
To find the Solidity of a Cylindrical Ring, - - 261—262

# ELEMENTARY

# GEOMETRY.

---

## BOOK I.

### DEFINITIONS AND REMARKS.

1. *Extension* has three dimensions, length, breadth, and thickness.

*Geometry* is the science which has for its object:

1st. The measurement of extension; and 2dly, To discover, by means of such measurement, the properties and relations of geometrical figures.

2. A *Point* is that which has place, or position, but not magnitude.

3. A *Line* is length, without breadth or thickness.

4. A *Straight Line* is one which lies in the same direction between any two of its points.

5. A *Curve Line* is one which changes is direction at every point.

The word *line* when used alone, will designate a straight line; and the word *curve*, a curve line.

6. A *Surface* is that which has length and breadth, without height or thickness.

7. A *Plane Surface* is that which lies even throughout its whole extent, and with which a straight line, laid in any direction, will exactly coincide in its whole length.

8. A *Curved Surface* has length and breadth without thickness, and like a curve line is constantly changing its direction.

9. A *Solid* or *Body* is that which has length, breadth, and thickness. Length, breadth, and thickness are called dime

sions. Hence, a solid has three dimensions, a surface two, and a line one. A point has no dimensions, but position only

10. *Geometry* treats of lines, surfaces, and solids.

11. A *Demonstration* is a course of reasoning which establishes a truth.

12. An *Hypothesis* is a supposition on which a demonstration may be founded.

13. A *Theorem* is something to be proved by demonstration.

14. A *Problem* is something proposed to be done.

15. A *Proposition* is something proposed either to be done or demonstrated—and may be either a problem or a theorem.

16. A *Corollary* is an obvious consequence, deduced from something that has gone before.

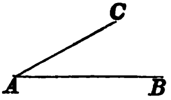
17. A *Scholium* is a remark on one or more preceding propositions.

18. An *Axiom* is a self evident proposition.

### OF ANGLES.

19. An *Angle* is the portion of a plane included between two straight lines which meet at a common point. The two straight lines are called the *sides* of the angle, and the common point of intersection, the *vertex.*

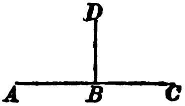
Thus, the part of the plane included between *AB* and *AC* is called an *angle:* *AB* and *AC* are its *sides*, and *A* its *vertex.*



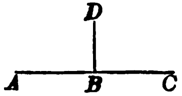
An angle is generally read, by placing the letter at the vertex in the middle. Thus, we say, the angle *CAB*. We may, however, say simply, the angle *A*.

20. One line is said to be perpendicular to another when it inclines no more to the one side than to the other

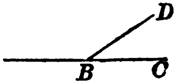
The two angles formed are then equal to each other. Thus, if the line *DB* is perpendicular to *AC*, the angle *DBA* will be equal to *DBC*.



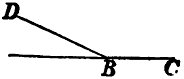
21. When two lines are perpendicular to each other, the angles which they form are called right angles. Thus, *DBA* and *DBC* are called right angles.



22. An acute angle is less than a right angle. Thus, *DBC* is an acute angle.

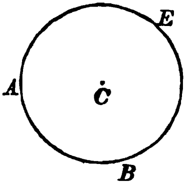


23. An obtuse angle is greater than a right angle. Thus, *DBC* is an obtuse angle.



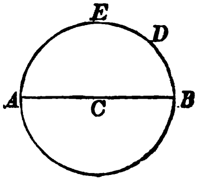
24. The circumference of a circle is a curve line all the points of which are equally distant from a certain point within called the centre.

Thus, if all the points of the curve *AEB* are equally distant from the centre *C*, this curve will be the circumference of a circle.

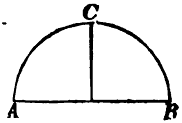


25. Any portion of the circumference, as *AED*, is called an *arc*.

26. The diameter of a circle is a straight line passing through the centre and terminating at the circumference. Thus, *ACB* is a diameter.



27. One half of the circumference, as *ACB* is called a *semicircumference;* and one quarter of the circumference, as *AC*, is called a *quadrant*



sions. Hence, a solid has three dimensions, a surface two, and a line one. A point has no dimensions, but position only

10. *Geometry* treats of lines, surfaces, and solids.

11. A *Demonstration* is a course of reasoning which establishes a truth.

12. An *Hypothesis* is a supposition on which a demonstration may be founded.

13. A *Theorem* is something to be proved by demonstration.

14. A *Problem* is something proposed to be done.

15. A *Proposition* is something proposed either to be done or demonstrated—and may be either a problem or a theorem.

16. A *Corollary* is an obvious consequence, deduced from something that has gone before.

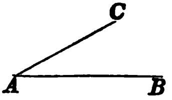
17. A *Scholium* is a remark on one or more preceding propositions.

18. An *Axiom* is a self evident proposition.

### OF ANGLES.

19. An *Angle* is the portion of a plane included between two straight lines which meet at a common point. The two straight lines are called the *sides* of the angle, and the common point of intersection, the *vertex.*

Thus, the part of the plane included between *AB* and *AC* is called an *angle:* *AB* and *AC* are its *sides*, and *A* its *vertex.*



An angle is generally read, by placing the letter at the vertex in the middle. Thus, we say, the angle *CAB.* We may, however, say simply, the angle *A.*

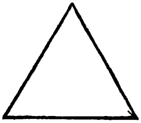
20. One line is said to be perpendicular to another when it inclines no more to the one side than to the other

### OF PLANE FIGURES.

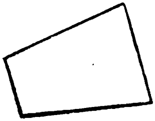
35. A Plane Figure is a portion of a plane terminated on all sides by lines, either straight or curved.

36. If the lines which bound a figure are straight, the space which they inclose is called a *rectilineal* figure, or *polygon*. The lines themselves, taken together, are called the *perimeter* of the polygon. Hence, the perimeter of a polygon is the sum of all its sides.

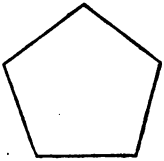
37. A polygon of three sides is called a triangle.



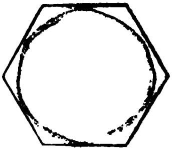
38. A polygon of four sides is called a quadrilateral.



39. A polygon of five sides is called a pentagon.



40. A polygon of six sides is called a hexagon.



41. A polygon of seven sides is called a heptagon.

42. A polygon of eight sides is called an octagon.

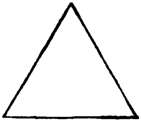
43. A polygon of nine sides is called a nonagon.

44. A polygon of ten sides is called a decagon.

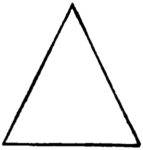
45. A polygon of twelve sides is called a dodecagon.

46. There are several kinds of triangles.

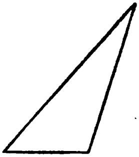
*First.* An equilateral triangle, which has its three sides all equal.



*Second.* An isosceles triangle, which has two of its sides equal.

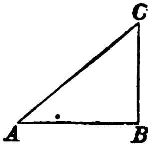


*Third.* A scalene triangle, which has its three sides all unequal.



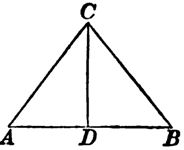
*Fourth.* A right angled triangle, which has one right angle.

In the right angled triangle $ABC$, the side $AC$, opposite the right angle, is called the hypothenuse.

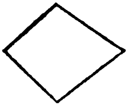


47. The base of a triangle is the side on which it stands. Thus, $AB$ is the base of the triangle $ACB$.

The altitude of a triangle is a line drawn from the angle opposite the base and perpendicular to the base. Thus, $CD$ is the altitude of the triangle $ACB$.



**48.** There are three kinds of quadrilaterals.

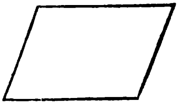


**1.** The *trapezium*, which has none of its sides parallel.

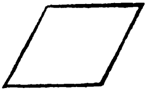
**2.** The *trapezoid*, which has only two of its sides parallel.

**3.** The *parallelogram*, which has its opposite sides parallel.

**49.** There are four kinds of parallelograms:

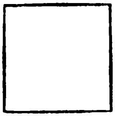


**1.** The *rhomboid*, which has no right angle.

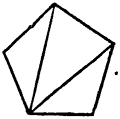


**2.** The *rhombus*, or *lozenge*, which is an equilateral rhomboid.

**3.** The *rectangle*, which is an equiangular parallelogram.



**4.** The *square*, which is both equilateral and equiangular.



50. A DIAGONAL of a figure is a line which joins the vertices of two angles not adjacent.

51. The base of a figure is the side on which it is supposed to stand; and the altitude is a line drawn from the opposite side or angle, perpendicular to the base.

## AXIOMS.

1. Things which are equal to the same thing are equal to each other.

2. If equals be added to equals, the wholes will be equal.

3. If equals be taken from equals, the remainders will be equal.

4. If equals be added to unequals, the wholes will be unequal.

5. If equals be taken from unequals, the remainders will be unequal.

6. Things which are double of equal things, are equal to each other.

7. Things which are halves of the same thing, are equal to each other.

8. The whole is greater than any of its parts

9. The whole is equal to the sum of all its parts.

10. All right angles are equal to each other.

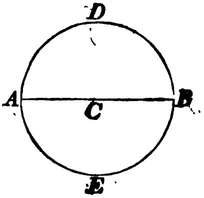
11. A straight line is the shortest distance between two points.

12. Magnitudes, which being applied to each other, coincide throughout their whole extent, are equal.

## PROPERTIES OF POLYGONS.

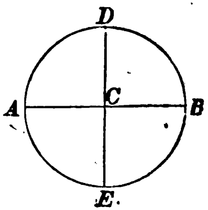
### THEOREM I.

*Every diameter of a circle divides the circumference into two equal parts.*



Let $ADBE$ be the circumference of a circle, and $ACB$ a diameter: then will the part $ADB$ be equal to the part $AEB$.

For, suppose the part $AEB$ to be turned around $AB$, until it shall fall on the part $ADB$. The curve $AEB$ will then exactly coincide with the curve $ADB$, or else there would be some point in the curve $AEB$ or $ADB$, unequally distant from the centre $C$, which is contrary to the definition of a circumference (Def. 24). Hence, the two curves will be equal (Ax. ~~11~~ 12).

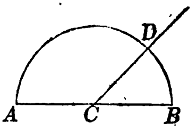


*Corollary* 1. If two lines, $AB$, $DE$, be drawn through the centre $C$ perpendicular to each other, each will divide the circumference into two equal parts; and the entire circumference will be divided into the equal quadrants $DB$, $DA$, $AE$, and $EB$.

*Cor.* 2. Hence, a right angle, as $DCB$, is measured by one quadrant, or 90 degrees; two right angles by a semicircumference, or 180 degrees; and four right angles by the whole circumference, or 360 degrees

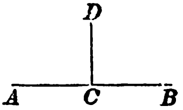
### THEOREM II.

*If one straight line meet another straight line, the sum of the two adjacent angles will be equal to two right angles.*

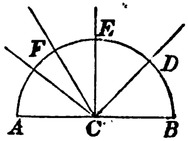


Let the straight line $CD$ meet the straight line $AB$, at the point $C$; then will the angle $DCB$ plus the angle $DCA$ be equal to two right angles.

About the centre $C$, with any radius as $CB$, suppose a semicircumference to be described. Then, the angle $DCB$ will be measured by the arc $BD$, and the angle $DCA$ by the arc $AD$. But the sum of the two arcs is equal to a semicircumference: hence, the sum of the two angles is equal to two right angles (Th. i, Cor. 2).

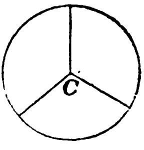


*Cor.* 1. If one of the angles, as $DCB$, is a right angle, the other angle, $DCA$ will also be a right angle.



*Cor.* 2. Hence, all the angles which can be formed at any point $C$, by any number of lines, $CD$, $CE$, $CF$, &c., drawn on the same side of $AB$, are equal to two right angles: for, they will be measured by a semicircumference.

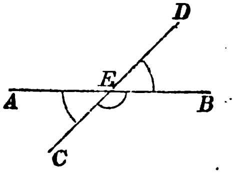
*Cor.* 3. If $DC$ meets two lines $CB$, $CA$, making $DCB$ plus $DCA$ equal to two right angles, $ACB$ will form one straight line.



*Cor.* 4. Hence, also, all the angles which can be formed round any point, as $C$, are equal to four right angles. For, the sum of all the arcs which measure them, is equal to the entire circumference, which is the measure of four right angles (Th. i. Cor. 2).

### THEOREM III.

*If two straight lines intersect each other, the opposite or vertical angles which they form, are equal.*



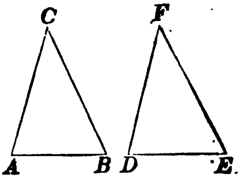
Let the two straight lines *AB* and *CD* intersect each other at the point *E*: then will the opposite angle *AEC* be equal to *DEB*, and *AED*=*CEB*.

For, since the line *AE* meets the line *CD*, the angle *AEC*+*AED*= two right angles. But since the line *DE* meets the line *AB*, we have *DEB*+*AED*= two right angles. Taking away from these equals the common angle *AED*, and there will remain the angle *AEC* equal to the angle *DEB* (Ax. 3).

In the same manner we may prove that the angle *AED* is equal to the angle *CEB*.

### THEOREM IV.

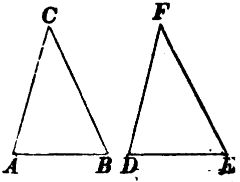
*If two triangles have two sides and the included angle of the one, equal to two sides and the included angle of the other, each to each, the two triangles will be equal.*



Let the triangles *ABC* and *DEF* have the side *AC* equal to *DF*, *CB* to *FE*, and the angle *C* equal to the angle *F*: then will the triangle *ACB* be equal to the triangle *DEF*.

For, suppose the side *AC*, of the triangle *ACB*, to be placed on *DF*, so that the extremity *C* shall fall on the extremity *F*: then, since the sides are equal, *A* will fall on *D*.

But since the angle *C* is equal to the angle *F*, the line *CB*

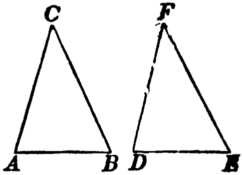


will fall on *FE*; and since *CB* is equal to *FE*, the extremity *B* will fall on *E*; and consequently the side *AB* will fall on the side *DE* (Ax. ~~11~~ 13). Hence, the two triangles will fill the same space, and consequently are equal (Ax. 12.).

*Scholium.* Two triangles are said to be equal, when being applied the one to the other they exactly coincide (Ax. 12). Hence, *equal* triangles have their like parts equal, each to each, since those parts coincide with each other. The converse of the proposition is also true, namely, that *two triangles which have all the parts of the one equal to the corresponding parts of the other, each to each, are equal:* for if applied the one to the other, the equal parts will coincide.

### THEOREM V.

*If two triangles have two angles and the included side of the one, equal to two angles and the included side of the other, each to each, the two triangles will be equal.*



Let the two triangles *ABC* and *DEF* have the angle *A* equal to the angle *D*, the angle *B* equal to the angle *E*, and the included side *AB* equal to the included side *DE*: then will the triangle *ABC* be equal to the triangle *DEF*.

For, let the side *AB* be placed on the side *DE*, the extremity *A* on the extremity *D*; and since the sides are equal, the point *B* will fall on the point *E*.

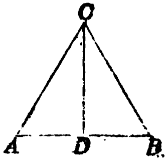
Then since the angle *A* is equal to the angle *D*, the side

*AC* will take the direction *DF:* and since the angle *B* is equal to the angle *E*, the side *BC* will fall on the side *EF:* hence, the point *C* will be found at the same time on *DF* and *EF*, and therefore will fall at the intersection *F:* consequently, all the parts of the triangle *ABC* will coincide with the parts of the triangle *DEF*, and therefore, the two triangles are equal.

### THEOREM VI.

*In an isosceles triangle the angles opposite the equal sides are equal to each other.*

Let *ABC* be an isosceles triangle, having the side *AC* equal to the side *CB:* then will the angle *A* be equal to the angle *B*.



For, suppose the line *CD* to be drawn dividing the angle *C* into two equal parts.

Then, the two triangles *ACD* and *DCB*, have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: that is, the side *AC* equal to *BC*, the side *CD* common, and the included angle *ACD* equal to the included angle *DCB:* hence the two triangles are equal (Th. iv); and hence, the angle *A* is equal to the angle *B*.

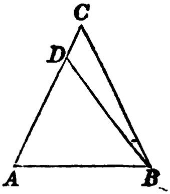
*Cor.* 1. Hence, the line which bisects the vertical angle of an isosceles triangle, bisects the base. It is also perpendicular to the base, since the angle *CDA* is equal to the angle *CDB*.

*Cor.* 2. Hence, also, every equilateral triangle, must also be equiangular: that is, have all its angles equal, each to each.

### THEOREM VII.

Conversely.—*If a triangle has two of its angles equal, the sides opposite those angles will also be equal.*

In the triangle *ABC*, let the angle *A* be equal to the angle *B*: then will the side *BC* be equal to the side *AC*.



For, if the two sides are not equal, one of them must be greater than the other. Suppose *AC* to be the greater side. Then take a part *AD* equal to *BC*,

Now, in the two triangles *ADB* and *ABC*, we have the side $AD=BC$, by hypothesis; the side *AB* common, and the angle *A* equal to the angle *B*: hence, the two triangles have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: hence, the two triangles are equal (Th. iv), that is, a part *ADB* is equal to the whole *ABC*, which is impossible (Ax. 8): consequently, the side *AC* cannot be greater than the side *CB*, and hence, the triangle is isosceles.

*Scholium* 1. The method of reasoning pursued in the last theorem, is called the "reductio ad absurdum," or a proof that leads to a known absurdity.

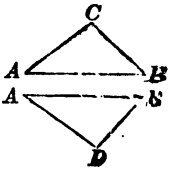
Let us analyze this method of reasoning. We wished to prove that the two sides *AC*, *CB* were equal. We supposed them unequal, and *AC* the greater—that was an hypothesis (See Def. 12). We then reasoned on the hypothesis, and proved a part equal to the whole, which we know to be false (Ax. 8). Hence, we conclude that the hypothesis is untrue, because after a correct chain of reasoning it leads to a result which we know to be absurd.

*Scholium* 2. Generally,—If the demonstration is based on known principles, previously proved, or admitted in the axioms, the conclusion will always be true. But, if the demonstration is based on an hypothesis, (as in the last theorem, that $AC$ was the greater side), and the conclusion is *contrary* to what has been previously proved, or admitted in the axioms, then, it follows, that the hypothesis cannot be true.

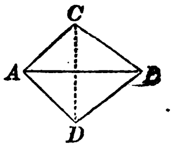
The former is called a *direct*, and the latter an *indirect* demonstration.

### THEOREM VIII.

*If two triangles have the three sides of the one equal to the three sides of the other, each to each, the three angles will also be equal, each to each.*



Let the two triangles $ABC$, $ABD$, have the side $AB$ equal to the side $AB$, the side $AC$ equal to $AD$, and the side $CB$ equal to $DB$: then will the corresponding angles also be equal, viz: the angle $A$ will be equal to the angle $A$, the angle $B$ to the angle $B$, and the angle $C$ to the angle $D$.



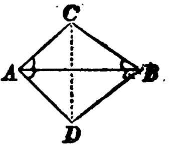
For, suppose the triangles to be joined by their longest equal sides $AB$, and the line $CD$ to be drawn.

Then, since the side $AC$ is equal to $AD$, by hypothesis, the triangle $ADC$ will be isosceles; and therefore, the angle $ACD$ will be equal to the angle $ADC$ (Th. vi). In like manner, in the triangle $CBD$, the side $CB$ is equal to $DB$: hence, the angle $BCD$ is equal to the angle $BDC$.

Now, by the addition of equals, we have

$$ACD+BCD=ADC+BDC$$

that is, the angle $ACB=ADB$.

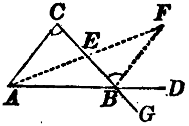


Now, the two triangles $ACB$ and $ADB$ have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each: hence, the remaining angles will be equal (Th. iv): consequently, the angle $CAB$ is equal to $BAD$, and the angle $CBA$ to the angle $ABD$.

*Sch.* The angles of the two triangles which are equal to each other, are those which lie opposite the equal sides.

### THEOREM IX.

*If one side of a triangle is produced, the outward angle is greater than either of the inward opposite angles.*



Let $ABC$ be a triangle, having the side $AB$ produced to $D$: then will the outward angle $CBD$ be greater than either of the inward opposite angles $A$ or $C$.

For, suppose the side $CB$ to be bisected at the point $E$. Draw $AE$, and produce it until $EF$ is equal to $AE$, and then draw $BF$.

Now, since the two triangles $AEC$ and $BEF$ have $AE=EF$ and $EC=EB$, and the included angle $AEC$ equal to the included angle $BEF$ (Th. iii), the two triangles will be equal in all respects (Th. iv): hence, the angle $EBF$ will be equal to the angle $C$. But the angle $CBD$ is greater than the angle $CBF$, consequently it is greater than the angle $C$.

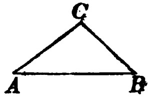
In like manner, if $CB$ be produced to $G$, and $AB$ be bisected, it may be proved that the outward angle $ABG$, or its equal $CBD$ (Th. iii), is greater than the angle $A$.

Cor. to X. ∴ Then The difference between any two sides of a triangle is less than the third side.

H. L. R.

### THEOREM X.

*The sum of any two sides of a triangle is greater than the third side.*

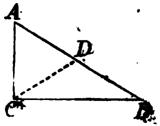


Let $ABC$ be a triangle: then will the sum of two of its sides, as $AC$, $CB$, be greater than the third side $AB$.

For, the straight line $AB$ is the shortest distance between the two points $A$ and $B$ (Ax. xi): hence, $AC+CB$ is greater than $AB$.

### THEOREM XI.

*The greater side of every triangle is opposite the greater angle: and conversely, the greater angle is opposite the greater side.*



*First.* In the triangle $CAB$, let the angle $C$ be greater than the angle $B$: then, will the side $AB$ be greater than the side $AC$.

For, draw $CD$, making the angle $BCD$ equal to the angle $B$. Then, the triangle $CBD$ will be isosceles: hence, the side $CD=DB$ (Th. vii.)

But, by the last theorem $AC$ is less than $AD+CD$; that is, less than $AD+DB$, and consequently less than $AB$.

*Secondly.* Let us suppose the side $AB$ to be greater than $AC$; then will the angle $C$ be greater than the angle $B$.

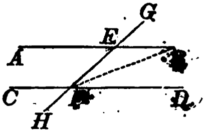
For if the angle $C$ were equal to $B$, the triangle $CAB$ would be isosceles, and the side $AC$ would be equal to $AB$ (Th. vii), which would be contrary to the hypothesis.

Again, if the angle $C$ were less than $B$, then, by the first part of the theorem, the side $AB$ would be less than $AC$, which is also contrary to the hypothesis Hence, since $C$

cannot be equal to $B$, nor less than $B$, it follows that it must be greater.

### THEOREM XII.

*If a straight line intersect two parallel lines, the alternate angles will be equal.*



If two parallel straight lines, $AB$ $CD$, are intersected by a third line $GH$, the angles $AEF$ and $EFD$ are called *alternate* angles. It is required to prove that these angles are equal.

If they are unequal one of them must be greater than the other. Suppose $EFD$ to be the greater angle.

Now conceive $FB$ to be drawn, making the angle $EFB$ equal to the angle $AEF$, and meeting $AE$ in $B$.

Then, in the triangle $FEB$ the outward angle $FEA$ is greater than either of the inward angles $B$ or $EFB$ (Th. ix.); and therefore, $EFB$ can never be equal to $AEF$ so long as $FB$ meets $EB$.

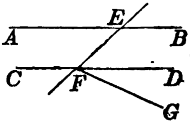
But since we have supposed $EFD$ to be greater than $AEF$, it follows that $EFB$ could not be equal to $AEF$, if $FB$ fell below $FD$. Therefore, if the angle $EFB$ is equal to the angle $AEF$, $FB$ cannot meet $AB$, nor fall below $FD$, and consequently must coincide with the parallel $CD$ (Def. 30): and hence, the alternate angles $AEF$ and $EFD$ are equal.

*Cor.* If a line be perpendicular to one of two parallel lines, it will also be perpendicular to the other.

### THEOREM XIII.

Conversely,—*If a line intersect two straight lines, making the alternate angles equal, those straight lines will be parallel.*

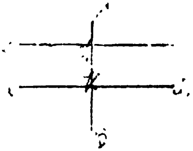
Let the line *EF* meet the lines *AB*, *CD*. making the angle *AEF* equal to the angle *EFD:* then will the lines *AB* and *CD* be parallel.



For, if they are not parallel, suppose through the point *F* the line *FG* to be drawn parallel to *AB*.

Then, because of the parallels *AB*, *FG*, the alternate angles, *AEF* and *EFG* will be equal (Th. xii). But, by hypothesis, the angle *AEF* is equal to *EFD*: hence, the angle *EFD* is equal to the angle *EFG* (Ax. 1); that is, a part is equal to the whole, which is absurd (Ax. 8): therefore, no line but *CD* can be parallel to *AB*.

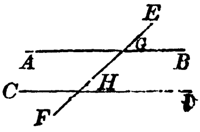
*Cor* I. If two lines are perpendicular to the same line, they will be parallel to each other.



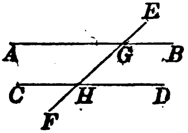
### THEOREM XIV.

*If a line cut two parallel lines, the outward angle is equal to the inward opposite angle on the same side; and the two inward angles, on the same side, are equal to two right angles.*

Let the line *EF* cut the two parallels *AB*, *CD:* then will the outward angle *EGB* be equal to the inward opposite angle *EHD;* and the two inward angles, *BGH* and *GHD*, will be equal to two right angles.



*First.* Since the lines *AB*, *CD*, are parallel, the angle *AGH* is equal to the alternate angle *GHD* (Th. xii); but the angle *AGH* is equal to the opposite angle *EGB:* hence, the angle *EGB* is equal to the angle *EHD* (Ax. 1).



*Secondly.* Since the two adjacent angles *EGB* and *BGH* are equal to two right angles (Th. ii); and since the angle *EGB* has been proved equal to *EHD*, it follows that the sum of *BGH* plus *GHD*, is also equal to two right angles.

*Cor.* 1. Conversely, if one straight line meets two other straight lines, making the angles on the same side equal to each other, those lines will be parallel.

*Cor.* 2. If a line intersect two other lines, making the sum of the two inward angles equal to two right angles, those two lines will be parallel.

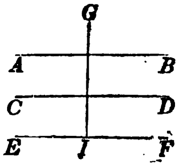
*Cor.* 3. If a line intersect two other lines, making the sum of the two inward angles less than two right angles, those lines will not be parallel, but will meet if sufficiently produced.

### THEOREM XV.

***All straight lines which are parallel to the same line, are parallel to each other.***

Let the lines *AB* and *CD* be each parallel to *EF:* then will they be parallel to each other.

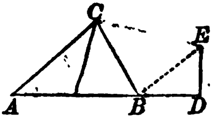
For, let the line *GI* be drawn perpendicular to *EF:* then will it also be perpendicular to the parallels *AB*, *CD* (Th. xii Cor.).



Then, since the lines $AB$ and $CD$ are perpendicular to the line $GI$, they will be parallel to each other (Th. xiii. Cor).

### THEOREM XVI.

*If one side of a triangle be produced, the outward angle will be equal to the sum of the inward opposite angles.*



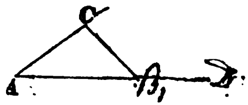
In the triangle $ABC$, let the side $AB$ be produced to $D$: then will the outward angle $CBD$ be equal to the sum of the inward opposite angles $A$ and $C$.

For, conceive the line $BE$ to be drawn parallel to the side $AC$. Then, since $BC$ meets the two parallels $AC$, $BE$, the alternate angles $ACB$ and $CBE$ will be equal (Th. xii).

And since the line $AD$ cuts the two parallels $BE$ and $AC$ the angles $EBD$ and $CAB$ are equal to each other (Th. xiv). Therefore, the inward angles $C$ and $A$, of the triangle $ABC$, are equal to the angles $CBE$ and $EBD$; and consequently, the sum of the two angles, $A$ and $C$, is equal to the outward angle $CBD$ (Ax. 1).

### THEOREM XVII.

*In any triangle the sum of the three angles is equal to two right angles.*



Let $ABC$ be any triangle: then will the sum of the three angles

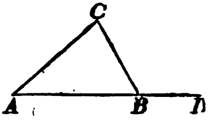
$$A+B+C=\text{two right angles.}$$

For, let the side $AB$ be produced to $D$. Then, the outward angle

$$CBD = A+C \text{ (Th. xvi).}$$

To each of these equals add the angle $CBA$, and we shall have

$$CBD+CBA=A+C+B.$$



But the sum of the two angles $CBD$ and $CBA$, is equal to two right angles (Th. ii): hence

$$A+B+C=\text{two right angles (Ax. 1)},$$

*Cor.* 1. If two angles of one triangle be equal to two angles of another triangle, the third angles will also be equal (Ax. 3).

*Cor.* 2. If one angle of one triangle be equal to one angle of another triangle, the sum of the two remaining angles in each triangle, will also be equal (Ax. 3).

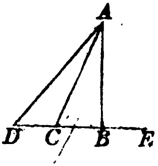
*Cor.* 3. If one angle of a triangle be a right angle, the sum of the other two angles will be equal to a right angle; and each angle singly, will be acute.

*Cor.* 4. No triangle can have more than one right angle, nor more than one obtuse angle; otherwise, the sum of the three angles would exceed two right angles: hence, at least two angles of every triangle must be acute.

### THEOREM XVIII.

I. *A perpendicular is the shortest line that can be drawn from a given point to a given line.*

II. *If any number of lines be drawn from the same point, those which are nearest the perpendicular are less than those which are more remote.*



Let $A$ be a given point, and $DE$ a straight line. Suppose $AB$ to be drawn perpendicular to $DE$, and suppose the oblique lines $AC$ and $AD$ also to be drawn: Then, *AB* will be shorter than either of the oblique lines, and *AC* will be less than *AD*

*First.* Since the angle *B*, in the triangle *ACB*, is a right angle, the angle *C* will be acute (Th. xvii. Cor. 3): and since the greater side of every triangle is opposite the greater angle (Th. xi), the side *AC* will be greater than *AB*.

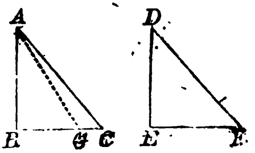
*Secondly.* Since the angle *ACB* is acute, the adjacent angle *ACD* will be obtuse (Th. ii): consequently, the angle *D* is acute (Th. xvii. Cor. ~~3~~4), and therefore less than the angle *ACD*. And since the greater side of every triangle is opposite the greater angle, it follows that *AD* is greater than *AC*.

*Cor.* A perpendicular is the shortest distance from a point to a line.

### THEOREM XIX.

*If two right angled triangles have the hypothenuse and a side of the one equal to the hypothenuse and a side of the other, the remaining parts will also be equal, each to each.*

Let the two right angled triangles *ABC* and *DEF*, have the hypothenuse *AC* equal to *DF*, and the side *AB* equal to *DE:* then will the remaining parts be equal, each to each.



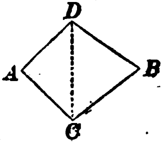
For, if the side *BC* is equal to *EF*, the corresponding angles of the two triangles will be equal (Th. viii). If the sides are unequal, suppose *BC* to be the greater, and take a part, *BG*, equal to *EF*, and draw *AG*.

Then, in the two triangles *ABG* and *DEF*, the angle *B* is equal to the angle *E*, the side *AB* to the side *DE*, and the side *BG* to the side *EF:* hence, the two triangles are equal in all respects (Th. iv) and consequently, the side *AG* is equal to

*DF*. But *DF* is equal to *AC*, by construction ~~hypothesis~~; therefore, *AG* is equal to *AC* (Ax. 1). But this is impossible (Th. xviii); hence, the sides *BC* and *EF* cannot be unequal; consequently, the ~~triangles~~ are equal (Th. viii).

### THEOREM XX.

*The sum of the four angles of every quadrilateral is equal to four right angles.*



Let *ACBD* be a quadrilateral: then will $A+B+C+D=$four right angles.

Let the diagonal *DC* be drawn dividing the quadrilateral *AB*, into two triangles, *BDC*, *ADC*.

Then, because the sum of the three angles of each triangle is equal to two right angles (Th. xvii), it follows that the sum of the angles of both triangles is equal to four right angles. But the sum of the angles of both triangles, make up the angles of the quadrilateral. Hence, the sum of the four angles of the quadrilateral is equal to four right angles.

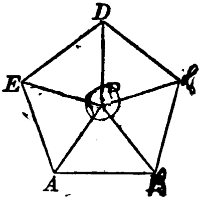
*Cor.* 1. If then three of the angles be right angles, the fourth angle will also be a right angle.

*Cor.* 2. If the sum of two of the four angles be equal to two right angles, the sum of the remaining two will also be equal to two right angles.

*Cor.* 3. Since all the angles of a square or rectangle, are equal to each other (Def. 4), and their sum equal to four right angles, it follows that each angle is equal to one right angle.

### THEOREM XXI.

*The sum of all the interior angles of any polygon is equal to twice as many right angles, wanting four, as the figure has sides*



Let $ABCDE$ be any polygon: then will the sum of its inward angles

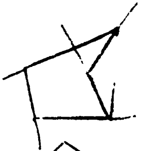
$$A+B+C+D+E$$

be equal to twice as many right angles, wanting four, as the figure has sides.

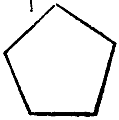
For, from any point $P$, within the polygon, draw the lines $PA$, $PB$, $PC$, $PD$, $PE$, to each of the angles, dividing the polygon into as many triangles as the figure has sides.

Now, the sum of the three angles of each of these triangles is equal to two right angles (Th. xvii): hence, the sum of the angles of all the triangles is equal to twice as many right angles as the figure has sides.

But the sum of all the angles about the point $P$ is equal to four right angles (Th. ii. Cor. 4); and since this sum makes no part of the inward angles of the polygon, it must be subtracted from the sum of all the angles of the triangles, before found. Hence, *the sum of the interior angles of the polygon is equal to twice as many right angles, wanting four, as the figure has sides.* Explain with [illegible]



*Sch.* This proposition is not applicable to polygons which have *re-entrant* angles.

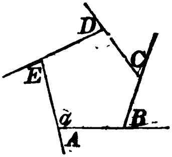


The reasoning is limited to polygons with salient angles, which may properly be named *convex polygons.*

### THEOREM XXII.

*If every side of a polygon be produced out,* [illegible] direction *the sum of all the outward angles thereby formed, will be equal to four right angles.*

Let $A$, $B$, $C$, $D$, and $E$, be the outward angles of a polygon formed by producing all the sides. Then will



$A+B+C+D+E=$four right angles.

For, each interior angle, plus its exterior angle, as $A+a$, is equal to two right angles (Th. ii). But there are as many exterior as interior angles, and as many of each as there are sides of the polygon, hence, the sum of all the interior and exterior angles will be equal to twice as many right angles as the polygon has sides.

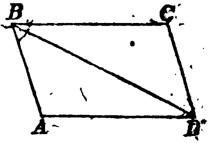
But the sum of all the interior angles together with four right angles, is equal to twice as many right angles as the polygon has sides (Th. xxi): that is, equal to the sum of all the inward and outward angles taken together.

From each of these equal sums take away the inward angles, and there will remain, the outward angles equal to four right angles (Ax. 3).

## THEOREM XXIII.

*The opposite sides and angles of every parallelogram are equal, each to each: and a diagonal divides the parallelogram into two equal triangles.*

Let $ABCD$ be any parallelogram, and $DB$ a diagonal: then will the opposite sides and angles be equal to each other, each to each, and the diagonal $DB$ will divide the parallelogram into two equal triangles.



For, since the figure is a parallelogram, the sides $AB$, $DC$ are parallel, as also the sides $AD$, $BC$. Now, since the parallels are cut by the diagonal *DB*, the alternate angles will be equal (Th. xii): that is the angle

$$ADB = DBC \quad \text{and} \quad BDC = ABD.$$

Hence, the two triangles *ADB BDC*, having two angles in the one equal to two angles in the other, will have their third angles equal (Th. xvii. Cor. 1), viz. the angle *A* equal to the angle C, and these are two of the opposite angles of the parallelogram.

Also, if to the equal angles *ADB*, *DBC*, we add the equals *BDC*, *ABD*, the sums will be equal (Ax. 2): viz. the whole angle *ADC* to the whole angle *ABC*, and these are the other two opposite angles of the parallelogram.

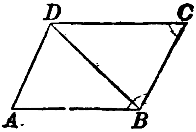
Again, since the two triangles *ADB*, *DBC*, have the side *DB* common, and the two adjacent angles in the one equal to the two adjacent angles in the other, each to each, the two triangles will be equal (Th. v): hence, the diagonal divides the parallelogram into two equal triangles.

*Cor.* 1. If one angle of a parallelogram be a right angle, each of the angles will also be a right angle, and the parallelogram will be a rectangle.

*Cor.* 2. Hence, also, the sum of either two adjacent angles of a parallelogram, will be equal to two right angles.

### THEOREM XXIV.

*If the opposite sides of a quadrilateral, are equal, each to each, the equal sides will be parallel, and the figure will be a parallelogram.*



Let $ABCD$ be a quadrilateral, having its opposite sides respectively equal, viz.

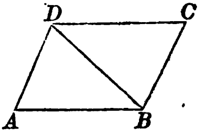
$AB=CD$ and $AD=BC$

Then will these sides be parallel, and the figure will be a parallelogram.

For, draw the diagonal $BD$. Then, the two triangles $ABD$, $BDC$, have all the sides of the one equal to all the sides of the other, each to each: therefore, the two triangles are equal (Th. viii); hence, the angle $ADB$, opposite the side $AB$, is equal to the angle $DBC$ opposite the side $DC$; therefore, the sides $AD$, $BC$, are parallel (Th. xiii). For a like reason $DC$ is parallel to $AB$, and the figure $ABCD$ is a parallelogram.

### THEOREM XXV.

*If two opposite sides of a quadrilateral are equal and parallel the remaining sides will also be equal and parallel, and the figure will be a parallelogram.*

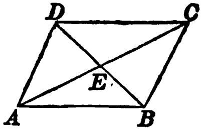


Let $ABCD$ be a quadrilateral, having the sides $AB$, $CD$, equal and parallel: then will the figure be a parallelogram.

For, draw the diagonal $DB$, dividing the quadrilateral into two triangles. Then, since $AB$ is parallel to $DC$, the alternate angles, $ABD$ and $BDC$ are equal (Th. xii): moreover, the side $BD$ is common; hence the two triangles have two sides and the included angle of the one, equal to two sides and the included angle of the other: the triangles are therefore equal, and consequently, $AD$ is equal to $BC$, and the angle $ADB$ to the angle $DBC$; and consequently, $AD$ is also parallel to $BC$ (Th. xiii). Therefore, the figure $ABCD$ is a parallelogram.

### THEOREM XXVI.

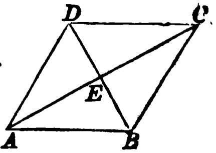
*The two diagonals of a parallelogram divide each other into equal parts, or mutually bisect each other.*



Let $ABCD$ be a parallelogram, and $AC$, $BD$ its two diagonals intersecting at $E$. Then will

$$AE=EC \quad \text{and} \quad BE=ED.$$

Comparing the two triangles $AED$ and $BEC$, we find the side $AD=BC$ (Th. xxiii), the angle $ADE=EBC$ and $EAD=ECB$: hence, the two triangles are equal (Th. v): therefore, $AE$, the side opposite $ADE$, is equal to $EC$, the side opposite $EBC$; and $ED$ is equal to $EB$



*Sch.* In the case of a rhombus (Def. 48), the sides $AB$, $BC$ being equal, the triangles $AEB$ and $BEC$ have all the sides of the one equal to the corresponding sides of the other, and are therefore equal. Whence it follows that the angles $AEB$ and $BEC$ are equal. Therefore, the diagonals of a rhombus bisect each other at right angles.

3. All radii of the same or equal circles are equal.

# GEOMETRY.

---

## BOOK II,

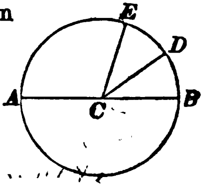
### OF THE CIRCLE

#### DEFINITIONS.

1. THE circumference of a circle is a curve line, all the points of which are equally distant from a certain point within called the centre.

2. The circle is the space bounded by this curve line.

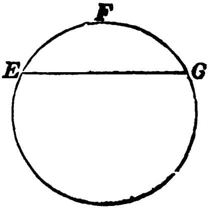
3. Every straight line, *CA*, *CD*, *CE*, drawn from the centre to the circumference, is called a *radius* or *semidiameter*. Every line which, like *AB*, passes through the centre and terminates in the circumference, is called a *diameter*.



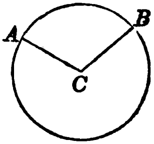
4. Any portion of the circumference, as *EFG*, is called an *arc*.

5. A straight line, as *EG*, joining the extremities of an arc, is called a *chord*.

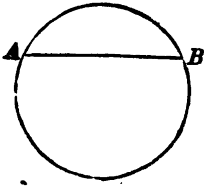
6. A *segment* is the surface or portion of a circle included between an arc and its chord. Thus *EFG* is a segment.



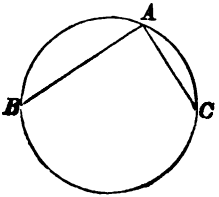
7. A *sector* is the part of the circle included between an arc and the two radii drawn through its extremities. Thus, *CAB* is a sector



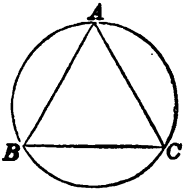
8. A straight line is said to be inscribed in a circle, when its extremities are in the circumference. Thus, the line *AB* is inscribed in a circle.



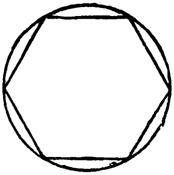
9. An inscribed angle is one which is formed by two chords that intersect each other in the circumference. Thus, *BAC* is an inscribed angle.



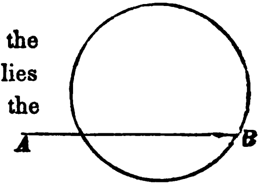
10. An inscribed triangle is one which has its three angular points in the circumference. Thus, *ABC* is an inscribed triangle.

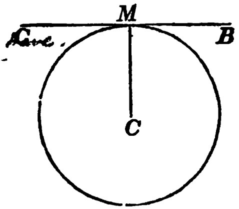


11. Any polygon is said to be inscribed in a circle when the vertices of all the angles are in the circumference. The circle is then said to circumscribe the polygon.



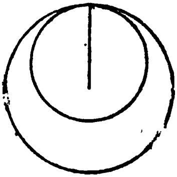
12. A *secant* is a line which cuts. ~~meets~~ the circumference ~~in two points~~, and lies partly within and partly without the circle. Thus *AB* is a secant.



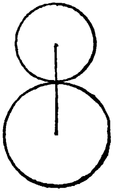


13. A *tangent* is a line which ~~has~~ Can. Curve. but one point in common with the circumference. Thus, *CMB* is a tangent.

14. Two circles are said to touch each other internally, when one lies within the other, and their circumferences have but one point in common.



15. Two circles are said to touch each other externally, when ~~one~~ lies ~~without~~ the other, and their circumferences have but one point in common

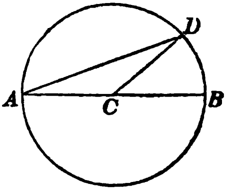


### THEOREM I.

[illegible] then [illegible]

*~~Every chord~~ is less than a ~~diameter~~.*

Let $AD$ be any chord. Draw the radii $CA$, $CD$ to its extremities. We shall then have, $AD$ less than $AC+CD$ (Book I. Th. x*). But $AC+CD$ is equal to the diameter $AB$: hence, the chord $AD$ is less than the diameter.



### THEOREM II.

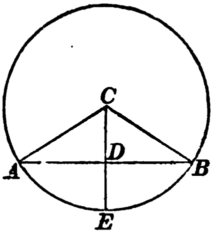
*If from the centre of a circle a line be drawn to the middle of a chord,*

I. *It will be perpendicular to the chord;*

[illegible]

II. *And it will bisect the arc of the chord.*

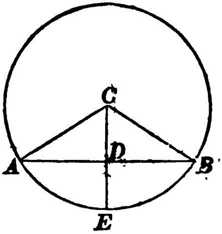
Let $C$ be the centre of a circle, and $AB$ any chord. Draw $CD$ through $D$, the middle point of the chord, and produce it to $E$: then will $CD$ be perpendicular to the chord, and the arc $AE$ equal to $EB$.



*First.* Draw the two radii $CA$, $CB$. Then the two triangles $ACD$ $DCB$, have the three sides of the one equal to the three sides of the

---

*Note. When reference is made from one theorem to another, in the same Book, the number of the theorem referred to is alone given: but when the theorem referred to is found in a preceding Book, the number of the Book is also given.



other, each to each: viz. $AC$ equal to $CB$, being radii, $AD$ equal to $DB$, by hypothesis, and $CD$ common: hence, the corresponding angles are equal (Book I. Th. viii): that is, the angle $CDA$ equal to $CDB$, and the angle $ACD$ equal to the angle $DCB$.

But, since the angle $CDA$ is equal to the angle $CDB$, the radius $CE$ is perpendicular to the chord $AB$ (Bk. I. Def. 20).

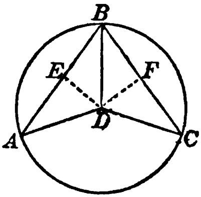
*Secondly.* Since the angle $ACE$ is equal to $BCE$, the arc $AE$ will be equal to the arc $EB$, for equal angles must have equal [illegible] measures (Bk. I. Def. 29).

Hence, the radius drawn through the middle point of a chord, is perpendicular to the chord, and bisects the arc of the chord.

*Cor.* Hence, a line which bisects a chord at right angles, bisects the arc of the chord, and passes through the centre of the circle. Also, a line drawn through the centre of the circle and perpendicular to the chord, bisects it.

### THEOREM III.

*If more than two equal lines can be drawn from any point within a circle to the circumference, that point will be the centre.*



Let $D$ be any point within the circle $ABC$. Then, if the three lines $DA$, $DB$, and $DC$, drawn from the point $D$ to the circumference, are equal, the point $D$ will be the centre.

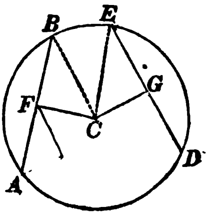
For, draw the chords $AB$, $BC$, bisect them at the points $E$ and $F$, and join $DE$ and $DF$

Then, since the two triangles $DAE$ and $DEB$ have the side $AE$ equal to $EB$, $AD$ equal to $DB$, and $DE$ common, they will be equal in all respects; and consequently, the angle $DEA$ is equal to the angle $DEB$ (Bk. I. Th. viii); and therefore, $DE$ is perpendicular to $AB$ (Bk. I. Def. 20) But, if $DE$ bisects $AB$ at right angles, it will pass through the centre of the circle (Th. ii. Cor).

In like manner, it may be shown that $DF$ passes through the centre of the circle, and since the centre is found in the two lines $ED$, $DF$, it will be found at their common intersection $D$.

### THEOREM IV.

*Any chords which are equally distant from the centre of a circle, are equal.*



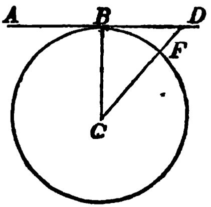
Let $AB$ and $ED$ be two chords equally distant from the centre $C$: then will the two chords $AB$, $ED$ be equal to each other.

Draw $CF$ perpendicular to $AB$, and $CG$ perpendicular to $ED$, and since these perpendiculars measure the distances from the centre, they will be equal. Also draw $CB$ and $CE$.

Then, the two right angled triangles $CFB$ and $CEG$ having the hypothenuse $CB$ equal to the hypothenuse $CE$, and the side $CF$ equal to $CG$, will have the third side $BF$ equal to $EG$ (Bk. I Th. xix). But, $BF$ is the half of $BA$, and $EG$ he half of $DE$ (Th. ii. Cor); hence $BA$ is equal to $DE$ (Ax 6).

### THEOREM V.

*A line which is perpendicular to a radius at its extremity, is tangent to the circle.*



Let the line *ABD* be perpendicular to the radius *CB* at the extremity *B*: then will it be tangent to the circle at the point *B*.

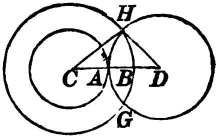
For, from any other point of the line, as *D*, draw *DFC* to the centre, cutting the circumference in *F*.

Then, because the angle *B*, of the triangle *CDB*, is a right angle, the angle at *D* is acute (Bk. I. Th. xvii. Cor. 3), and consequently less than the angle *B*. But the greater side of every triangle is opposite to the greater angle (Bk. I. Th. xi); therefore, the side *CD* is greater than *CB*, or its equal *CF*. Hence, the point *D* is without the circle, and the same may be shown for every other point of the line *AD*. Consequently, the line *ABD* has but one point in common with the circumference of the circle, and therefore is tangent to it at the point *B* (Def. 13)

*Cor.* Hence, if a line is tangent to a circle, and a radius be drawn through the point of contact, the radius will be perpendicular to the tangent.

### THEOREM VI.

*If the distance between the centres of two circles is equal to the sum of their radii, the two circles will touch each other externally.*

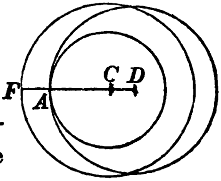


Let $C$ and $D$ be the two centres, and suppose the distance between them to be equal to the sum of the radii, that is, to $CA+AD$.

The circumferences of the circles will evidently have the point $A$ common, and they will have no other. Because, if they had two points common, that, is if they cut each other in two points, $G$ and $H$, the distance $CD$ between their centres would be less than the sum of their radii $CH$, $HD$ (Bk. I. Th. x); but this would be contrary to the supposition.

### THEOREM VII.

*If the distance between the centres of two circles is equal to the difference of their radii, the two circles will touch each other internally.*



Let $C$ and $D$ be the centres of two circles at a distance from each other equal to $AD-AC=CD$.

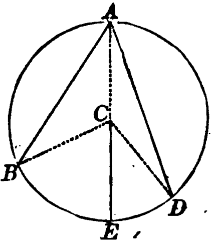
Now, it is evident, as in the last theorem, that the circumferences will have the point $A$ common; and they can have no other. For, if they had two points common, the difference between the radii $AD$ and $FC$ would not be equal to $CD$, the distance between their centres: therefore, they cannot have two points in common when the difference of their radii is equal to the distance between their centres: hence, they are tangent to each other.

*Sch* If two circles touch each other, either externally or internally, their centres and the point of contact will be in the same straight line

1st [illegible] the centre lies on one of the sides of the
2nd " " " " within the angle
3rd " " " " without the angle

## THEOREM VIII.

*An angle at the circumference of a circle is measured by half the arc that subtends it.*



Let *BAD* be an inscribed angle: then will it be measured by half the arc *BED*, which subtends it.

For, through the centre *C* draw the diameter *ACE*, and draw the radii *BC*, *CD*.

Then, in the triangle *ABC*, the exterior angle *BCE* is equal to the sum of the interior angles *B* and *A* (Bk. I. Th. xvi). But since the triangle *BAC* is isosceles, the angles *A* and *B* are equal (Bk. I. Th. vi); therefore, the exterior angle *BCE* is equal to double the angle *BAC*.

But, the angle *BCE* is measured by the arc *BE*, which subtends it; and consequently, the angle *BAE*, which is half of *BCE*, is measured by half the arc *BE*.

It may be shown, in like manner, that the angle *EAD* is measured by half the arc *ED*: and hence, by the addition of equals, it would follow that, the angle *BAD* is measured by half the arc *BED*, which subtends it.

*Cor*. 1. Hence, if an angle at the centre, and an angle at the circumference, both stand on the same arc, the angle at the centre will be double the angle at the circumference.

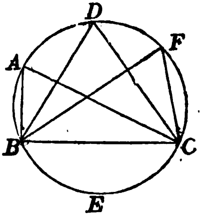
*Cor*. 2. If two angles at the circumference stand on equal arcs they will be equal to each other.

angle.

### THEOREM IX.

*All angles at the circumference, which stand upon the same arc are equal to each other.*

Let the angles $BAC$, $BDC$, $BFC$, have their vertices in the circumference, and stand on the same arc $BEC$: then will they be equal to each other.

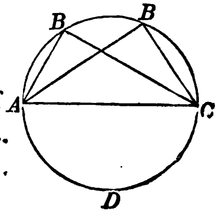


For, each angle is measured by half the arc $BEC$ (Th. viii); hence, the angles are all equal.

### THEOREM X.

*An angle in a semicircle, is a right angle.*

Let $ABBC$ be a semicircle: then will every angle, as $B$, $B$, inscribed in it, be a right angle.

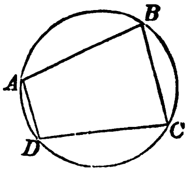


For, each angle is measured by half the semicircumference $ADC$, that is, by a quadrant, which measures a right angle. (Bk I. Th. i. Cor. 2).

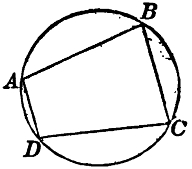
### THEOREM XI.

*If a quadrilateral be inscribed in a circle, the sum of either two of its opposite angles is equal to two right angles.*

Let $ABCD$ be any quadrilateral inscribed in a circle; then will the sum of the two opposite angles, $A$ and $C$, or $B$ and $D$, be equal to two right angles.



For, the angle $A$ is measured by half the arc $DCB$, which subtends it (Th. viii);



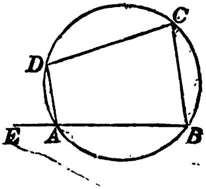
and the angle $C$ is measured by half the arc $DAB$, which subtends it. Hence, the sum of the two angles, $A$ and $C$, is measured by half the entire circumference. But half the entire circumference is the measure of two right angles; therefore, the sum of the opposite angles $A$ and $C$ is equal to two right angles.

In like manner, it may be shown, that the sum of the two angles $B$ and $D$ is equal to two right angles.

### THEOREM XII.

*If the side of a quadrilateral, inscribed in a circle, be produced out, the exterior angle will be equal to the inward opposite angle.*

Let the side $BA$, of the quadrilateral $ABCD$ be produced to $E$, then will the outward angle $DAE$ be equal to the inward opposite angle $C$.



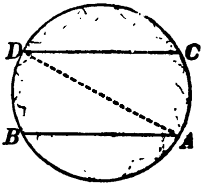
For, the angle $DAB$ plus the angle $C$, is equal to two right angles (Th. xi). But $DAB$ plus $DAE$ is also equal to two right angles (Bk. I. Th. ii). Taking from each the common angle $DAB$, and we shall have the angle $DAE$ equal to the interior opposite angle $C$.

### THEOREM XIII.

*Two parallel chords intercept equal arcs.*

Let the chords $AB$ and $CD$ be parallel: then will the arcs $AC$ and $BD$ be equal.

For, draw the line $AD$. Then, because the lines $AB$ and $CD$ are parallel, the alternate angles $ADC$ and $DAB$ will be equal (Bk. I. Th. xii). But the angle $ADC$ is measured by half the arc $AC$, and the angle $DAB$ by half the arc $BD$ (Th. viii): hence the two arcs $AC$ and $BD$ are themselves equal.



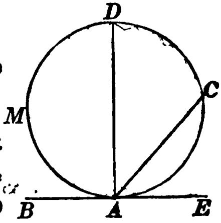
### THEOREM XIV.

*The angle formed by a tangent and a chord, is measured by half the arc of the chord.*

Let $BAE$ be tangent to the circle at the point $A$, and $AC$ any chord.

From $A$, the point of contact, draw the diameter $AD$.

Then, the angle $BAD$ will be a right angle (Th. v. Cor), and therefore will be measured by half the semi~~circle~~ $AMD$ (Bk. I, Th. i. Cor. 2).

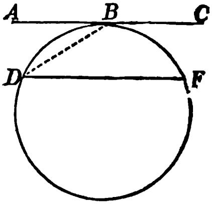


But the angle $DAC$ being at the circumference, is measured by half the arc $DC$; hence, by the addition of equals, the two angles $BAD$ and $DAC$, or the entire angle $BAC$ will be measured by half the arc $AMDC$.

It may be shown, by taking the difference between the two angles $DAE$ and $DAC$, that the angle $CAE$ is measured by half the arc $AC$ included between its sides.

### THEOREM XV.

*If a tangent and a chord are parallel to each other, they will intercept equal arcs.*

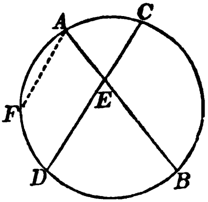


Let the tangent *ABC* be parallel to the chord *DF*: then will the intercepted arcs *DB*, *BF*, be equal to each other.

For, draw the chord *DB*. Then, since *AC* and *DF* are parallel, the angle *ABD* will be equal to the angle *BDF*. But *ABD* being formed by a tangent and a chord, will be measured by half the arc *DB*; and *BDF* being an angle at the circumference will be measured by half the arc *BF* (Th. viii). But since the angles are equal, the arcs will be equal: hence *DB* is equal to *BF*. ×

### THEOREM XVI.

*The angle formed within a circle by the intersection of two chords, is measured by half the sum of the intercepted arcs.*

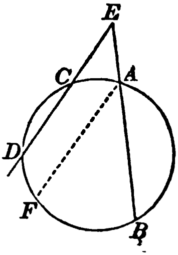


Let the two chords *AB* and *CD* intersect each other at the point *E*: then will the angle *AEC*, or its equal *DEB*, be measured by half the sum of the intercepted arcs *AC*, *DB*.

For, draw the chord *AF* parallel to *CD*. Then because of the parallels, the angle *DEB* will be equal to the angle *FAB* (Bk I. Th. xiv), and the arc *FD* to the arc *AC*. But the angle *FAB* is measured by half the arc *FDB*, that is, by half the sum of the arcs *FD*, *DB*. Now, since *FD* is equal to *AC*, it follows that the angle *DEB*, or its equal *AEC*, will be measured by half the sum of the arcs *DB* and *AC*.

### THEOREM XVII.

*The angle formed without a circle by the intersection of two secants is measured by half the difference of the intercepted arcs.*



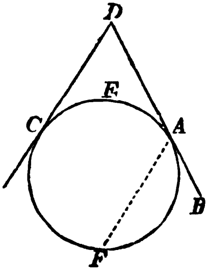
Let the two secants $DE$ and $EB$ intersect each other at $E$: then will the angle $DEB$ be measured by half the intercepted arcs $CA$ and $DB$.

Draw the chord $AF$ parallel to $ED$. Then, because $AF$ and $ED$ are parallel, and $EB$ cuts them, the angles $FAB$ and and $DEB$ are equal (Bk. I. Th. xiv).

But the angle $FAB$, at the circumference, is measured by half the arc $FB$ (Th. viii), which is the difference of the arcs $DFB$ and $CA$: hence, the equal angle $E$ is also measured by half the difference of the intercepted arcs $DFB$ and $CA$

### THEOREM XVIII.

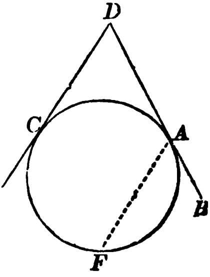
*An angle formed by two tangents is measured by half the difference of the intercepted arcs.*

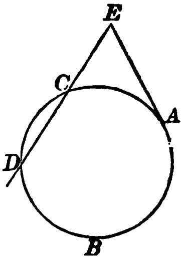


Let $CD$ and $DA$ be two tangents to the circle at the points $C$ and $A$: then will the angle $CDA$ be measured by half the difference of the intercepted arcs $CEA$ and $CFA$.

For, draw the chord $AF$ parallel to the tangent $CD$. Then, because the lines $CD$ and $AF$ are parallel, the angle $BAF$ will be equal to the angle $BDC$ (Bk. I. Th. xiv). But the angle $BAF$, formed by a tangent and a chord, is measured by half the arc $AF$, that is, by half the difference of $CFA$ and $CF$.

But since the tangent $DC$ and the chord $AF$ are parallel, the arc $CF$ is equal to the arc $CA$: hence the angle $BAF$, or its equal $BDC$, which is measured by half the difference of $CFA$ and $CF$, is also measured by half the difference of the intercepted arcs $CFA$ and $CA$.

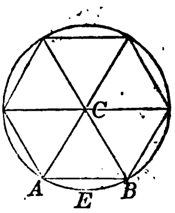




*Ccr.* In like manner it may be proved that the angle $E$, formed by a tangent and secant, is measured by half the difference of the intercepted arcs $AC$ and $DBA$.

### THEOREM XIX.

*The chord of an arc of sixty degrees is equal to the radius of the circle.*



Let $AEB$ be an arc of sixty degrees and $AB$ its chord: then will $AB$ be equal to the radius of the circle.

For, draw the radii $CB$ and $CA$. Then, since the angle $ACB$ is at the centre, it will be measured by the arc $AEB$: that is, it will be equal to sixty degrees (Bk. I. Def. 29).

Again, since the sum of the three angles of a triangle is equal to one hundred and eighty degrees (Bk. I. Th. xvii), it follows that the sum of the two angles *A* and *B* will be equal to one hundred and twenty degrees. But the triangle *CAB* is isosceles: hence, the angles at the base are equal (Bk. I. Th. vi): hence, each angle is equal to sixty degrees, and consequently, the side *AB* is equal to *AC* or *CB* (Bk. I. Th. vi).

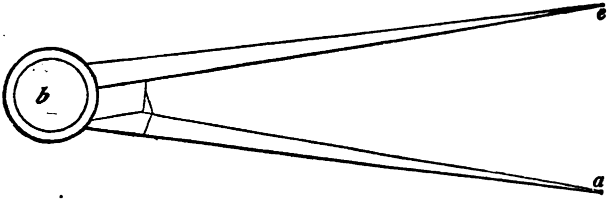
## PROBLEMS

### RELATING TO THE FIRST AND SECOND BOOKS.

The Problems of Geometry explain the methods of constructing or describing the geometrical figures.

For these constructions, a straight ruler and the common compasses or dividers, are all the instruments that are absolutely necessary.

DIVIDERS OR COMPASSES.



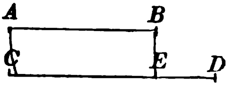
The dividers consist of the two legs *ba*, *be*, which turn easily about a common joint at *b*. The legs of the dividers are extended or brought together by placing the forefinger on the joint at *b*, and pressing the thumb and fingers against the legs.

### PROBLEM I.

*On any line, as CD, to lay off a distance equal to AB.*

Take up the dividers with the thumb and second finger, and place the forefinger on the joint at *b*.

Then, set one foot of the dividers at *A*, and extend the legs with the thumb and fingers, until the other foot reaches *B*.



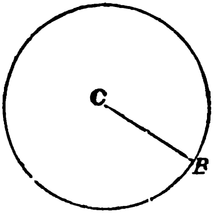
Then, raise the dividers, place one foot at *C*, and mark with the other the distance *CE*: and this distance will evidently be equal to *AB*.

### PROBLEM. II.

*To describe from a given centre the circumference of a circle having a given radius.*

Let *C* be the given centre, and *CB* the given radius.

Place one foot of the dividers at *C*, and extend the other leg until it reaches to B. Then, turn the dividers around the leg at *C*, and the other leg will describe the required circumference.



### OF THE RULER.

A ruler of a convenient size, is about twenty inches in ength, two inches wide, and one fifth of an inch in thickness. t should be made of a hard material, and perfectly straight and smooth.

### PROBLEM III.

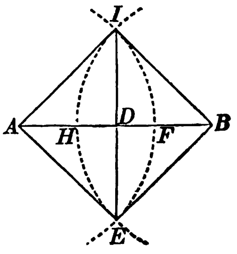
*To draw a straight line through two given points A and B.*

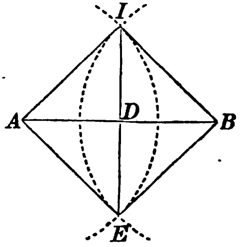
Place one edge of the ruler on *A* and slide the ruler around until the same edge falls on *B*. Then, with a pen, or pencil, draw the line *AB*.

### PROBLEM IV.

*To bisect a given line: that is, to divide it into two equal parts.*

Let *AB* be the given line to be divided. With *A* as a centre, and radius greater than half of *AB*, describe an arc *IFE*. Then, with *B* as a centre, and an equal radius *BI* describe the arc *IHE*. Join the points *I* and *E* by the line *IE*: the point *D*, where it intersects *AB*, will be the middle point of the line *AB*.

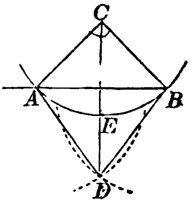




For, draw the radii $AI$, $AE$ $BI$, and $BE$. Then, since these radii are equal, the triangles $AIE$ and $BIE$ have all the sides of the one equal to the corresponding sides of the other; hence, their corresponding angles are equal (Bk. I. Th. viii); that is, the angle $AIE$ is equal to the angle $BIE$ Therefore, the two triangles $AID$ and $BID$, have the side $AI=IB$, the angle $AID=BID$, and $ID$ common: hence, they are equal (Bk. I. Th. iv), and $AD$ is equal to $DB$.

### PROBLEM V.

*To bisect a given angle or a given arc.*



Let $ACB$ be the given angle, and $AEB$ the given arc.

From the points $A$ and $B$, as centres, describe with the same radius two arcs cutting each other in $D$. Through $D$ and the centre $C$, draw $CED$, and it will divide the angle $ACB$ into two equal parts, and also bisect the arc $AEB$ at $E$.

For, draw the radii $AD$ and $BD$. Then, in the two triangles $ACD$, $CBD$, we have

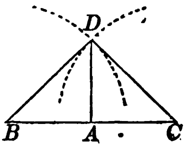
$$AC=CB, \qquad AD=BD$$

and $CD$ common: hence, the two triangles have their corresponding angles equal (Bk I. Th. viii), and consequently, $ACD$ is equal to $BCD$. But since $ACD$ is equal to $BCD$, it follows that the arc $AE$, which measures the former, is equal to the arc $BE$, which measures the latter

### PROBLEM VI.

*At a given point in a straight line to erect a perpendicular to the line.*

Let $A$ be the given point, and $BC$ the given line.



From $A$ lay off any two distances, $AB$ and $AC$, equal to each other Then, from the points $B$ and $C$, as centres, with a radius greater than $AB$, describe two arcs intersecting each other at $D$: draw $DA$, and it will be the perpendicular required.

For, draw the equal radii $BD$, $DC$. Then, the two triangles, $BDA$, and $CDA$, will have

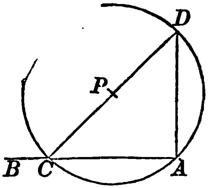
$$AB=AC \qquad BD=DC$$

and $AD$ common: hence, the angle $DAB$ is equal to the angle $DAC$ (Bk. I. Th. viii), and consequently, $DA$ is perpendicular to $BC$. (Bk. I Def. 21).

### SECOND METHOD.

*When the point A is near the extremity of the line.*

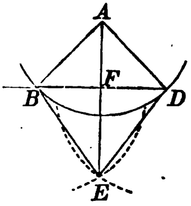
Assume any centre, as $P$, out of the given line. Then with $P$ as a centre, and radius from $P$ to $A$, describe the circumference of a circle Through $C$, where the circumference cuts $BA$, draw $CPD$. Then, through $D$, where $CP$ produced meets the circumference, draw $DA$: then will $DA$ be perpendicular to $BA$, since $CAD$ is an angle in a emicircle (Bk. II. Th. x).



### PROBLEM VII.

*From a given point without a straight line to let fall a perpendicular on the line.*

Let $A$ be the given point, and $BD$ the given line.

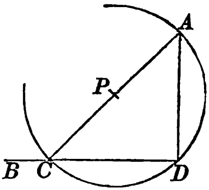


From the point $A$ as a centre, with a radius greater than the shortest distance to $BD$, describe an arc cutting $BD$ in the points $B$ and $D$. Then, with $B$ and $D$ as centres, and the same radius, describe two arcs intersecting each other at $E$. Draw $AFE$, and it will be the perpendicular required.

For, draw the equal radii $AB$, $AD$, $BE$ and $DE$. Then, the two triangles $EAB$ and $EAD$ will have the sides of the one equal to the sides of the other, each to each; hence, their corresponding angles will be equal (Bk. I. Th. viii), viz. the angle $BAE$ to the angle $DAE$. Hence, the two triangles $BAF$ and $DAF$ will have two sides and the included angle of the one, equal to two sides and the included angle of the other, and therefore, the angle $AFB$ will be equal to the angle $AFD$ (Bk. I. Th. iv): hence, $AFE$ will be perpendicular to $BD$.

SECOND METHOD.

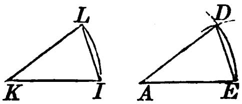
When the given point $A$ is nearly opposite the extremity of the line.



Draw $AC$, to any point $C$ of the line $BD$. Bisect $AC$ at $P$. Then, with $P$ as a centre and $PC$ as a radius, describe the semicircle $CDA$; draw $AD$, and it will be perpendicular to $CD$, since $CDA$ is an angle in a semicircle (Bk. II. Th. x).

### PROBLEM VIII.

*At a given point in a given line, to make an angle equal to a given angle.*



Let $A$ be the given point, $AE$ the given line, and $IKL$ the given angle.

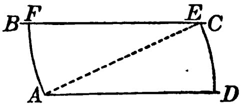
From the vertex $K$, as a centre, with any radius, describe the arc $IL$, terminating in the two sides of the angle: and draw the chord $IL$.

From the point $A$, as a centre, with a distance $AE$, equal to $KI$, describe the arc $DE$; then with $E$, as a centre, and a radius equal to the chord $IL$, describe an arc cutting $DE$ at $D$; draw $AD$, and the angle $EAD$ will be equal to the angle $K$.

For, draw the chord $DE$. Then the two triangles $IKL$ and $EAD$, having the three sides of the one equal to the three sides of the other, each to each, the angle $EAD$ will be equal to the angle $K$ (Bk. I. Th. viii).

### PROBLEM IX.

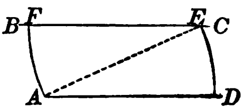
*Through a given point to draw a line that shall be parallel to a given line.*



Let $A$ be the given point and $BC$ the given line.

With $A$ as a centre, and any radius greater than the shortest distance from $A$ to $BC$, describe the indefinite arc $DE$. From the point $E$, as a centre, with the same radius, describe the arc $AF$: then, make $ED$ equal to $AF$ and draw $AD$, and it will be the required parallel.

For, since the arcs *AF* and *ED* are equal, the angles *EAD* and *AEF*, which they measure, are equal: hence, the line *AD* is parallel to *BC* (Bk I. Th. xiii).

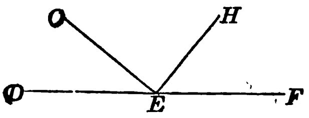


### PROBLEM X.

*Two angles of a triangle being given or known, to find the third.*

Draw the indefinite line *DEF*.

At any point, as *E*, make the angle *DEC* equal to one of the given angles, and then *CEH* equal to a second, by Prob. VIII; then will the angle *HEF* be equal to the third angle of the triangle.



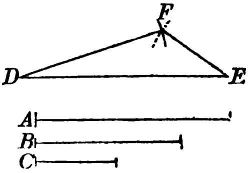
For, the sum of the three angles of a triangle is equal to two right angles (Bk. I. Th. xvii); and the sum of the three angles on the same side of the line *DE* is equal to two right angles (Bk. I. Th. ii. Cor. 2); hence, if *DEC* and *CEH* are equal to two of the angles, the angle *HEF* will be equal to the remaining angle of the triangle.

### PROBLEM XI.

*Three sides of a triangle being given, to describe the triangle.*

Let *A*, *B*, and *C*, be the given sides.

Draw *DE*, and make it equal to the side *A*. From the point *D*, as a centre, with a radius equal to the second side *B*, describe an arc:



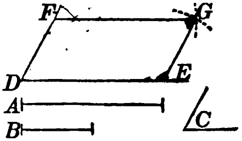
from $E$ as a centre, with the third side $C$, describe another arc intersecting the former in $F$: draw $DF$ and $FE$: then will $DEF$ be the required triangle.

For, the three sides are respectively equal to the three lines $A$, $B$~~, and~~ [illegible].

Sch) The sum of any two sides must be greater than the third side.

### PROBLEM XII.

*The adjacent sides of a parallelogram, with the angle which they contain, being given, to describe the parallelogram.*



Let $A$ and $B$ be the given sides and $C$ the given angle.

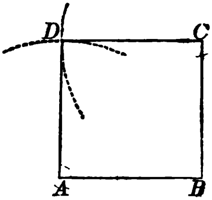
Draw the line $DE$ and make it equal to $A$. At the point $D$ make the angle $EDF$ equal to the angle $C$. Make the side $DF$ equal to $B$. Then describe two arcs, one from $F$, as a centre, with a radius $FG$ equal to $DE$, the other from $E$, as a centre, with a radius $EG$ equal to $DF$. Through the point $G$, the point of intersection, draw the lines $EG$ and $FG$, and $DEGF$ will be the required parallelogram.

For, in the quadrilateral $DFGE$, the opposite sides $DE$ and $FG$ are each equal to $A$: the opposite sides $DF$ and $EG$ are each equal to $B$, and the angle $EDF$ is equal to $C$. But, since the opposite sides are equal, they are also parallel (Bk. I. Th. xxiv), and therefore the figure is a parallelogram.

### PROBLEM XIII.

*To describe a square on a given line.*

Let $AB$ be the given line.



At the point $B$ draw $BC$ perpendicular to $AB$, by Problem VI, and then make it equal to $AB$.

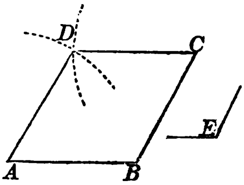
Then, with $A$ as a centre, and radius equal to $AB$, describe an arc; and with $C$ as a centre, and the same radius $AB$, describe another arc; and through $D$, their poin of intersection, draw $AD$ and $CD$: then will $ABCD$ be the required square.

For, since the opposite sides are equal, the figure will be a parallelogram (Bk. I. Th. xxiv): and since one of the angles is a right angle, the others will also be right angles (Bk. I. Th. xxiii. Cor. 1); and since the sides are all equal, the figure will be a square.

### PROBLEM XIV.

*To construct a rhombus, having given the length of one of the equal sides, and one of the angles.*

Let $AB$ be equal to the given side, and $E$ the given angle.

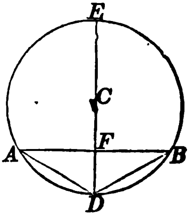


At $B$ lay off an angle, $ABC$, equal to $E$, by Prob. VIII. and make $BC$ equal to $AB$. Then, with $A$ and $C$ s centres, and a radius equal to $AB$, describe two arcs. Through $D$, their point of intersection, draw the lines $AD$, $CD$: then will $ABCD$ be the required rhombus.

For, since the opposite sides are equal, they will be parallel (Bk. I., Th. xxiv). But they are each equal to $AB$, and the angle $B$ is equal to the angle $E$: hence, $ABCD$ is the required rhombus.

### PROBLEM XV.

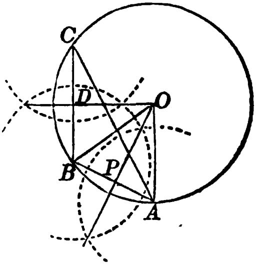
*To find the centre of a circle*



Draw any chord, as $AB$, and bisect it by Problem IV. Then, through $F$, the middle point, draw $DCE$, perpendicular to $AB$, by Problem VI. Then $DCE$ will be a diameter of the circle (Bk. II. Th. ii. Cor.). Then bisect $DE$ at $C$, and $C$ will be the centre of the circle.

### PROBLEM XVI.

*To describe the circumference of a circle through three given points not in the same straight line.*

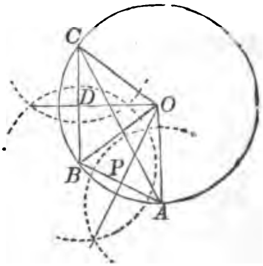


Let $A$, $B$, $C$, be the given points. Join these points by the straight lines $AC$ $AB$, $BC$.

Then, bisect any two of these straight lines, as $AB$, $BC$, by the perpendiculars $OD$, $OP$ (Prob. iv); and the point $O$, where these perpendiculars intersect each other, will be the centre of the circle.

Then with $O$ as a centre, and a radius equal to $OA$, describe the circumference of a circle, and it will pass through the points $A$, $B$, and $C$.

For, the two right angled triangles $OAP$ and $OBP$ have the side $AP$ equal to the side $BP$, $OP$ common, and the included angles $OPA$ and $OPB$ equal, being right angles; hence, the side $OB$ is equal to $OA$ (Bk. I. Th. iv).

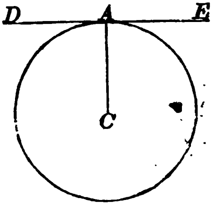


In like manner it may be shown, that $OC$ is equal to $OB$. Hence, a circumference described with the radius $OA$, will pass through the points $B$ and $C$.

*Sch.* This problem enables us to describe the circumference of a circle about a given triangle. For, we may consider the vertices of the three angles as the three points through which the circumference is to pass.

### PROBLEM XVII.

*Through a given point in the circumference of a circle, to draw a tangent line to the circle.*

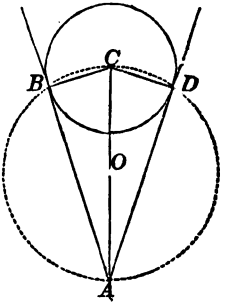


Let $A$ be the given point

Through $A$, draw the radius $AC$ to the centre, and then draw $DAE$ perpendicular to $AC$, by Problem VI. Then will $DAE$ be tangent to the circle at the point $A$ (Bk. II. Th. v).

### PROBLEM XVIII.

*Through a given point without the circumference, to draw a tangent line to the circle.*



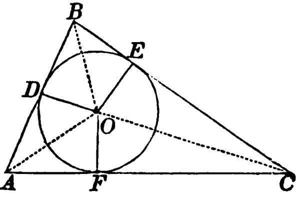
Let $C$ be the centre of the circle, and $A$ the given point without the circle.

Join $A$ and the centre $C$, and on $AC$, as a diameter, describe a circumference. Through the points $B$ and $D$ where the two circumferences intersect each other, draw the lines $AB$ and $AD$: these lines will be tangent to the circle whose centre is $C$.

For, since the angles $ABC$ and $ADC$ are each inscribed in a semicircle, they will be right angles (Bk. II. Th. x). Again, since the lines $AB$, $AD$, are each perpendicular to a radius at its extremity, they will be tangent to the circle (Bk. II. Th. v).

### PROBLEM XIX

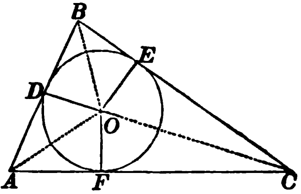
*To inscribe a circle in a given triangle.*



Let $ABC$ be the given triangle.

Bisect the angles $A$ and $B$ by the lines $AO$ and $BO$, meeting at the point $O$. From $O$, let fall the perpendiculars $OD$, $OE$, $OF$, on the three sides of the triangle—these perpendiculars will be equal to each other.

For, in the two right angled triangles $DAO$ and $FAO$, we have the right angle $D$ equal the right angle $F$, the angle $FAO$ equal to $DAO$, and consequently, the third angles $AOD$ and $AOF$ are equal (Bk. I. Th. xvii. Cor 1). But the two triangles have a common side $AO$, hence, they are equal (Bk. I. Th. v), and consequently, $OD$ is e ual to $OF$

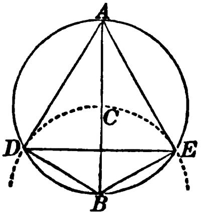


In a similar manner, it may be proved that $OE$ and $OD$ are equal; hence, the three perpendiculars, $OD$, $OF$, and $OE$, are all equal.

Now, if with $O$ as a centre, and $OF$ as a radius, we describe the circumference of a circle, it will pass through the points $D$ and $E$. and since the sides of the triangle are perpendicular to the radii $OF$, $OD$, $OE$, they will be tangent to the circumference (Bk. II. Th. v). Hence, the circle will be inscribed in the triangle.

### PROBLEM XX.

*To inscribe an equilateral triangle in a circle.*



Through the centre $C$ draw any diameter, as $ACB$. From $B$ as a centre, with a radius equal to $BC$, describe the arc $DCE$. Then, draw $AD$, $AE$, and $DE$, and $DAE$ will be the required triangle.

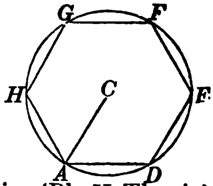
For, since the chords $BD$, $BE$, are each equal to the radius $CB$, the arcs $BD$, $BE$, are each equal to sixty degrees (Bk. II. Th. XIX), and the arc $DBE$ to one hundred and twenty degrees; hence, the angle $DAE$ is equal to sixty degrees (Bk. II. Th. viii).

Again, since the arc $BD$ is equal to sixty degrees, and the arc $BDA$ equal to one hundred and eighty degrees, it follows that $DA$ will be equal to one hundred and twenty degrees: hence, the angle $DEA$ is equal to sixty degrees, and consequently, the third angle $ADE$, is equal to sixty degrees.

Therefore, the triangle $ADE$ is equilateral (Bk. I. Th. vi Cor. 2).

### PROBLEM XXI.

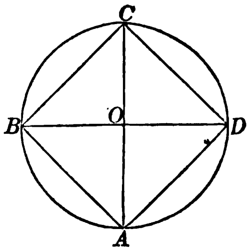
*To inscribe a regular hexagon in a circle.*



Draw any radius, as $AC$. Then apply the radius $AC$ around the circumference, and it will give the chords $AD$, $DE$, $EF$, $FG$, $GH$, and $HA$, which will be the sides of the regular hexagon. For, the side of a hexagon is equal to the radius (Bk. II. Th. xix).

### PROBLEM XXII.

*To inscribe a square in a given circle.*

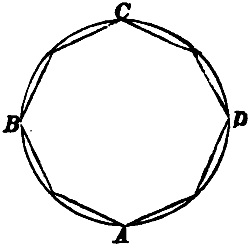


Let $ABCD$ be the given circle. Draw the two diameters $AC$, $BD$, at right angles to each other, and through the points $A$, $B$, $C$ and $D$ draw the lines $AB$, $BC$, $CD$, and $DA$: then will $ABCD$ be the required square.

For, the four right angled triangles, $AOB$, $BOC$, $COD$, and $DOA$ are equal, since the sides $AO$, $OB$, $OC$, and $OD$ are equal, being radii of the circle; and the angles at $O$ are equal in each, being right angles: hence, the sides $AB$, $BC$, $CD$, and $DA$ are equal (Bk. I. Th. iv).

But each of the angles $ABC$, $BCD$, $CDA$, $DAB$, is a right angle, being an angle in a semicircle (Bk. II. Th. x): hence, the figure $ABCD$ is a square (Bk. I. Def. 48)

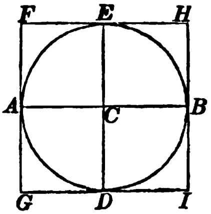
*Sch.* If we bisect the arcs *AB*, *BC*, *CD*, *DA*, and join the points, we shall have a regular octagon inscribed in the circle. If we again bisect the arcs, and join the points of bisection, we shall have a regular polygon of sixteen sides.



### PROBLEM XXIII.

*To describe a square about a given circle.*

Draw the diameters *AB*, *DE*, at right angles to each other. Through the extremities *A* and *B* draw *FAG* and *HBI* parallel to *DE*, and through *E* and *D*, draw *FEH* and *GDI* parallel to *AB*: then will *FGIH* be the required square.



For, since *ACDG* is a parallelogram, the opposite sides are equal (Bk. I. Th. xxiii): and since the angle at *C* is a right angle, all the other angles are right angles (Bk. I. Th. xxiii. Cor. 1): and as the same may be proved of each of the figures *CI*, *CH* and *CF*, it follows that all the angles, *F*, *G*, *I*, and *H*, are right angles, and that the sides *GI*, *IH*, *HF*, and *FG*, are equal, each being equal to the diameter of the circle. Hence the figure *GIHF* is a square (Bk. I. Def. 48).

$\frac{6}{3} = 2$

# GEOMETRY.

---

## BOOK III.

### OF RATIOS AND PROPORTIONS.

#### DEFINITIONS.

1. *Ratio* is the quotient arising from dividing one quantity by another quantity of the same kind. Thus, if the numbers 3 and 6 have the same unit, the ratio of 3 to 6 will be expressed by

$$\frac{6}{3}=2.$$

And in general, if $A$ and $B$ represent quantities of the same kind, the ratio of $A$ to $B$ will be expressed by

$$\frac{B}{A}.$$

2. If there be four numbers, 2, 4, 8, 16, having such values that the second divided by the first is equal to the fourth divided by the third, the numbers are said to be in proportion. And in general, if there be four quantities $A$, $B$, $C$, and $D$, having such values that

$$\frac{B}{A}=\frac{D}{C},$$

then, $A$ is said to have the same *ratio* to $B$, that $C$ has to $D$, or, the ratio of $A$ to $B$ is equal to the ratio of $C$ to $D$ When four quantities have this relation to each other, they are said to be in proportion. Hence, the proportion of four quantities results from an equality of their ratios taken two and two.

To express that the ratio of $A$ to $B$ is equal to the ratio of $C$ to $D$, we write the quantities thus:

$$A : B :: C : D;$$

and read, $A$ is to $B$, as $C$ to $D$.

The quantities which are compared together are called the *terms* of the proportion. The first and last terms are called the *extremes*, and the second and third terms, the *means*. Thus, $A$ and $D$ are the extremes, and $B$ and $C$ the means.

3. Of four proportional quantities, the first and third are called the *antecedents*, and the second and fourth the *consequents;* and the last is said to be a fourth proportional to the other three taken in order. Thus, in the last proportion, $A$ and $C$ are the antecedents, and $B$ and $D$ the consequents.

4. Three quantities are in proportion when the first has the same ratio to the second, that the second has to the third; and then the middle term is said to be a mean proportional between the other two. For example,

$$3 : 6 :: 6 : 12;$$

and 6 is a mean proportional between 3 and 12.

5. Quantities are said to be in proportion by *inversion*, or *inversely*, when the consequents are made the antecedents, and the antecedents the consequents.

Thus, if we have the proportion

$$3 : 6 :: 8 \cdot 16.$$

the inverse proportion would be

$$6 : 3 :: 16 : 8.$$

6. Quantities are said to be in proportion by *alternation*, or *alternately*, when antecedent is compared with antecedent and consequent with consequent.

Thus, if we have the proportion

$$3 : 6 :: 8 : 16,$$

the alternate proportion would be

$$3 : 8 :: 6 : 16.$$

7. Quantities are said to be in proportion by *composition*, when the sum of the antecedent and consequent is compared either with antecedent or consequent.

Thus, if we have the proportion

$$2 : 4 :: 8 : 16,$$

the proportion by composition would be

$$2+4 : 4 :: 8+16 : 16;$$

that is, $6 : 4 :: 24 : 16.$

8. Quantities are said to be in proportion by *division*, when the difference of the antecedent and consequent is compared either with the antecedent or consequent.

Thus, if we have the proportion

$$3 : 9 :: 12 : 36,$$

the proportion by division will be

$$9-3 : 9 :: 36-12 : 36;$$

that is, $6 : 9 :: 24 : 36.$

9. Equimultiples of two or more quantities are the products which arise from multiplying the quantities by the same number.

Thus, if we have any two numbers, as 6 and 5, and multiply them both by any number, as 9, the equimultiples will be 54 and 45; for

$$6\times9=54 \quad \text{and} \quad 5\times9=45.$$

Also, $m\times A$ and $m\times B$ are equimultiples of $A$ and $B$, the common multiplier being $m$.

10. Two variable quantities, $A$ and $B$, are said to be *reciprocally proportional*, or *inversely proportional*, when one increases in the same ratio as the other diminishes. When this relation exists, either of them is equal to a constant quantity divided by the other.

Thus, if we had any two numbers, as 2 and 4, so related to each other that if we divided one by any number we must multiply the other by the same number, one would increase in the same ratio as the other would diminish, and their product would not be changed.

### THEOREM I.

*If four quantities are in proportion, the product of the two extremes will be equal to the product of the two means.*

If we have the proportion

$$A : B :: C : D$$

we have, by Def. 2,

$$\frac{B}{A}=\frac{D}{C}$$

and by clearing the equation of fractions, we have

$$BC=AD$$

*Sch.* The general principle is verified in the proportion between the numbers

$$2 : 10 :: 12 : 60$$

which gives

$$2\times60=10\times12=120$$

### THEOREM II.

*If four quantities are so related to each other, that the product of two of them is equal to the product of the other two; then two of them may be made the means, and the other two the extremes of a proportion.*

Let $A$, $B$, $C$, and $D$, have such values that

$$B \times C = A \times D$$

Divide both sides of the equation by $A$, and we have

$$\frac{B}{A} \times C = D$$

Then divide both sides of the last equation by $C$, and we have

$$\frac{B}{A} = \frac{D}{C}$$

hence, by Def. 2, we have

$$A : B :: C : D.$$

*Sch.* The general truth may be verified by the numbers

$$2 \times 18 = 9 \times 4$$

which give

$$2 : 4 :: 9 : 18$$

### THEOREM III.

*If three quantities are in proportion, the product of the two extremes will be equal to the square of the middle term.*

Let us suppose that we have

$$A : B :: B : C$$

Then, by Def. 2, we have

$$\frac{B}{A} = \frac{C}{B}$$

and by clearing the equation of its fractions, we have

$$B^2 = C \times A$$

*Sch.* The proposition may be verified by the numbers

$$3 : 6 :: 6 : 12$$

which give

$$3 \times 12 = 6 \times 6 = 36$$

### THEOREM IV.

*If four quantities are in proportion, they will be in proportion when taken alternately.*

Let $A : B :: C : D$

Then, by Def. 2, we have

$$\frac{B}{A} = \frac{D}{C}$$

Multiplying both members of this equation by $\frac{C}{B}$, we have

$$\frac{C}{A} = \frac{D}{B}$$

and consequently,

$$A : C :: B : D.$$

*Sch.* The theorem may be verified by the proportion

$$10 : 15 :: 20 : 30$$

For, we have, by alternation,

$$10 : 20 :: 15 : 30.$$

### THEOREM V.

*If there be two sets of proportions, having an antecedent and a consequent in the one, equal to an antecedent and a consequent in the other; then, the remaining terms will be proportional.*

If we have

$A : B :: C : D$, and $A : B :: E : F$;

shall have

$$\frac{B}{A}=\frac{D}{C} \quad \text{and} \quad \frac{B}{A}=\frac{F}{E}$$

Hence, by Ax. 1, we have

$$\frac{D}{C}=\frac{F}{E}$$

and consequently,

$$C : D :: E : F.$$

*Sch.* The proposition may be verified by the following proportions,

$$2 : 6 :: 8 : 24 \quad \text{and} \quad 2 : 6 :: 10 : 30$$

which give

$$8 : 24 :: 10 : 30.$$

### THEOREM VI.

*If four quantities are in proportion, they will be in proportion when taken inversely.*

If we have the proportion

$$A : B :: C : D$$

we have, by Th. I,

$$A\times D=B\times C,$$

or

$$B\times C=A\times D.$$

Hence, we have, by Th. II,

$$B : A :: D : C.$$

*Sch.* The proposition may be verified by the proportion

$$7 : 14 :: 8 : 16;$$

which, when taken inversely, gives

$$14 : 7 :: 16 : 8.$$

### THEOREM VII.

*If four quantities are in proportion, they will be in proportion by composition.*

Let us suppose that we have

$$A : B :: C : D$$

we shall then have

$$A\times D=B\times C.$$

To each of these equals, add $B\times D$, and we have

$$(A+B)\times D=(C+D)\times B;$$

and by separating the factors by Th. II, we have

$$A+B : B :: C+D : D.$$

*Sch.* The proposition may be verified by the following proportion,

$$9 : 27 :: 16 : 48.$$

We shall have, by composition,

$$9+27 : 27 :: 16+48 : 48,$$

that is, $36 : 27 :: 64 : 48,$

in which the ratio is three fourths.

### THEOREM VIII.

*If four quantities are in proportion, they will be in proportion by division.*

Let us suppose that we have

$$A : B :: C : D;$$

we shall then have

$$A\times D=B\times C.$$

From each of these equals let us subtract $B\times D$, and we have

$$(A-B)\times D=(C-D)\times B;$$

and by separating the factors by Th. II, we have,

$$A-B : B :: C-D : D.$$

*Sch* The proposition may be verified by the proportion,

$$24 : 8 :: 48 : 16.$$

We have, by division,

$$24-8 \ : \ 8 \ :: \ 48-16 \ : \ 16;$$

that is,

$$16 \ : \ 8 \ :: \ 32 \ : \ 16;$$

in which the ratio is one-half.

End.

### THEOREM IX.

*Equal multiples of two quantities have the same ratio as the quantities themselves.*

If we have the proportion

$$A \ : \ B \ :: \ C \ : \ D$$

we shall have

$$\frac{B}{A}=\frac{D}{C}$$

Now, let $M$ be any number, and by it multiply the numerator and denominator of the first member of the equation which will not change its value: we shall then have

$$\frac{M\times B}{M\times A}=\frac{D}{C}$$

and hence we have

$$M\times A \ : \ M\times B \ :: \ C \ : \ D,$$

that is, the equal multiples $M\times A$ and $M\times B$, have the same ratio as $A$ to $B$.

*Sch* The proposition may be verified by the proportion,

$$5 \ : \ 10 \ :: \ 12 \ : \ 24;$$

for, by multiplying the first antecedent and consequent by any number, as 6, we have

$$30 \ : \ 60 \ :: \ 12 \ : \ 24,$$

in which the ratio is still 2.

### THEOREM X.

*If four quantities are proportional, and one antecedent and its consequent be augmented by quantities which have the same ratio as the antecedent and consequent, the four quantities will still be in proportion.*

Let us take the proportions

$A : B :: C : D$, and $A : B :: E : F$,
which give

$$A\times D=B\times C \quad \text{and} \quad A\times F=B\times E;$$

adding these equals we have

$$A\times(D+F)=B\times(C+E);$$

and by Th. II, we have

$$A : B :: C+E : D+F$$

in which the antecedent $C$ and its consequent $D$, are augmented by the quantities $E$ and $F$, which have the same ratio.

*Sch.* The proposition may be verified by the proportion,

$$9 : 18 :: 20 : 40,$$

in which the ratio is 2.

If we augment the antecedent and its consequent by 15 and 30, which have the same ratio, we have

$$9 : 18 :: 20+15 : 40+30$$

that is, $9 : 18 :: 35 \cdot 70$,
in which the ratio is still 2.

### THEOREM XI.

*If four quantities are proportional, and one antecedent and its consequent be diminished by quantities which have the same ratio as the antecedent and consequent, the four quantities will still be in proportion*

Let us take the proportions

$A : B :: C : D$, and $A : B :: E : F$, which give

$$A \times D = B \times C \quad \text{and} \quad A \times F = B \times E.$$

By subtracting these equalities, we have

$$A \times (D - F) = B \times (C - E);$$

and by Th. II, we obtain

$$A : B :: C - E : D - F,$$

in which the antecedent and consequent, $C$ and $D$, are diminished by $E$ and $F$, which have the same ratio.

*Sch.* The proposition may be verified by the proportion,

$$9 : 18 :: 20 : 40,$$

for, by diminishing the antecedent and consequent by 15 and 30, we have

$$9 : 18 :: 20 - 15 : 40 - 30;$$

that is $9 : 18 :: 5 : 10$

in which the ratio is still 2.

### THEOREM XII.

*If we have several sets of proportions, having the same ratio, any antecedent will be to its consequent, as the sum of the antecedents to the sum of the consequents.*

If we have the several proportions,

$A : B :: C : D$ which gives $A \times D = B \times C$

$A : B :: E : F$ which gives $A \times F = B \times E$

$A : B :: G : H$ which gives $A \times H = B \times G$

We shall then have, by addition,

$$A \times (D + F + H) = B \times (C + E + G);$$

and consequently, by Th. II.

$$A : B :: C + E + G : D + F + H.$$

*Sch.* The proposition may be verified by the following proportions: viz.

2 : 4 :: 6 : 12 and 1 : 2 :: 3 : 6

Then, 2 : 4 :: 6+3 : 12+6;

that is, 2 : 4 :: 9 : 18,

in which the ratio is still 2.

### THEOREM XIII.

*If four quantities are in proportion, their squares or cubes will also be proportional.*

If we have the proportion

$$A : B :: C : D,$$

it gives

$$\frac{B}{A}=\frac{D}{C}$$

Then, if we square both members, we have

$$\frac{B^2}{A^2}=\frac{D^2}{C^2}$$

and if we cube both members, we have

$$\frac{B^3}{A^3}=\frac{D^3}{C^3}$$

and then, changing these equalities into a proportion, we have for the first,

$$A^2 : B^2 :: C^2 : D^2;$$

and for the second

$$A^3 : B^3 :: C^3 : D^3.$$

*Sch.* We may verify the proposition by the proportion,

2 : 4 :: 6 : 12,

and by squaring each term we have,

4 : 16 :: 36 : 144

numbers which are still proportional, and in which the ratio is 4.

If we cube the numbers we have,

$$2^3 \;:\; 4^3 \;::\; 6^3 \;:\; 12^3$$

that is, $\quad 8 \;:\; 64 \;::\; 216 \;:\; 1728,$

in which the ratio is 8.

### THEOREM XIV.

*If we have two sets of proportional quantities, the products of the corresponding terms will be proportional.*

Let us take the proportions,

$$A \;:\; B \;::\; C \;:\; D \quad \text{which gives} \quad \frac{B}{A}=\frac{D}{C}$$

$$E \;:\; F \;::\; G \;:\; H \quad \text{which gives} \quad \frac{F}{E}=\frac{H}{G}$$

Multiplying the equalities together, we have

$$\frac{B\times F}{A\times E}=\frac{D\times H}{C\times G}$$

and this by Th. II, gives

$$A\times E \;:\; B\times F \;::\; C\times G \;:\; D\times H.$$

*Sch.* The proposition may be verified by the following proportions:

$$8 \;:\; 12 \;::\; 10 \;:\; 15,$$

and $\quad 3 \;:\; 4 \;::\; 6 \;:\; 8;$

we shall then have

$$24 \;:\; 48 \;::\; 60 \;:\; 120$$

which are proportional, the ratio being 2.

# GEOMETRY.

---

## BOOK IV

### OF THE MEASUREMENT OF AREAS, AND THE PROPORTIONS OF FIGURES.

#### DEFINITIONS.

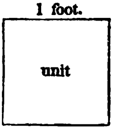
1. Similar figures, are those which have the angles of the one equal to the angles of the other, each to each, and the sides about the equal angles proportional.

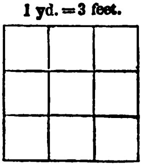
2. Any two sides, or any two angles, which are like placed in the two similar figures, are called *homologous* sides or angles.

3. A polygon which has all its angles equal, each to each, and all its sides equal, each to each, is called a *regular polygon.* A regular polygon is both equiangular and equilateral.

4. If the length of a line be computed in feet, one foot is the unit of the line, and is called the *linear unit.* If the length of a line be computed in yards, one yard is the linear unit

5. If we describe a square on the unit of length, such square is called the unit of surface. Thus, if the linear unit is one foot, one square foot will be the unit of surface, or superficial unit.





6. If the linear unit is one yard, one square yard will be the unit of surface; and this square yard contains nine square feet.

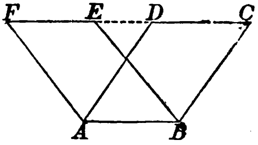
7. The *area* of a figure is the measure of its surface. The unit of the number which expresses the area, is a square, the side of which is the unit of length.

8. Figures have equal areas, when they contain the same measuring unit an equal number of times.

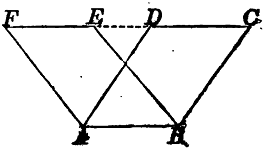
9. Figures which have equal areas are called *equivalent*. The term *equal*, when applied to figures, implies an equality in all respects. The term *equivalent*, implies an equality in one respect only: viz. an equality in their areas. The sign ≎, denotes equivalency, and is read, *is equivalent to*.

### THEOREM I.

*Parallelograms which have equal bases and equal altitudes, are equivalent.*



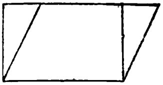
Place the base of one parallelogram on that of the other, so that $AB$ shall be the common base of the two parallelograms $ABCD$ and $ABEF$. Now, since the parallelograms have the same altitude, their upper bases, $DC$ and $FE$, will fall on the same line $FEDC$, parallel to $AB$. Since the opposite sides of a parallelogram are equal to each other (Bk. I. Th. xxiii), $AD$ is equal to $BC$. Also, $DC$ and $FE$ are each equal to $AB$: and consequently, they are equal to each



other (Ax. 1). To each, add $ED$: then will $CE$ be equal to $DF$.

But since the line $FC$ cuts the two parallels $CB$ and $DA$, the angle $BCE$ will be equal to the angle $ADF$ (Bk. I. Th. xiv): hence, the two triangles $ADF$ and $BCE$ have two sides and the included angle of the one equal to two sides and the included angle of the other, each to each; consequently, they are equal (Bk. I. Th. iv).

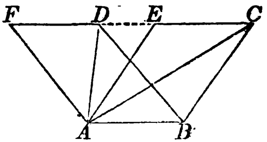
If then, from the whole space $ABCF$ we take away the triangle $ADF$, there will remain the parallellogram $ABCD$; but if we take away the equal triangle $BEC$, there will remain the parallelogram $ABEF$: hence, the parallelogram $ABEF$ is equivalent to the parallelogram $ABCD$ (Ax. 3).



*Cor.* A parallelogram and a rectangle, having equal bases and equal altitudes, are equivalent.

### THEOREM II.

*Triangles which have equal bases and equal altitudes, are equivalent.*



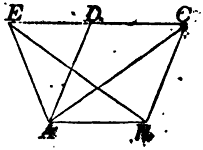
Place the base of one triangle on that of the other, so that $ABC$ and $ABD$ shall be two triangles, having a common base $AB$, and for their altitude, the distance between the two parallels $AB$, $FC$: then will the triangle $ABC$ be equivalent to the triangle $ADB$.

For, through $A$ draw $AE$ parallel to $BC$, and $AF$ parallel to $BD$, forming the two parallelograms $BE$ and $BF$ Then, since these parallelograms have a common base and equal altitudes, they will be equivalent (Th. i).

But the triangle $ABC$ is half the parallelogram $BE$ (Bk. I. Th. xxiii); and $ABD$ is half the equal parallelogram $BF$. hence, the triangle $ABC$ is equivalent to the triangle $ABD$.

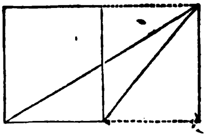
## THEOREM III.

*If a triangle and a parallelogram have equal bases and equal altitudes, the triangle will be half the parallelogram.*



Place the base of the triangle on the base of the parallelogram, so that $AB$ shall be the common base of the triangle and parallelogram: then will the triangle $ABE$ be half the parallelogram $BD$.

For, draw the diagonal $AC$. Then, since the altitude of the triangle $AEB$ is equal to that of the parallelogram, the vertex will be found some where in $CD$, or in $CD$ produced. Now the two triangles $ABC$ and $ABE$, having the same base $AB$, and equal altitudes, are equivalent (Th. ii). But the triangle $ABC$ is half the parallelogram $BD$ (Bk. I. Th. xxiii): hence, the triangle $ABE$ is half the parallelogram $BD$ (Ax. 1).

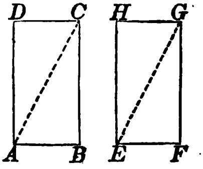


*Cor.* Hence, if a triangle and a rectangle have equal bases and equal altitudes, the triangle will be half the rectangle.

For the rectangle would be equivalent to a parallelogram of the same base and altitude (Th. i. Cor.), and since the triangle is half the parallelogram, it is also equivalent to half the rectangle.

### THEOREM IV.

*Rectangles which are described on equal lines are equivalent.*



Let *BD* and *FH* be two rectangles, having the sides *AB*, *BC*, equal to the two sides *EF*, *FG*, each to each: then will the rectangle *ABCD*, described on the lines *AB*, *BC*, be equivalent to the rectangle *EFGH*, described on the lines *EF*, *FG*.

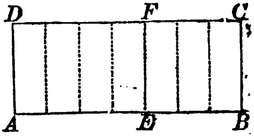
For, draw the diagonals *AC*, *EG*, dividing each parallelogram into two equal parts.

Then the two triangles, *ABC*, *EFG*, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, are equal (Bk. I. Th. iv). But these equal triangles are halves of the respective rectangles (Th. iii. Cor.): hence, the rectangles are equal (Ax. 7); and consequently equivalent.

*Cor.* The squares on equal lines are equal. For a square is but a rectangle having its sides equal.

### THEOREM V.

*Two rectangles having equal altitudes are to each other as their bases.*



Let *AEFD* and *EBCF* be two rectangles having the common altitude *AD*; then will they be to each other as the bases *AE* and *EB*.

For, suppose the base *AE* to be to the base *EB*, as any two numbers, say the numbers 4 and 3. Let *AE* be then divided into four equal parts, and *EB* into three equal parts, and through the points of division draw parallels to *AD* We shall thus form seven rectangles, all equivalent to each other since they have equal bases and equal altitudes (Th. iv).

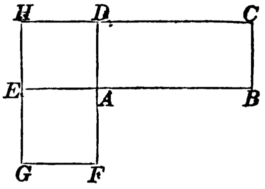
But the rectangle *AEFD* will contain four of these partial rectangles, while the rectangle *EBCF* will contain three; hence, the rectangle *AEFD* will be to the rectangle *EBCF* as 4 to 3; that is, as the base *AE* to the base *EB*.

The same reasoning may be applied to any other rectangles whose bases are whole numbers: hence,

$$AEFD : EBCF :: AE : EB.$$

### THEOREM VI.

*Any two rectangles are to each other as the products of their bases and altitudes.*

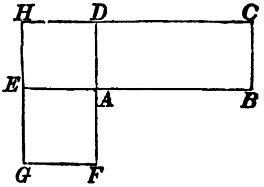


Let *ABCD* and *AEGF* be two rectangles: then will $ABCD : AEGF :: AB \times AD : AF \times AE$

For, having placed the two rectangles so that *BAE* and *DAF* shall form straight lines, produce the sides *CD* and *GE* until they meet in *H*.

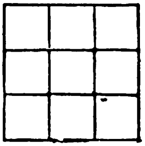
Then, the two rectangles *ABCD*, *AEHD*, having the common altitude *AD*, are to each other as their bases *AB* and *AE* (Th. v). In like manner, the two rectangles *AEHD* *AEGF*, having the same altitude *AE*, are to each other as their bases *AD* and *AF*. Thus, we have the proportions

$$ABCD : AEHD :: AB : AE,$$
$$AEHD : AEGF :: AD : AF.$$



If, now, we multiply the corresponding terms together, the products will be proportional (Bk. III. Th. xiv.); and the common multiplier $AEHD$ may be omitted (Bk. III. Th. ix.): hence, we shall have

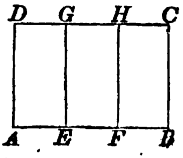
$$ABCD : AEGF :: AB \times AD : AE \times AF.$$



*Sch.* Hence, the product of the base by the altitude may be assumed as the measure of a rectangle. This product will give the number of superficial units in the surface: because, for one unit in height, there are as many superficial units as there are linear units in the base; for two units in height, twice as many; for three units in height, three times as many, &c.

### THEOREM VII.

*The sum of the rectangles contained by one line, and the several parts of another line any way divided, is equivalent to the rectangle contained by the two whole lines.*



Let $AD$ be one line, and $AB$ the other, divided into the parts $AE$, $EF$, $FB$: then will the rectangles contained by $AD$ and $AE$, $AD$ and $EF$, $AD$ and $FB$, be equivalent to the rectangle $AC$ which is contained by the lines $AD$ and $AB$.

For, through $E$ and $F$ draw

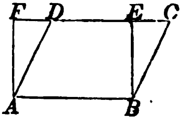
be equal to the ~~rectangle~~ of $AD \times AE$; $EH$ will be equal to $EG \times EF$, or to $AD \times EF$; and $FC$ will be equal to $FH \times FB$, or to $AD \times FB$.

But the rectangle $AC$ is equal to the sum of the partial rectangles: hence,

$$AD \times AB = AD \times AE + AD \times EF + AD \times FB.$$

### THEOREM VIII.

*The area of any parallelogram is equal to the product of its base by its altitude.*



Let $ABCD$ be any parallelogram, and $BE$ its altitude: then will its area be equal to $AB \times BE$.

For, draw $AF$ perpendicular to the base $AB$, and produce $CD$ to $F$. Then, the parallelogram $BD$ and the rectangle $BF$, having the same base and altitude are equivalent (Th. i. Cor.). But the area of the rectangle $BF$ is equal to the product of its base $AB$ by the altitude $AF$ (Th. vi. Sch.): hence, the area of the parallelogram is equal to $AB \times BE$.

*Cor.* Parallelograms of equal bases are to each other as their altitudes; and if their altitudes are equal, they are to each other as their bases.

For, let $B$ be the common base, and $C$ and $D$ the altitudes of two parallelograms. Then, by the theorem, their areas are to each other, as

$$B \times C \quad : \quad B \times D,$$

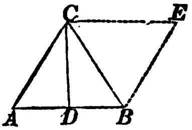
that is, (Bk. III. Th ix), as $\quad C : D.$

If $A$ and $B$ be their bases, and $C$ their common altitude, then they will be to each other as

$A \times C \quad B \times C \cdot$ that is, as $\quad A : B.$

### THEOREM IX.

*The area of a triangle is equal to half the product of its base by its altitude.*



Let $ABC$ be any triangle and $CD$ its altitude: then will its area be equal to half the product of $AB \times CD$.

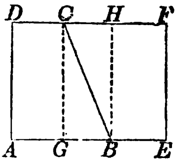
For, through $B$ draw $BE$ parallel to $AC$, and through $C$ draw $CE$ parallel to $AB$: we shall then form the parallelogram $AE$, having the same base and altitude as the triangle $ABC$.

But the area of the parallelogram is equal to the product of the base $AB$ by its altitude $DC$; and since the parallelogram is double the triangle (Th. iii), it follows that the area of the triangle is equal to half this product: that is, to half the product of $AB \times CD$.

*Cor.* Two triangles of the same altitude are to each other as their bases; and two triangles of the same base are to each other as their altitudes. And generally, triangles are to each other as the products of their bases and altitudes.

### THEOREM X.

*The area of a trapezoid is equal to half the product of its altitude multiplied by the sum of its parallel sides.*



Let $ABCD$ be a trapezoid, $CG$ its altitude, and $AB$, $DC$ its parallel sides: then will its area be equal to half the product of

$$\tfrac{1}{2} CG \times (AB + DC).$$

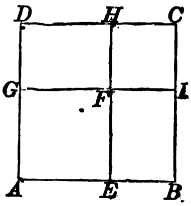
For, produce $AB$ until $BE$ is equal to $DC$, and complete the rectangle $AF$; also, draw $BH$ perpendicular to $AB$.

Then, the rectangle $AC$ will be equivalent to $BF$, since they have equal bases and equal altitudes (Th. iv). The diagonal $BC$ will divide the rectangle $GH$ into two equal triangles; and hence, the trapezoid $ABCD$ will be equivalent to the trapezoid $BEFC$; and consequently, the rectangle $AF$, is double the trapezoid $ABCD$.

But the rectangle $AF$ is equivalent to the product of $AD\times AE$; that is, to $CG\times(AB+DC)$; and consequently, the trapezoid $ABCD$ is equal to half that product.

### THEOREM XI.

*If a line be divided into two parts, the square described on the whole line is equivalent to the sum of the squares described on the two parts, together with twice the rectangle contained by the parts.*



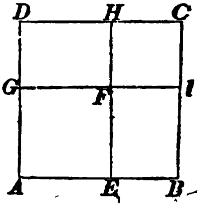
Let the line $AB$ be divided into two parts at the point $E$: then will the square described on $AB$ be equivalent to the two squares described on $AE$ and $EB$, together with twice the rectangle contained by $AE$ and $EB$: that is

$$\overline{AB}^2 \asymp \overline{AE}^2+\overline{EB}^2+2AE\times EB.$$

For, let $AC$ be a square on $AB$, and $AF$ a square on $AE$, and produce the sides $EF$ and $GF$ to $H$ and $I$.

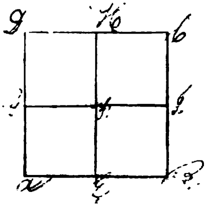
Then, since $EH$ is equal to $AD$, being the opposite side of a rectangle, it is also equal to $AB$; and $GI$ is likewise equal to $AB$. If, therefore, from these equals we take away $EF$ a-

*GF*, there will remain *FH* equal to *FI*, and each will be equal to *HC* or *IC*; and since the angle at *F* is a right angle, it follows that *FC* is equal to a square described on *EB*. It also follows, that *DF* and *FB* are each equal to the rectangle of *AE* into *EB*.



But the square *ABCD* is made up of four parts, viz., the square on *AE*; the square on *EB*; the rectangle *DF*, and the rectangle *FB*. Hence, the square on *AB* is equivalent to the square on *AE* plus the square on *EB*, plus twice the rectangle contained by *AE* and *EB*.

*Cor.* If the line *AB* be divided into two equal parts, the rectangles *DF* and *FB* would become squares, and the square described on the whole line would be equivalent to four times the square described on half the line.

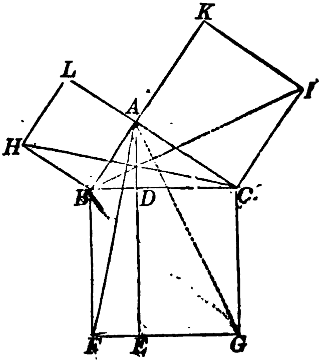


*Sch.* The property may be expressed in the language of algebra, thus,

$$(a+b)^2=a^2+2ab+b.^2$$

### THEOREM XII.

*The square described on the hypothenuse of a right angled triangle, is equivalent to the sum of the squares described on the other two sides.*



Let *BAC* be a right angled triangle, right angled at *A*: then will the square described on the hypothenuse *BC*, be equivalent to the two squares described on *BA* and *AC*.

Having described the squares *BG*, *BL* and *AI*, let fall from *A*, on the hypothenuse, the perpendicular *AD*, and produce it to *E*; then draw the diagonals *AF*, *CH*.

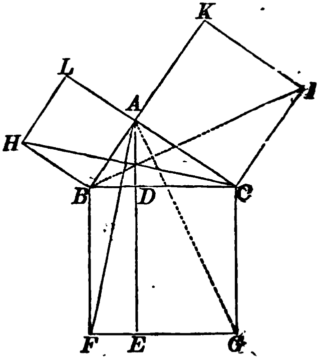
Now, the angle *ABF* is made up of the right angle *FBC* and the angle *CBA*; and the angle *CBH* is made up of the right angle *ABH* and the same angle *CBA*: hence, the angle *ABF* is equal to *CBH*. But *FB* is equal to *BC*, being sides of the same square; and for a like reason, *BA* is equal to *HB*. Therefore, the two triangles *ABF* and *CBH*, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, are equal (Bk. I. Th. iv).

Since the angles *BAC* and *BAL* are right angles, as also the angle *ABH*, it follows that *CAL* is a straight line parallel to *BH*. (Bk. I. Th. ii. Cor. 3). Hence, the square *HA* and the triangle *HBC*, stand on the same base and between the same parallels; therefore, the triangle is half the square (Th. iii. Cor.). For a like reason, the triangle *ABF* is half the rectangle *BE*.

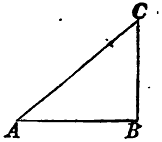
But it has already been proved that the triangle *ABF* is equal to the triangle *CBH*: hence, the rectangle *BE*, which is double the former, is equivalent to the square *BL*, which is double the latter (Ax. 6

In the same manner it may be proved, that the rectangle *DG* is equivalent to the square *CK*.

But the two rectangles *BE*, *DG*, make up the square *BG*: therefore, the square *BG*, described on the hypothenuse, is equivalent to the squares *BL* and *CK*, described on the other two sides.



*Cor.* Hence, the square of either side of a right angled triangle is equivalent to the square of the hypothenuse diminished by the square of the other side. That is, in the right angled triangle *ABC*

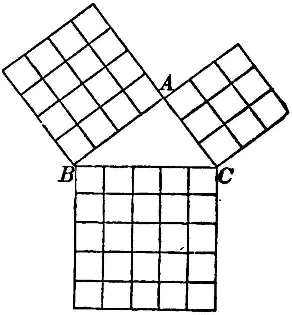


$$\overline{AB}^2 \asymp \overline{AC}^2 - \overline{BC}^2$$

or

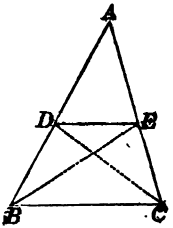
$$\overline{BC}^2 \asymp \overline{AC}^2 - \overline{AB}^2.$$

*Sch.* The last theorem may be illustrated by describing a square on the hypothenuse *BC*, equal to 5, also on the sides *BA*, *AC*, respectively equal to 4 and 3; and observing that the number of small squares in the large square is equal to the number in the two small squares.



### THEOREM XIII.

*If a line be drawn parallel to the base of a triangle, it will divide the other two sides proportionally.*



Let $ABC$ be any triangle, and $DE$ a straight line drawn parallel to the base $BC$: then will

$$AD : DB :: AE : EC.$$

For, draw $BE$ and $DC$. Then, the two triangles $BDE$ and $DCE$ have the same base $DE$, and the same altitude, since their vertices $B$ and $C$, lie in the line $BC$ parallel to $DE$: hence, they are equivalent (Th. ii).

Again, the triangles $ADE$ and $BDE$, have a common vertex $E$, and the same altitude; consequently, they are to each other as their bases (Th. ix. Cor.); hence, we have

$$ADE : BDE :: AD : DB.$$

But the triangles $ADE$ and $CDE$, having a common vertex $D$, are to each other as their bases $AE$ and $EC$: hence, we have

$$ADE : CDE :: AE : EC.$$

But the triangles $BDE$ and $CDE$ have been proved equivalent: hence, in the two proportions, the first antecedent and consequent in each are equal: therefore, by (Bk. III. Th. v) we have

$$AD : BD :: AE : EC.$$

*Cor.* The sides $AB$, $AC$, are also proportional to the parts $AD$, $AE$, or to $BD$, $CE$.

For, by composition (Bk. III. Th. vii), we have

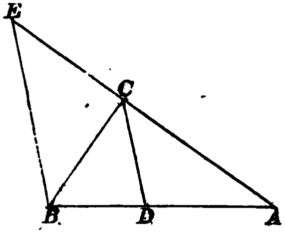
$$AD+BD : BD :: AE+EC : EC.$$

Then, by alternation (Bk. III. Th. iv).

$AB : AC :: BD : EC$, hence, also, $AB : AC :: AD : AE$

### THEOREM XIV.

*A line which bisects the vertical angle of a triangle divides the base into two segments which are proportional to the adjacent sides.*



Let $ACB$ be a triangle, having the angle $C$ bisected by the ine $CD$: then will

$AD : DB :: AC : CB$.

For, draw $BE$ parallel to $CD$ and produce $AC$ to $E$. Then, since $CB$ cuts the two parallels $CD$, $EB$, the alternate angles $BCD$ and $CBE$ are equal (Bk. I. Th. xii): hence, $CBE$ is equal to angle $ACD$.

But, since $AE$ cuts the two parallels $CD$, $BE$, the angle $ACD$ is equal to $CEB$ (Bk. I. Th. xiv): consequently, the angle $CBE$ is equal to the angle $CEB$ (Ax. 1): hence, the side $CB$ is equal to $CE$ (Bk. I. Th. vii).

Now, in the triangle $ABE$ the line $CD$ is drawn parallel to $BE$: hence, by the last theorem, we have

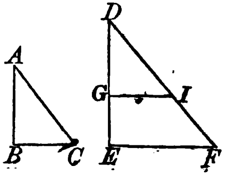
$$AD : DB :: AC : CE,$$

and by placing for $CE$, its equal $CB$, we have

$$AD : DB :: AC : CB.$$

### THEOREM XV

*Equiangular triangles have their sides proportional, and are similar.*



Let $ABC$ and $DEF$ be two equiangular triangles, having the angle $A$ equal to the angle $D$, the angle $C$ to the angle $F$, and the angle $B$ to the angle $E$: then will

$AB : AC :: DE . DF$,

For, on the sides of the larger triangle $DEF$, make $DI$ equal to $AC$ and $DG$ equal to $AB$, and join $IG$. Then the two triangles $ABC$ and $DIG$, having two sides and the included angle of the one equal to two sides and the included angle of the other, each to each, will be equal (Bk. I Th. iv) Hence, the angles $I$ and $G$ are equal to $C$ and $B$, and consequently, to the angles $F$ and $E$: therefore, $IG$ is parallel to $EF$ (Bk. I. Th. xiv, Cor. 1).

Now, in the triangle $DEF$, since $IG$ is parallel to the base, we have (Th. xiii).

$$DG : DI :: DE : DF,$$

that is, $$AB : AC :: DE : DF.$$

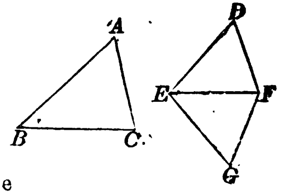
### THEOREM XVI.

*Two triangles which have their sides proportional are equiangular and similar.*

Let $BAC$ and $EDF$ be two triangles having

$$BC : EF :: AB : ED,$$

and $$BC : EF :: AC : DF;$$



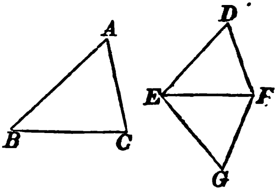
then will they have the corresponding angles equal, viz., the angle

$$B=E, \quad A=D \quad \text{and} \quad C=F.$$

For, at the point $E$ make $FEG$ equal to the angle $B$, and at $F$ make the angle $EFG$ equal to the angle $C$: Then will the angle at $G$ be equal to $A$, and the two triangles $BAC$ and $EGF$ will be equiangular (Bk. I. Th. xvii. Cor 1).

Therefore, by the last theorem we shall have

$$BC : EF :: AB : EG;$$



but by hypothesis,

$$BC : EF :: AB : DE:$$

hence, $EG$ is equal to $ED$.

By the last theorem we also have

$$BC : EF :: AC : FG.$$

and by hypothesis,

$$BC : EF :: AC : DF;$$

hence, $FG$ is equal to $DF$.

Therefore, the triangles $DEF$ and $EGF$, having their three sides equal, each to each, are equiangular (Bk. I. Th. viii). But, by construction, the triangle $EFG$ is equiangular with $BAC$: hence, the triangles $BAC$ and $EDF$ are equiangular, and consequently they are similar.

*Sch.* By Theorem XV, it appears that if the corresponding angles of two triangles are equal, each to each, the corresponding sides will be proportional; and in the last theorem it was proved that if the sides are proportional, the corresponding angles will be equal.

Now, these proportions do not hold good in the quadrilaterals. For, in the square and rectangle, the corresponding angles are equal, but the sides are not proportional; and the angles of a parallelogram or quadrilateral, may be varied at pleasure, without altering the lengths of the sides.

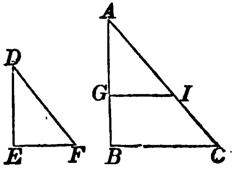
## THEOREM XVII.

*If two triangles have an angle in the one equal to an angle in the other, and the sides containing these angles proportional the two triangles will be equiangular and similar.*

Let $ABC$ and $DEF$ be two triangles having the angle $A$ equal to the angle $D$, and

$$AB : DE : AC : DF;$$

then will the two triangles be similar.



For, lay off $AG$ equal to $DE$, and through $G$ draw $GI$ parallel to $BC$. Then the angle $AGI$ will be equal to the angle $ABC$ (Bk. I. Th. xiv); and the triangles $AGI$ and $ABC$ will be equiangular. Hence, we shall have

$$AB : AG :: AC : AI.$$

But, by hypothesis, we have

$$AB : DE :: AC : DF,$$

and by construction, $AG$ is equal to $DE$; therefore, $AI$ is equal to $DF$, and consequently, the two triangles $AGI$ and $DEF$ are equal in all their parts (Bk. I. Th. iv). But the triangle $ABC$ is similar to $AGI$, consequently it is similar to $DEF$.

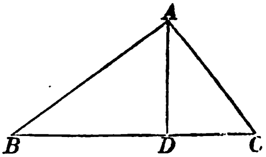
### THEOREM XVIII.

*If from the right angle of a right angled triangle, a perpendicular be let fall on the hypothenuse, then*

I. *The two partial triangles thus formed will be similar to each other and to the whole triangle.*

II. *Either side including the right angle will be a mean proportional between the hypothenuse and the adjacent segment.*

III. *The perpendicular will be a mean proportional between th segments of the hypothenuse.*



Let $ABC$ be a right angled triangle, and $AD$ perpendicular to the hypothenuse.

The two triangles $BAC$ and $BAD$ having the common angle $B$, and the right angle $BAC$ equal to the right angle at $D$, will be equiangular (Bk. I. Th. xvii Cor. 1); and, consequently, similar (Th. xv). For a like reason the triangles $BAC$ and $CAD$ are similar.

Now, from the triangles $BAC$ and $BAD$, we have

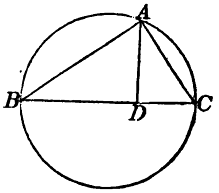
$$BC : BA :: BA : BD.$$

From the triangles $BAC$ and $CAD$, we have

$$BC : CA :: CA : CD;$$

and from the triangles $BAD$ and $DAC$, we have

$$BD : AD :: AD : DC.$$



*Cor.* If from a point $A$, in the circumference of a circle, $AD$ be drawn perpendicular to any diameter as $BC$, and the chords $AB$ $AC$ be also drawn, then the angle $BAC$ will be a right angle (Bk. II. Th. x): and by the theorem we shall have,

1st The perpendicular $AD$ a mean proportional between the segments $BD$ and $DC$.

2d Each chord will be a mean proportional between the diameter and the adjacent segment.

That is,

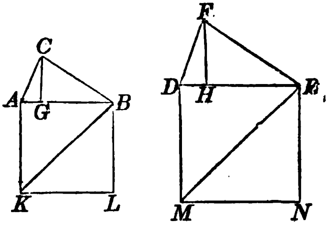
$$\overline{AD}^2 = BD \times DC$$
$$\overline{AB}^2 = BC \times BD$$
$$\overline{AC}^2 = BC \times CD.$$

### THEOREM XIX.

*Similar triangles are to each other as the squares described on their homologous sides*

Let *ABC* and *DEF* be two similar triangles, and *AL* and *DN* the squares described on the homologous sides *AB*, *DE*: then will the triangle

*ABC* : *DEF* : : *AL* : *DN*.



For, draw *CG* and *FH* perpendicular to the bases *AB*, *DE*, and draw the diagonals *BK* and *EM*.

Then, the similar triangles *ABC* and *DEF*, having their homologous sides proportional, we have

$$AC : DF :: AB : DE;$$

and the two *ACG*, *DFH*, give

$$AC : DF :: CG : FH;$$

hence, (Bk. III. Th. v), we have

$$AB : DE :: CG : FH,$$

or (Bk. III. Th. iv),

$$AB : CG :: DE : FH.$$

Now, the two triangles *ABC* and *AKB* have the common base *AB*; and the triangles *DEF* and *DEM* have the common base *DE*; and since triangles on equal bases are to each other as their altitudes (Th. ix, Cor.), we have

the triangle

$$ABC : ABK :: CG : AK \text{ or } AB$$

and the triangle,

$$DEF : DME :: FH : DM \text{ or } DE.$$

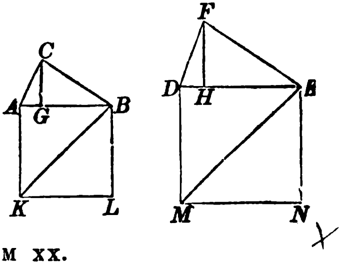
But we have proved

$$CG : AB :: FH : DE;$$

hence, $ABC : ABK :: DEF : DME$,

or, alternately,

$$ABC : DEF :: ABK : DME.$$



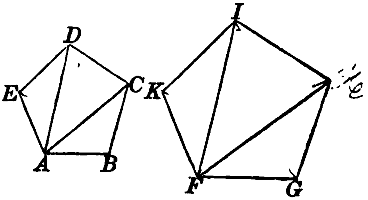
But the squares $AL$ and $DN$, being each double of the triangles $AKB$ and $DME$ have the same ratio; hence,

$$ABC : DEF :: AL : DN.$$

### THEOREM XX.

***Two similar polygons may be divided into an equal number of triangles, similar each to each, and similarly placed.***

Let $ABCDE$ and $FGHIK$ be two similar polygons.



From the angle $A$ draw the diagonals $AC$, $AD$: and from the homologous angle $F$, draw $FH$, $FI$.

Now, since the polygons are similar, the homologous angles $B$ and $G$ will be equal, and the sides about the equal angles proportional (Def. 1): that is,

$$AB : BC :: FG : GH.$$

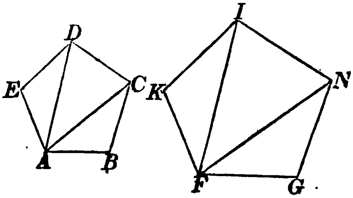
Hence, the triangles $ABC$ and $FGH$ have an angle in each equal, and the sides about the equal angles proportional: therefore, they are similar (Th. xvii), and consequently, the angle $ACB$ is equal to $FHG$. Taking these from the equal angles $BCD$ and $GHI$, there will remain $ACD$ equal to $FHI$. The two triangles $ACD$ and $FHI$ will then have an angle in each equal, and the sides about the equal angles proportional: hence, they will be similar.

In the same manner it may be shown that the triangles $AED$ and $FKI$ are similar: and, hence, whatever be the number of sides of the polygons, they may be divided into an equal number of similar triangles.

### THEOREM XXI.

*Similar polygons are to each other as the squares described on their homologous sides.*

Let $ABCDE$ and $FGNIK$, be two similar polygons; then will they be to each other as the squares described on $AB$, $FG$, or any other two homologous sides.



For, let the polygons be divided, as in the last theorem, into an equal number of similar triangles. Then, by Theorem XIX, we have the triangles

$$ABC : FGN :: \overline{AB}^2 : \overline{FG}^2$$

$$ADC : FIN :: \overline{DC}^2 \quad \overline{IN}^2$$

$$ADE : FIK :: \overline{DE}^2 : \overline{IK}^2$$

But since the polygons are similar, the ratio of the last antecedent to its consequent, in each of the proportions, is the same: hence, we have (Bk. III. Th. xii).

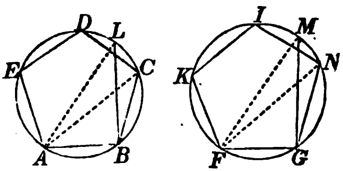
$$ABC+ADC+ADE : FGN+FIN+FIK :: \overline{AB}^2 : \overline{FG}^2;$$

that is, $ABCDE : FGNIK :: \overline{AB}^2 : \overline{FG}^2;$

Hence, the areas of similar polygons are to each other as the squares described on their homologous sides.

## THEOREM XXII.

*If similar polygons are inscribed in circles, their homologous sides, and also their perimeters, will have the same ratio to each other as the diameters of the circles in which they are inscribed.*



Let $ABCDE$, $FGNIK$, be two similar figures, inscribed in the circles whose diameters are $AL$ and $FM$: then will each side, $AB$, $BC$, &c., of the one, be to the homologous side $FG$, $GN$, &c., of the other, as the diameter $AL$ to the diameter $FM$. Also, the perimeter $AB+BC+CD$ &c., will be to the perimeter $FG+GN+NI$ &c., as the diameter $AL$ to the diameter $FM$

For, draw the two corresponding diagonals $AC$, $FN$, as also the lines $BL$ and $GM$.

Then, the two triangles $ACB$ and $FNG$ will be similar (Th. xx); and therefore, the angle $ACB$ is equal to the angle $FNG$. But, the angle $ACB$ is equal to the angle $ALB$, and the angle $FNG$ to the angle $FMG$ (Bk. II. Th. ix): hence, the angle $ALB$ is equal to the angle $FMG$ (Ax. 1); and since $ABL$ and $FGM$ are right angles (Bk. II. Th. x), the two triangles $ALB$ and $FMG$ will be equiangular (Bk. I. Th. xvii. Cor. 1), and consequently similar (Th. xv).

Therefore,

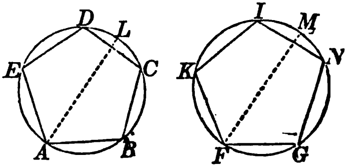
$$AB : FG :: AL : FM.$$

Again, since any two homologous sides are to each other in the same ratio as $AL$ to $FM$, we have (Bk. III. Th xii),

$$AB+BC+CD \text{ \&c.} : FG+GN+NI \text{ \&c.} :: AL : FM.$$

### THEOREM XXIII.

*Similar polygons inscribed in circles are to each other as the squares of the diameters of the circles.*



Let *ABCDE*, *FGNIK*, be two polygons inscribed in the circles whose diameters are *AL* and *FM*: then will the polygon *ABCDE*, be to the polygon *FGNIK* as the square of *AL* to the square of *FM*.

For, the polygons being similar, are to each other as the squares of their like sides (Th. xxi); that is, as $\overline{AB}^2$ to $\overline{FG}^2$

But, by the last theorem,

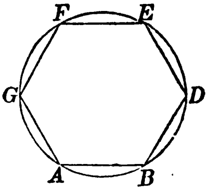
$$AB : FG :: AL : FM;$$

therefore (Bk III. Th. xiii),

$$\overline{AB}^2 : \overline{FG}^2 :: \overline{AL}^2 : \overline{FM}^2;$$

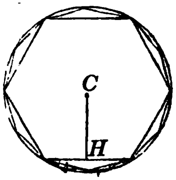
consequently,

$$ABCDE : FGNIK :: \overline{AL}^2 : \overline{FM}^2.$$



*Sch.* If any regular polygon, *ABDEFG*, be inscribed in a circle, and then the arcs *AB*, *BE*, &c., be bisected, and lines be drawn through these points of bisection, a new polygon will be formed having double the number of sides. It is plain that this new polygon will differ less from the circle than the first polygon, and its sides will lie nearer the circumference than the sides of the first polygon.

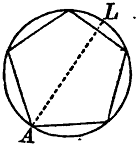
If now, we suppose the number of sides to be continually increased, the length of each side will constantly diminish until finally the polygon will become equal to the circle, and the perimeter will coincide with the circumference. When this takes place, the line *CH*, drawn perpendicular to one of the sides, will become equal to the radius of the circle.

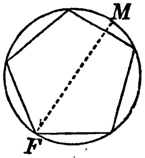


### THEOREM XXIV.

*The circumferences of circles are to each other as their diameters.*

Let there be two circles whose diameters are *AL* and *FM:* then will their circumferences be to each other as *AL* to *FM.*



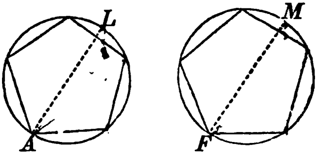


For, suppose two similar polygons to be inscribed in the circles: their perimeters will be to each other as *AL* to *FM* (Th. xxii).

Let us now suppose the arcs which subtend the sides of the polygons to be bisected, and new polygons of double the number of sides to be formed: their perimeters will still be to each other as *AL* to *FM,* and if the number of sides be increased until the perimeters coincide with the circumference, we shall have the circumferences to each other as the diameters *AL* and *FM.*

### THEOREM XXV.

*The areas of circles are to each other as the squares of their diameters.*



Let there be two circles whose diameters are $AL$ and $FM$: then will their areas be to each other as the square of $AL$ to the square of $FM$.

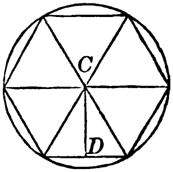
For, suppose two similar polygons to be inscribed in the circles: then will they be to each other as $\overline{AL}^2$ to $\overline{FM}^2$ (Th. xxiii).

Let us now suppose the number of sides of the polygons to be increased, by bisecting the arcs, until their perimeters shall coincide with the circumferences of the circles. The polygons will then become equal to the circles, and hence, the areas of the circles will be to each other as the squares of their diameters.

*Cor.* Since the circumferences of circles are to each other as their diameters (Th. xxiv), it follows, that the areas which are proportional to the squares of the diameters, will also be proportional to the squares of the circumferences.

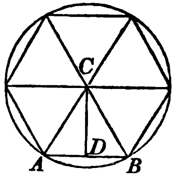
### THEOREM XXVI.

*The area of a regular polygon inscribed in a circle, is equal to half the product of the perimeter and the perpendicular let fall from the centre on one of the sides.*



Let $C$ be the centre of a circle circumscribing the regular polygon, and $CD$ a perpendicular to one of its sides: then will its area be equal to half the product of $CD$ by the perimeter.

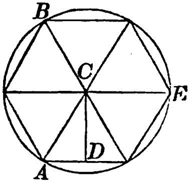
For, from $C$ draw radii to the vertices of the angles, forming as many



equal triangles as the polygon has sides, in each of which the perpendicular on the base will be equal to *CD*. Now, the area of one of them, as *ACB*, will be equal to half the product of *CD* by the base *AB*; and the same will be true for each of the other triangles: hence, the area of the polygon will be equal to half the product of *CD* by the perimeter

### THEOREM XXVII.

*The area of a circle is equal to half the product of the radius by the circumference.*



Let *C* be the centre of a circle: then will its area be equal to half the product of the radius *AC* by the circumference *ABE*.

For, inscribe within the circle a regular hexagon, and draw *CD* perpendicular to one of its sides. Then, the area of the polygon will be equal to half the product of *CD* multiplied by the perimeter (Th. xxvi).

Let us now suppose the number of sides of the polygon to be increased, until the perimeter shall coincide with the circumference; the polygon will then become equal to the circle, and the perpendicular *CD* to the radius *CA*. Hence, the area of the circle will be equal to half the product of the radius by the circumference.

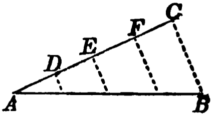
# PROBLEMS

## RELATING TO THE FOURTH BOOK.

### PROBLEM I.

*To divide a line into any proposed number of equal parts.*

Let $AB$ be the line, and let it be required to divide it into four equal parts.

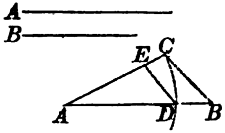


Draw any other line, $AC$, forming an angle with $AB$, and take any distance, as $AD$, and lay it off four times on $AC$. Join $C$ and $B$, and through the points $D$, $E$, and $F$, draw parallels to $CB$. These parallels to $BC$ will divide the line $AB$ into parts proportional to the divisions on $AC$ (Th. xiii): that is, into equal parts.

### PROBLEM II.

*To find a third proportional to two given lines.*

Let $A$ and $B$ be the given lines.



Make $AB$ equal to $A$, and draw $AC$, making an angle with it. On $AC$ lay off $AC$ equal to $B$, and join $BC$: then lay off $AD$, also equal to $B$, and through $D$ draw $DE$ parallel to $BC$: then will $AE$ be the third proportional sought.

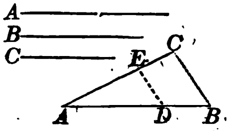
For, since $DE$ is parallel to $BC$, we have (Th. xiii)

$$AB \ : \ AC \ : : \ AD \text{ or } AC \ : \ AE;$$

therefore, $AE$ is the third proportional sought.

### PROBLEM III.

*To find a fourth proportional to the lines $A$, $B$, and $C$.*



Place two of the lines forming an angle with each other at $A$; that is, make $AB$ equal to $A$, and $AC$ equal $B$; also, lay off $AD$ equal to $C$. Then join $BC$, and through $D$ draw $DE$ parallel to $BC$, and $AE$ will be the fourth proportional sought.

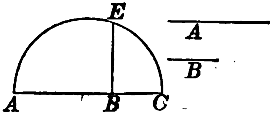
For, since $DE$ is parallel to $BC$, we have

$$AB : AC :: AD : AE;$$

therefore, $AE$ is the fourth proportional sought.

### PROBLEM IV.

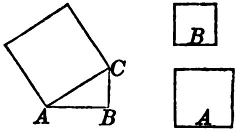
*To find a mean proportional between two given lines, $A$ and $B$.*



Make $AB$ equal to $A$, and $BC$ equal to $B$: on $AC$ describe a semicircle. Through $B$ draw $BE$ perpendicular to $AC$, and it will be the mean proportional sought (Th. xviii. Cor).

### PROBLEM V.

*To make a square which shall be equivalent to the sum of two given squares.*

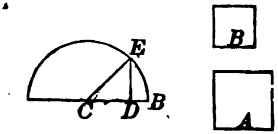


Let $A$ and $B$ be the sides of the given squares.

Draw an indefinite line $AB$, and make $AB$ equal to $A$. At $B$ draw $BC$ perpendicular to $AB$, and make $BC$ equal to $B$: then draw $AC$ and the square described on $AC$ will be equivalent to the squares on $A$ and $B$ (Th. xii).

### PROBLEM VI.

*To make a square which shall be equivalent to the difference between two given squares.*



Let $A$ and $B$ be the sides of the given squares.

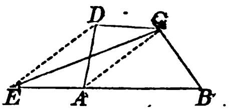
Draw an indefinite line, and make $CB$ equal to $A$, and $CD$ equal to $B$. At $D$ draw $DE$ perpendicular to $CB$, and with $C$ as a centre, and $CB$ as a radius, describe a semicircle meeting $DE$ in $E$, and join $CE$: then will the square described on $ED$ be equal to the difference between the given squares.

For, $CE$ is equal to $CB$, that is, equal to $A$, and $CD$ is equal to $B$: and by (Th. xii. Cor.),

$$\overline{ED}^2=\overline{CE}^2-\overline{CD}^2.$$

### PROBLEM VII.

*To make a triangle which shall be equivalent to a given quadrilateral.*



Let $ABCD$ be the given quadrilateral.

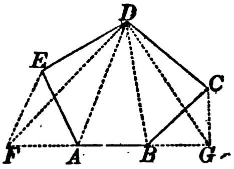
Draw the diagonal $AC$, and through $D$ draw $DE$ parallel to $AC$, meeting $BA$ produced at $E$. Join $EC$: then will the triangle $CEB$ be equivalent to the quadrilateral $BD$.

For, the two triangles $ACE$ and $ADC$, having the same base $AC$, and the vertices of the angles $D$ and $E$ in the same line $DE$ parallel to $AC$, are equivalent (Th. ii). If to each, we add $ACB$, we shall then have the triangle $ECB$ equivalent to the quadrilateral $BD$ (Ax. 2).

### PROBLEM VIII.

*To make a triangle which shall be equivalent to a given polygon.*

Let *ABCDE* be the polygon.



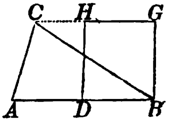
Draw the diagonals *AD*, *BD*. Produce *AB* in both directions, and through *C* and *E* draw *CG* and *EF*, respectively parallel to *AD* and *BD*: then join *FD* and *DG*, and the triangle *FDG* will be equivalent to the polygon *ABCDE*.

For, the triangle *AED* is equivalent to the triangle *AFD*, and *DBC* to *DBG* (Th. ii); and by adding *ADB* to the equals, we shall have the triangle *FDG* equivalent to the polygon *ABCDE*.

### PROBLEM IX.

*To make a rectangle that shall be equivalent to a given triangle.*

Let *ABC* be the given triangle.

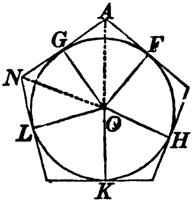


Bisect the base *AB* at *D*, and draw *DH* perpendicular to *AB*. Through *C*, the vertex of the triangle, draw *CHG* parallel to *AB*, and draw *BG* perpendicular to it: then will the rectangle *DG* be equivalent to the triangle *ABC*.

For, the triangle would be half a rectangle having the same base and altitude: hence, it is equivalent to *DG*, whose base is the half of *AB*, and altitude equal to that of the triangle.

### PROBLEM X.

*To inscribe a circle in a regular polygon.*



Bisect any two sides of the polygon by the perpendiculars $GO$, $FO$, and with their point of intersection $O$, as a centre, and $OG$ as a radius describe the circumference of a circle—this circle will touch all the sides of the polygon.

For, draw $OA$. Then in the two right angled triangles $OAG$ and $OAF$, the side $AO$ is common, and $AG$ is equal to $AF$, since each is half of one of the equal sides of the polygon: hence, $OG$ is equal to $OF$ (Bk. I. Th. xix). In the same manner it may be shown that $OH$, $OK$ and $OL$ are all equal to each other: hence, a circle described with the centre $O$ and radius $OF$ will be inscribed in the polygon.

*Cor.* Hence, also the lines $OA$, $ON$ &c., drawn to the angles of the polygon are equal.

## APPENDIX

### OF THE REGULAR POLYGONS.

1. In a regular polygon the angles are all equal to each other (Def. 3). If then, the sum of the inward angles of a regular polygon be divided by the number of angles, the quotient will be the value of one of the angles.

But the sum of the inward angles is equal to twice as many right angles, wanting four, as the polygon has sides, and we shall find the value in degrees by simply placing 90° for the right angle.

2. Thus, for the sum of all the angles of an equilateral triangle, we have

$$6\times 90^\circ-4\times 90^\circ=540^\circ-360^\circ=180^\circ$$

and for each angle

$$180^\circ\div 3=60^\circ:$$

Hence, each angle of an equilateral triangle, is equal to 60 degrees.

3. For the sum of all the angles of a square, we have

$$8\times 90^\circ-4\times 90^\circ=720^\circ-360^\circ=360^\circ,$$

and for each of the angles

$$360^\circ\div 4=90^\circ$$

4. For the sum of all the angles of a regular pentagon, we have

$$10\times 90^\circ-4\times 90^\circ=900^\circ-360^\circ=540^\circ,$$

and for each angle

$$540^\circ\div 5=108^\circ.$$

5. For the sum of all the angles of a regular hexagon, we have

$$12\times 90^\circ-4\times 90^\circ=1080^\circ-360^\circ=720^\circ,$$

and of each angle

$$720^\circ\div 6=120^\circ.$$

6. For the sum of the angles of a regular heptagon, we have

$$14\times 90^\circ-4\times 90^\circ=1260^\circ-360^\circ=900^\circ:$$

and for one of the angles

$$900^\circ\div 7=128^\circ\ 34'+.$$

7. For the sum of the angles of a regular octagon, we have

$$16\times 90^\circ-4\times 90^\circ=1440^\circ-360^\circ=1080^\circ:$$

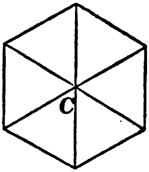
and for each angle

$$1080^\circ\div 8=135^\circ$$

8. Since the sum of the angles about any point is equal to four right angles (Bk. I. Th. ii. Cor. 3), it may be observed that there are only three kinds of regular polygons, which can be arranged around any point, as *C*, so as exactly to fill up the space. These are,

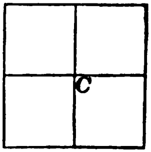
*First.*—Six equilateral triangles, in which each angle about *C* is equal to 60°, and their sum to

60°×6=360.



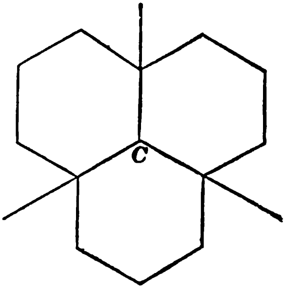
*Second.*—Four squares, in which each angle is equal to 90°, and their sum to

90°×4=360°



*Third.*—Three hexagons, in which each angle is equal to 120, and the sum of the three to

20°×3=360°.



# GEOMETRY.

---

## BOOK V.

### OF PLANES AND THEIR ANGLES.

#### DEFINITIONS.

1. A straight line is *perpendicular to a plane*, when it is perpendicular to every straight line of the plane which it meets. The point at which the perpendicular meets the plane, is called the *foot* of the perpendicular.

2. If a straight line is perpendicular to a plane, the plane is also said to be perpendicular to the line.

3. A line is parallel to a plane when it will not meet that plane, to whatever distance both may be produced. Conversely, the plane is then parallel to the line.

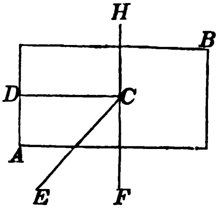
4. Two planes are parallel to each other, when they will not meet, to whatever distance both are produced.

5. If two planes are not parallel, they intersect each other in a line that is common to both planes: such line is called their *common intersection*.

6. The space included between two planes is called a *diedral angle:* the planes are the *faces* of the angle, and their intersection the *edge.* A diedral angle is measured by two lines, one in each plane, and both perpendicular to the common intersection at the same point.

This angle may be acute, obtuse, or a right angle. When it is a right angle, the planes are said to be perpendicular to each other.

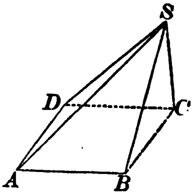
Let $AB$ be a plane coinciding with the plane of the paper, and $ECF$ a plane intersecting it in the line $FH$. Now, if from any point of the common intersection as $C$, we draw $CD$ in the plane $AB$, and $CE$ in the plane $ECF$, and both perpendicular to $CF$ at $C$, then will the angle $DCE$ measure the inclination between the two planes.



It should be remembered that the line $EC$ is directly over the line $CD$.

7. A polyedral angle is the angular space included between several planes meeting at the same point.

Thus, the polyedral angle $S$ is formed by the meeting of the planes $ASB$, $BSC$, $CSD$, $DSA$.

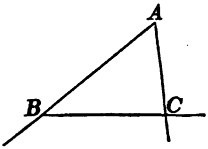


8. The angle formed by three planes is called a *triedral angle.*

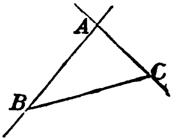
### THEOREM I.

*Two straight lines which intersect each other, lie in the same plane, and determine its position.*

Let $AB$ and $AC$ be two straight lines which intersect each other at $A$.

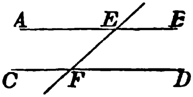


Through $AB$ conceive a plane to be passed, and let this plane be turned around $AB$ until it embraces the point $C$: the plane will then contain the two lines $AB$, $AC$, and if it be turned either way it will depart from the point $C$, and consequently from the line $AC$. Hence, the position of the plane is determined by the single condition of containing the two straight lines $AB$, $AC$.



*Cor.* 1. A triangle $ABC$, or three points $A$, $B$, $C$, not in a straight line, determine the position of a plane.

*Cor.* 2. Hence, also, two parallels $AB$, $CD$ determine the position of a plane. For drawing $EF$, we see that the plane of the two straight lines $AE$, $EF$ is that of the parallels $AB$, $CD$.

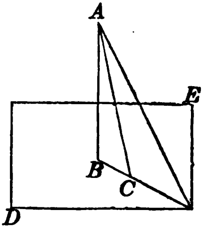


### THEOREM II.

*A perpendicular is the shortest line which can be drawn from a point to a plane.*

Let $A$ be a point above the plane $DE$, and $AB$ a line drawn perpendicular to the plane: then will $AB$ be shorter than any oblique line $AC$.

For, through $B$, the foot of the perpendicular, draw $BC$ to the point where the oblique line $AC$ meets the plane.



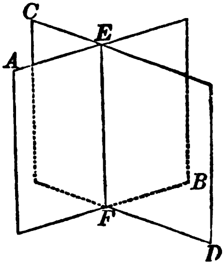
Now, since $AB$ is perpendicular to the plane, the angle $ABC$ will be a right angle (Def. 1.), and consequently less than the angle $C$: therefore, $AB$, opposite the angle $C$, will be less than $AC$, opposite the angle $B$ (Bk. I. Th. xi).

*Cor.* It is evident that if several lines be drawn from the point $A$ to the plane, that those which are nearest the perpendicular $AB$, will be less than those more remote.

*Sch.* The distance from a point to a plane is measured on the perpendicular: hence, when the *distance* only is named, the shortest distance is always understood.

### THEOREM III.

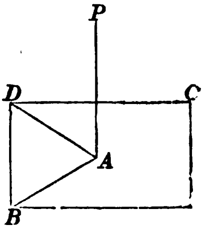
*The common intersection of two planes is a straight line.*



Let the two planes $AB$, $CD$, cut each other. Join any two points $E$ and $F$, in the common intersection, by the straight line $EF$. This line will lie wholly in the plane $AB$, and also wholly in the plane $CD$ (Bk. I. Def. 7); therefore, it will be in both planes at once, and consequently, is their common intersection.

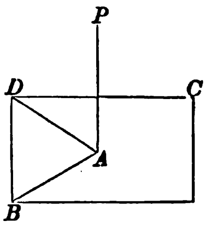
### THEOREM IV.

*A straight line which is perpendicular to two straight lines at their point of intersection, will be perpendicular to the plane of those lines.*



Let the line $PA$ be perpendicular to the two lines $AD$, $AB$: then will it be perpendicular to the plane $BC$ which contains them.

For, if $AP$ is not perpendicular to the plane $BC$, suppose a plane to be drawn through $A$, that shall be perpendicular to $AP$.

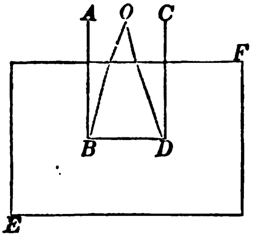


Now, every line drawn through $A$, and perpendicular to $AP$, will be a line of this last plane (Def. 1): hence, this last plane will contain the lines $AB$, $AD$, and consequently, a line which is perpendicular to two lines at the point of intersection, will be perpendicular to the plane of those lines.

## THEOREM V.

*If two straight lines are perpendicular to the same plane they will be parallel to each other.*

Let the two lines $AB$, $CD$, be perpendicular to the plane $EF$: then will they be parallel to each other.



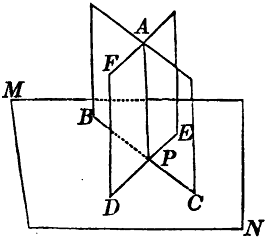
For, join $B$ and $D$, the points in which the lines meet the plane $EF$.

Then, because the lines $AB$, $CD$, are perpendicular to the plane $EF$, they will be perpendicular to the line $BD$ (Def. 1). Now, if $BA$ and $DC$ are not parallel, they will meet at some point as $O$: then, the triangle $OBD$ would have two right angles, which is impossible (Bk. I. Th. xvii. Cor. 4).

*Cor.* If two lines are parallel, and one of them is perpendicular to a plane, the other will also be perpendicular to the same plane.

### THEOREM VI.

*If two planes intersect each other at right angles, and a line be drawn in one plane perpendicular to the common intersection, this line will be perpendicular to the other plane.*

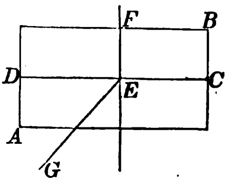


Let the plane $FE$ be perpendicular to $MN$, and $AP$ be drawn in the plane $FE$, and perpendicular to the common intersection $DE$: then will $AP$ be perpendicular to the plane $MN$.

For, in the plane $MN$ draw $CP$ perpendicular to the common intersection $DE$. Then, because the planes $MN$ and $FE$ are perpendicular to each other, the angle $APC$, which measures their inclination, will be a right angle (Def. 6). Therefore, the line $AP$ is perpendicular to the two straight lines $PC$ and $PD$; hence, it is perpendicular to their plane $MN$ (Th. iv).

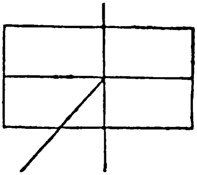
### THEOREM VII.

*If one plane intersect another plane, the sum of the angles on the same side will be equal to two right angles.*



Let the plane $GEF$ intersect the plane $AB$ in the line $FE$: then will the sum of the two angles on the same side be equal to two right angles.

For, from any point, as $E$, in the common intersection, draw the lines $EG$ and $DEC$, one in each plane, and both perpendicular to the common intersection at $E$. Then, the line $GE$ makes, with the line $DEC$, two angles, which together are equal to two right angles (Bk I. Th. ii): but these angles measure the inclination of the planes; therefore, the sum of the angles on the same side, which two planes make with each other, is equal to two right angles.

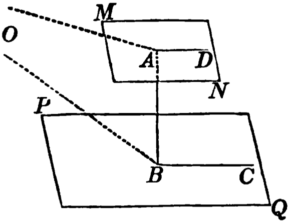


*Cor.* In like manner it may be demonstrated, that planes which intersect each other have their vertical or opposite angles equal.

### THEOREM VIII.

*Two planes which are perpendicular to the same straight line are parallel to each other.*

Let the planes *MN* and *PQ* be perpendicular to the line *AB*: then will they be parallel.

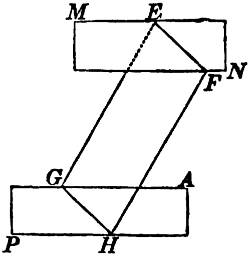


For, if they can meet any where, let *O* be one of their their common points, and draw *OB*, in the plane *PQ*, and *OA*, in the plane *MN*.

Now, since *AB* is perpendicular to both planes, it will be perpendicular to *OB* and *OA* (Def. 1): hence, the triangle *OAB* will have two right angles, which is impossible (Bk. I. Th. xvii. Cor. 4); therefore, the planes can have no point, as O, in common, and consequently, they are parallel (Def. 4).

### THEOREM IX.

*If a plane cuts two parallel planes, the lines of intersection will be parallel.*

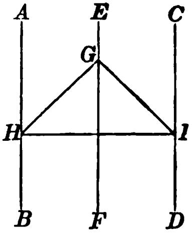


Let the parallel planes *MN* and *PA* be intersected by the plane *EH:* then will the lines of intersection *EF*, *GH*, be parallel.

For, if the lines *EF*, *GH*, were not parallel, they would meet each other if sufficiently produced, since they lie in the same plane. If this were so, the planes *MN*, *PA*, would meet each other, and, consequently, could not be parallel which would be contrary to the supposition.

### THEOREM X.

*If two lines are parallel to a third line, though not in the same plane with it, they will be parallel to each other.*



Let the lines *AB* and *CD* be each parallel to the third line *EF*, though not in the same plane with it: then will they be parallel to each other.

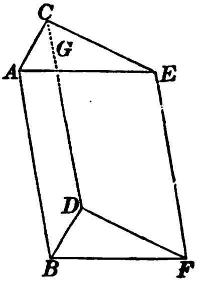
For, since *EF* and *CD* are parallel, they will lie in the same plane *FC* (Th. i. Cor. 2), and *AB*, *EF* will also lie in the plane *EB*.

At any point, *G*, in the line *EF*, let *GI* and *GH* be drawn in the planes *FC*, *BE*, and each perpendicular to *FE* at *G*.

Then, since the line *EF* is perpendicular to the lines *GH* *GI*, it will be perpendicular to the plane *HGI* (Th. iv). And since *FE* is perpendicular to the plane *HGI*, its parallels *AB* and *DC* will also be perpendicular to the same plane (Th. v). Hence, since the two lines *AB*, *CD*, are both perpendicular to the plane *HGI*, they will be parallel to each other.

### THEOREM XI.

*If two angles, not situated in the same plane, have their sides parallel and lying in the same direction, the angles will be equal.*



Let the angles $ACE$ and $BDF$ have the sides $AC$ parallel to $BD$, and $CE$ to $DF$: then will the angle $ACE$ be equal to the angle $BDF$.

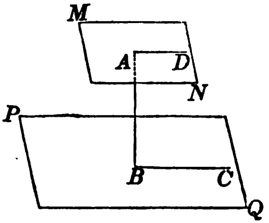
For, make $AC$ equal to $BD$, and $CE$ equal to $DF$, and join $AB$, $CD$, and $EF$; also, draw $AE$, $BF$.

Now since $AC$ is equal and parallel to $BD$, the figure $AD$ will be a parallelogram (Bk. I. Th. xxv); therefore, $AB$ is equal and parallel to $CD$.

Again, since $CE$ is equal and parallel to $DF$, $CF$ will be a parallelogram, and $EF$ will be equal and parallel to $CD$. Then, since $AB$ and $EF$ are both parallel to $CD$, they will be parallel to each other (Th. x); and since they are each equal to $CD$, they will be equal to each other. Hence, the figure $BAEF$ is a parallelogram (Bk. I. Th. xxv), and consequently, $AE$ is equal to $BF$. Hence, the two triangles $ACE$ and $BDF$ have the three sides of the one equal to the three sides of the other, each to each, and therefore the angle $ACE$ is equal to the angle $BDF$ (Bk. I. Th. viii).

### THEOREM XII.

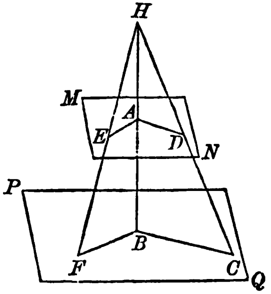
*If two planes are parallel, a straight line which is perpendicular to the one will also be perpendicular to the other.*



Let *MN* and *PQ* be two parallel planes, and let *AB* be perpendicular to *MN:* then will it be perpendicular to *PQ*.

For, draw any line, *BC*, in the plane *PQ*, and through the lines *AB*, *BC*, suppose the plane *ABC* to be drawn, intersecting the plane *MN* in the line *AD:* then, the intersection *AD* will be parallel to *BC* (Th. ix). But since *AB* is perpendicular to the plane *NM*, it will be perpendicular to the straight line *AD*, and consequently, to its parallel *BC* (Bk. I. Th. xii. Cor.)

In like manner, *AB* might be proved perpendicular to any other line of the plane *PQ*, which should pass through *B;* hence, it is perpendicular to the plane (Def. 1).



*Cor.* If from any point as *H*, any oblique lines, as *HEF*, *HDC*, be drawn, the parallel planes will cut these lines proportionally.

For, draw *HAB* perpendicular to the plane *MN:* then, by the theorem, it will also be perpendicular to *PQ*. Then draw *AD*, *AE*, *BC*, *BF*. Now, since *AE*, *BF*, are the intersections of the plane *FHB*, with the two parallel planes *MN*, *PQ*, they are parallel (Th. ix.); and so also are *AD*, *BC*.

Then, $HA : HB : HE : HF$,

and $HA : HB : HD : HC$,

hence, $HE : HF :: HD : HC$.

# GEOMETRY.

## BOOK VI.

### OF SOLIDS.

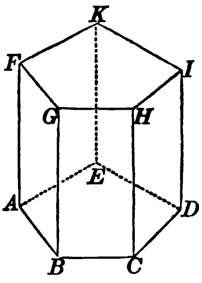
### DEFINITIONS.

1 Every solid bounded by planes is called a *polyedron.*

2. The planes which bound a polyedron are called *faces.* The straight lines in which the faces intersect each other, are called the *edges* of the polyedron, and the points at which the edges intersect, are called the *vertices* of the angles, or vertices of the polyedron.

3. Two polyedrons are similar, when they are contained by the same number of similar planes, and have their polyedral angles equal, each to each.

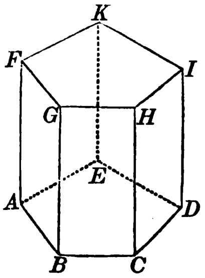
4. A prism is a solid, whose ends are equal polygons, and whose side faces are parallelograms.



Thus, the prism whose lower base is the pentagon *ABCDE*, terminates in an equal and parallel pentagon *FGHIK*, which is called the *upper base.* The side faces of the prism are the parallelograms *DH*, *DK*, *EF*, *AG*, and *BH.* These are called the *convex*, or *lateral* surface of the prism

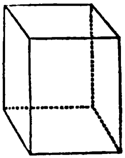
5. The altitude of a prism is the distance between its upper and lower bases: that is, it is a line drawn from a point of the upper base, perpendicular, to the lower base.

6, A right prism is one in which the edges *AF*, *BG*, *EK*, *HC*, and *DI*, are perpendicular to the bases. In the right prism, either of the perpendicular edges is equal to the altitude. In the oblique prism the altitude is less than the edge.

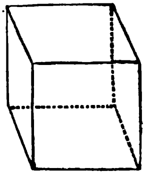


7. A prism whose base is a triangle, is called a *triangular* prism; if the base is a quadrangle, it is called a quadrangular prism; if a pentagon, a pentagonal prism; if a hexagon a hexagonal prism; &c.

8. A prism whose base is a parallelogram, and all of whose faces are also parallelograms, is called a parallelopipedon. If all the faces are rectangles, it is called a rectangular parallelopipedon.

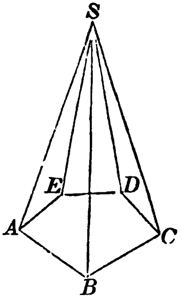


9. If the faces of the rectangular parallelopipedon are squares, the solid is called a *cube:* hence, the cube is a prism bounded by six equal squares



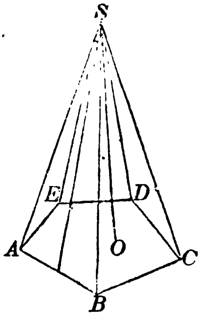
10. A pyramid is a solid, formed by several triangles united at the same point $S$, and terminating in the different sides of a polygon $ABCDE$.

The polygon $ABCDE$, is called the *base* of the pyramid; the point $S$, is called the *vertex*, and the triangles $ASB$, $BSC$, $CSD$, $DSE$, and $ESA$. form its *lateral*, or *convex* surface.



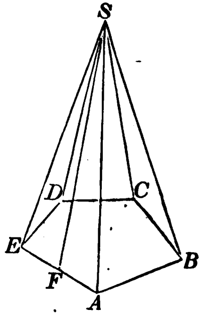
11. A pyramid whose base is a triangle, is called a ***triangular*** pyramid; if the base is a quadrangle, it is called a quadrangular pyramid; if a pentagon, it is called a petagonal pyramid; if the base is a hexagon, it is called a hexagonal pyramid; &c.

12. The ***altitude*** of a pyramid, is the perpendicular let fall from the vertex, upon the plane of the base. Thus, $SO$ is the altitude of the pyramid $S—ABCDE$.



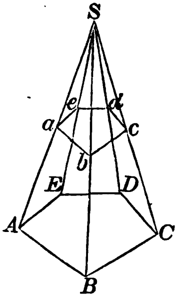
13. When the base of a pyramid is a regular polygon, and he perpendicular $SO$ passes through he middle point of the ase, the pyramid is called a right pyramid, and the line $SO$ is called the *axis*.

14. The *slant height* of a right pyramid, is a line drawn from the vertex, perpendicular to one of the sides of the polygon which forms its base. Thus, *SF* is the slant height of the pyramid *S—ABCDE*.



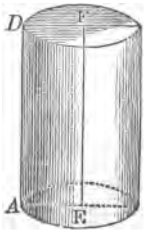
15. If from the pyramid *S—ABCDE* the pyramid S—*abcde* be cut off by a plane parallel to the base, the remaining solid, below the plane, is called the *frustum* of a pyramid.

The altitude of a frustum is the perpendicular distance between the upper and lower planes.



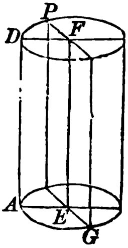
16. A *Cylinder* is a solid, described by the revolution of a rectangle, *AEFD*, about a fixed side, *EF*.

As the rectangle *AEFD*, turns around the side *EF*, like a door upon its hinges, the lines *AE* and *FD* describe circles, and the line *AD* describes the *convex surface* of the cylinder.



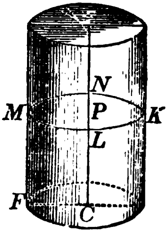
The circle described by the line *AE*, is called the *lower base* of the cylinder, and the circle described by *DF*, is called the *upper base*.

The immovable line *EF* is called the axis of the cylinder A cylinder, therefore, is a round body with circular ends

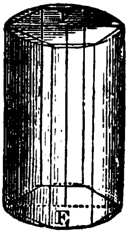


17. If a plane be passed through the axis of a cylinder, it will intersect the cylinder in a rectangle, *PG*, which is double the revolving rectangle *DE.*

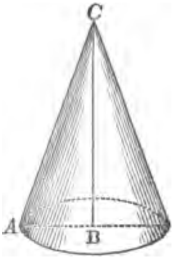
18. If a cylinder be cut by a plane parallel to the base, the section will be a circle equal to the base. For, while the side *FC*, of the rectangle *MC*, describes the lower base, the equal side *MP*, will describe the circle *MLKN*, equal to the lower base.



19. If a polygon be inscribed in the lower base of a cylinder, and a corresponding polygon be inscribed in the upper base, and their vertices be joined by straight lines, the prism thus formed is said to be *inscribed* in the cylinder.



20. A *cone* is a solid, described by the revolution of a right angled triangle, *ABC*, about one of its sides, *CB*.

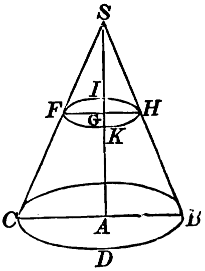


The circle described by the revolving side, *AB*, is called the *base* of the cone.

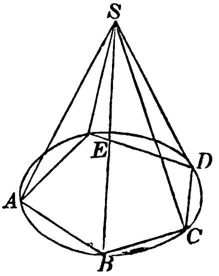
The hypothenuse, *AC*, is called the *slant height* of the cone, and the surface described by it, is called the *convex surface* of the cone.

The side of the triangle, *CB*, which remains fixed, is called the *axis*, or *altitude* of the cone, and the point *C*, the *vertex* of the cone.

21. If a cone be cut by a plane parallel to the base, the section will be a circle. For, while in the revolution of the right angled triangle *SAC*, the line *CA* describes the base of the cone, its parallel *FG* will describe a circle *FKHI*, parallel to the base. If from the cone *S—CDB*, the cone *S—FKH* be taken away, the remaining part is called the *frustum* of the cone.



22. If a polygon be inscribed in the base of a cone, and straight lines be drawn from its vertices to the vertex of the cone, the pyramid thus formed is said to be inscribed in the cone. Thus, the pyramid *S—ABCD* is inscribed in the cone.

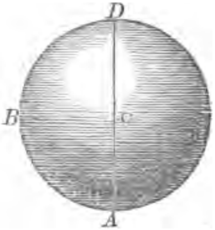


23. Two cylinders are similar, when the diameters of their bases are proportional to their altitudes.

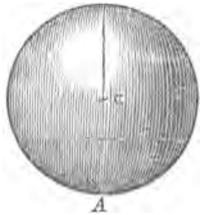
24. Two cones are also similar, when the diameters of their bases are proportional to their altitudes.

25. A *sphere* is a solid terminated by a curved surface, all the points of which are equally distant from a certain point within called the centre.

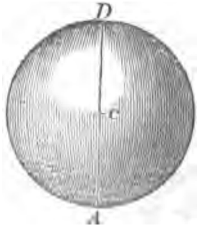
26. The sphere may be described by revolving a semicircle, *ABD*, about the diameter *AD*. The plane will describe the solid sphere, and the semicircumference *ABD* will describe the surface.



27. The *radius* of a sphere is a line drawn from the centre to any point of the circumference. Thus, *CA* is a radius.



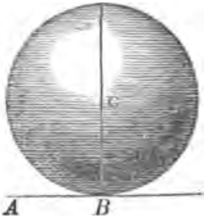
28. The *diameter* of a sphere is a line passing through the centre, and terminated by the circumference. Thus, *AD* is a diameter.



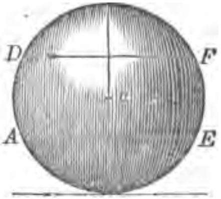
29. All diameters of a sphere are equal to each other; and each is double a radius.

30. The axis of a sphere is any line about which it revolves; and the points at which the axis meets the surface re called the *poles*.

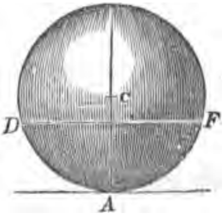
31. A plane is *tangent* to a sphere when it has but one point in common with it. Thus, *AB* is a tangent plane, touching the sphere at *B*.



32. A *zone* is a portion of the surface of a sphere, included between two parallel planes which form its bases. Thus, the part of the surface included between the planes *AE* and *DF* is a zone. The bases of this zone are the two circles whose diameters are *AE* and *DF*.



33. One of the planes which bound a zone may become tangent to the sphere; in which case the zone will have but one base. Thus, if one plane be tangent to the sphere at *A*, and another plane cut it in the circle *DF*, the zone included between them, will have but one base.



34. A *spherical segment* is a portion of the solid sphere included between two parallel planes. These parallel planes are its bases. If one of the planes is tangent to the sphere, the segment will have but one base.

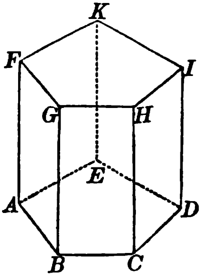
35. The *altitude* of a zone or segment, is the distance between the parallel planes which form its bases.

### THEOREM I.

*The convex surface of a right prism is equal to the perimeter of its base multiplied by its altitude.*

Let $ABCDE—K$ be a right prism: then will its convex surface be equal to

$$(AB+BC+CD+DE+EA)\times AF.$$



For, the convex surface is equal to the sum of the rectangles $AG$, $BH$, $CI$, $DK$, and $EF$, which compose it; and the area of each rectangle is equal to the product of its base by its altitude. But the altitude of each rectangle is equal to the altitude of the prism: hence, their areas, that is, the convex surface of the prism, is equal to

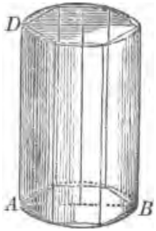
$$(AB+BC+CD+DE+EA)\times AF;$$

that is, equal to the perimeter of the base of the prism multiplied by its altitude.

### THEOREM II.

*The convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude.*

Let *DB* be a cylinder, and *AB* the diameter of its base: the convex surface will then be equal to the altitude *AD* multiplied by the circumference of the base.

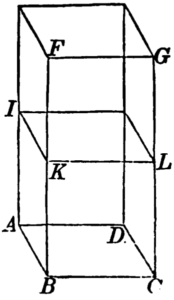


For, suppose a regular prism to be inscribed within the cylinder. Then, the convex surface of the prism will be equal to the perimeter of the base multiplied by the altitude (Th. i). But the altitude of the prism is the same as that of the cylinder; and if we suppose the sides of the polygon, which forms the base of the prism, to be indefinitely increased, the polygon will become the circle (Bk. IV. Th. xxiii. Sch.), in which case, its perimeter will become the circumference, and the prism will coincide with the cylinder. But its convex surface is still equal to the perimeter of its base multiplied by its altitude: hence, the convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude.

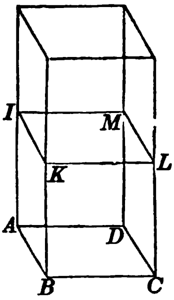
### THEOREM III.

*In every prism the sections formed by planes parallel to the base are equal polygons.*

Let *AG* be any prism, and *IL* a section made by a plane parallel to the base *AC*: then will the polygon *IL* be equal to *AC*.



For, the two planes *AC*, *IL*, being parallel, the lines *AB*, *IK*, in which they intersect the plane *AF*, will also be parallel (Bk. V. Th. ix). For a like reason, *BC* and *KL* will be parallel; also, *CD* will be parallel to *LM*, and *AD* to *IM*.



But, since *AI* and *BK* are parallel, the figure *AK* is a parallelogram: hence *AB* is equal to *IK* (Bk. I. Th. xxiii). In the same way it may be shown that *BC* is equal to *KL*, *CD* to *LM*, and *AD* to *IM*.

But, since the sides of the polygon *AC* are respectively parallel to the sides of the polygon *IL*, it follows that their corresponding angles are equal (Bk. V. Th. xi), viz., the angle *A* to the angle *I*, the angle *B* to *K*, the angle *C* to *L*, and the angle *M* to *D*; hence, the polygon *IL* is equal to *AC*.

*Sch.* It was shown in Definition 18, that the section of a cylinder, by a plane parallel to the base, is a circle equal to the base.

### THEOREM IV.

*If a pyramid be cut by a plane parallel to the base,*

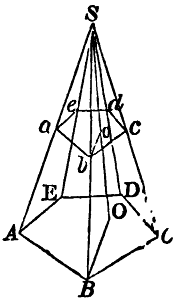
I. *The edges and altitude will be divided proportionally.*

II. *The section will be a polygon similar to the base.*

Let the pyramid *S—ABCDE*, of which *SO* is the altitude, be cut by the plane *abcde* parallel to the base: then will,

$$Sa : SA :: Sb : SB,$$

and the same for the other edges; and the polygon *abcde* will be similar to the base *ABCDE*.



*First.* Since the planes *ABC* and *abc* are parallel, their intersections, $AB$, $ab$, by the plane $SAB$, will also be parallel (Bk. V. Th. ix); hence, the triangles $SAB$, $sab$, are similar, and we have

$$SA : Sa :: SB : Sb;$$

for a similar reason, we have

$$SB : Sb :: SC : Sc;$$

and the same for the other edges: hence, the edges $SA$, $SB$, $SC$, &c., are cut proportionally at the points $a$, $b$, $c$, &c.

The altitude $SO$ is likewise cut proportionally at the point

The altitude $SO$ is likewise cut in the same proportion at the point $o$; for, since $BO$ is parallel to $bo$, we have

$$SO : So :: SB : Sb.$$

*Secondly.* Since $ab$ is parallel to $AB$, $bc$ to $BC$, $cd$ to $CD$, &c.; the angle $abc$ is equal to $ABC$, the angle $bcd$ to $BCD$, and so on (Bk. V. Th. xi).

Also, by reason of the similar triangles, $SAB$, $Sab$, we have

$$AB : ab : SB : Sb,$$

and by reason of the similar triangles $SBC$, $Sbc$, we have

$$SB : Sb :: BC : bc;$$

hence (Bk III. Th. v),

$$AB : ab :: BC : bc;$$

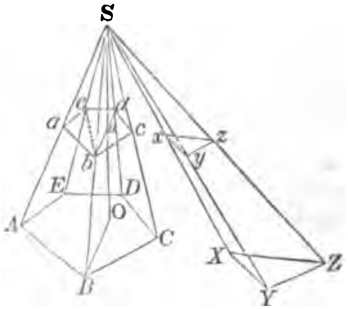
and for a similar reason, we also have

$$BC : bc :: CD : cd, \&c.$$

Hence, the polygons $ABCDE$, $abcde$, having their angles respectively equal, and their homologous sides proportional, are similar.

### THEOREM V.

*If two pyramids, having equal altitudes and their bases in the same plane, be intersected by planes parallel to the plane of the bases, the sections in each pyramid will be proportional to the bases*



Let $S$—$ABCDE$, and $S$—$XYZ$, be two pyramids, having a common vertex, and their bases situated in the same plane. If these pyramids are cut by a plane parallel to the plane of their bases, giving the sections *abcde*, *xyz*, then will the sections *abcde*, *xyz*, be to each other as the bases $ABCDE$, $XYZ$.

For, the polygons $ABCDE$, *abcde*, being similar, their surfaces are as the squares of the homologous sides $AB$, $ab$;

but $AB : ab :: SA : Sa$;

hence, $ABCDE : abcde :: \overline{SA}^2 : \overline{Sa}^2$

For the same reason,

$$XYZ : xyz :: \overline{SX}^2 : \overline{Sx}^2.$$

But since *abc* and *xyz* are in one plane, the lines $SA$, $Sa$, $SX$, $Sx$, are proportional to $SO$, $So$: (Bk. V. Th. xii. Cor.), therefore,

$$SA : Sa :: SX : Sx:$$

hence, $ABCDE : abcde :: XYZ : xyz$.

consequently, the sections *abcde*, *xyz*, are to each other as the bases $ABCDE$, $XYZ$.

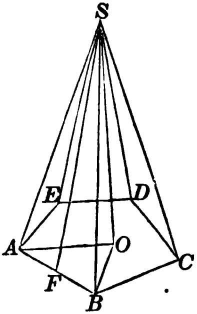
*Cor.* If the bases $ABCDE$, $XYZ$, are equivalent, any sections *abcde*, *xyz*, made at equal distances from the bases, will be also equivalent

### THEOREM VI.

*The convex surface of a right pyramid is equal to half the product of the perimeter of its base multiplied by the slant height.*

Let $S—ABCDE$ be a right pyramid, $SF$ its slant height: then will its convex surface be equal to half the product

$$SF \times (AB+BC+CD+DE+EA).$$



For, since the pyramid is right, the point $O$, in which the axis meets the base, is the centre of the polygon $ABCDE$; hence, the lines $OA$, $OB$, &c. drawn to the vertices of the base, are equal (Bk. IV. prob. x. Cor).

Now, in the right angled triangles $SAO$, $SBO$, the bases and perpendiculars are equal: hence, the hypothenuses are equal; and in the same way it may be proved that all the edges of the pyramid are equal. The triangles, therefore, which form the convex surface of the prism, are all equal to each other.

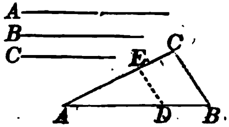
But the area of either of these triangles, as $SAB$, is equal to half the product of the base $AB$, by the slant height of the pyramid $SF$: hence, the area of all the triangles, which form the convex surface of the pyramid, is equal to half the product of the perimeter of the base by the slant height.

### THEOREM VII.

*The convex surface of the frustum of a regular pyramid is equal to half the sum of the perimeters of the upper and lower bases multiplied by the slant height.*

### PROBLEM III.

*To find a fourth proportional to the lines A, B, and C.*



Place two of the lines forming an angle with each other at $A$; that is, make $AB$ equal to $A$, and $AC$ equal $B$; also, lay off $AD$ equal to $C$. Then join $BC$, and through $D$ draw $DE$ parallel to $BC$, and $AE$ will be the fourth proportional sought.

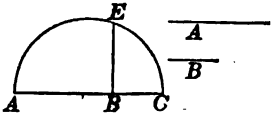
For, since $DE$ is parallel to $BC$, we have

$$AB : AC :: AD : AE;$$

therefore, $AE$ is the fourth proportional sought.

### PROBLEM IV.

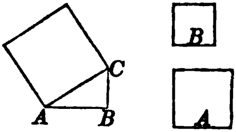
*To find a mean proportional between two given lines, A and B.*



Make $AB$ equal to $A$, and $BC$ equal to $B$: on $AC$ describe a semicircle. Through $B$ draw $BE$ perpendicular to $AC$, and it will be the mean proportional sought (Th. xviii. Cor).

### PROBLEM V.

*To make a square which shall be equivalent to the sum of two given squares.*

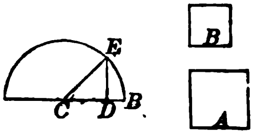


Let $A$ and $B$ be the sides of the given squares.

Draw an indefinite line $AB$, and make $AB$ equal to $A$. At $B$ draw $BC$ perpendicular to $AB$, and make $BC$ equal to $B$: then draw $AC$ and the square described on $AC$ will be equivalent to the squares on $A$ and $B$ (Th. xii).

### PROBLEM VI.

*To make a square which shall be equivalent to the difference between two given squares.*



Let $A$ and $B$ be the sides of the given squares.

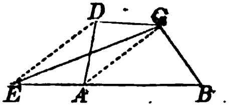
Draw an indefinite line, and make $CB$ equal to $A$, and $CD$ equal to $B$. At $D$ draw $DE$ perpendicular to $CB$, and with $C$ as a centre, and $CB$ as a radius, describe a semicircle meeting $DE$ in $E$, and join $CE$: then will the square described on $ED$ be equal to the difference between the given squares.

For, $CE$ is equal to $CB$, that is, equal to $A$, and $CD$ is equal to $B$: and by (Th. xii. Cor.),

$$\overline{ED}^2=\overline{CE}^2-\overline{CD}^2.$$

### PROBLEM VII.

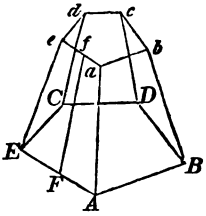
*To make a triangle which shall be equivalent to a given quadrilateral.*



Let $ABCD$ be the given quadrilateral.

Draw the diagonal $AC$, and through $D$ draw $DE$ parallel to $AC$, meeting $BA$ produced at $E$. Join $EC$: then will the triangle $CEB$ be equivalent to the quadrilateral $BD$.

For, the two triangles $ACE$ and $ADC$, having the same base $AC$, and the vertices of the angles $D$ and $E$ in the same line $DE$ parallel to $AC$, are equivalent (Th. ii). If to each, we add $ACB$, we shall then have the triangle $ECB$ equivalent to the quadrilateral $BD$ (Ax. 2).



Let *a—ABCDE* be the frustum of a regular pyramid: then will its convex surface be equal to half the product of the perimeter of its two bases multiplied by the slant height *Ff*.

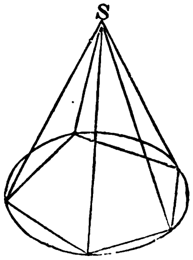
For, since the upper base *abcde*, is similar to the lower base *ABCDE* (Th. iv), and since *ABCDE* is a regular polygon, it follows that the sides *ab*, *bc*, *cd*, *de*, and *ea*, are all equal to each other.

Hence, the trapezoids *EAae*, *ABba*, &c., which form the convex surface of the frustum are equal. But the perpendicular distance between the parallel sides of these trapezoids is equal to *Ef*, the slant height of the frustum.

Now, the area of either of the trapezoids, as *AEea*, is equal to half the product of $Ff \times (EA + ea)$ (Bk. IV. Th. x): hence, the area of all of them, that is, the convex surface of the frustum, is equal to half the sum of the perimeters of the upper and lower bases, multiplied by the slant height.

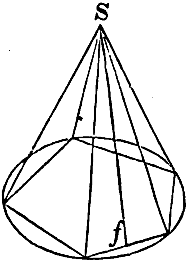
### THEOREM VIII.

*The convex surface of a cone is equal to half the product of the circumference of the base multiplied by the slant height.*



In the circle which forms the base of the cone, inscribe a regular polygon, and join the vertices with the vertex *S*, of the cone. We shall then have a right pyramid inscribed in the cone.

The convex surface of this pyramid will be equal to half the product of the perimeter of the base by the slant height (Th. vi).

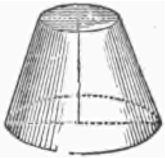


Let us now suppose the number of sides of the polygon to be indefinitely increased: the polygon will then coincide with the base of the cone, the pyramid will become the cone, and the line *Sf*, which measures the slant height of the pyramid, will then measure the slant height of the cone.

Hence, the convex surface of the cone is equal to half the product of the slant height by the circumference of the base.

### THEOREM IX.

*The convex surface of the frustum of a cone is equal to half the sum of the circumferences of its two bases multiplied by the slant height.*

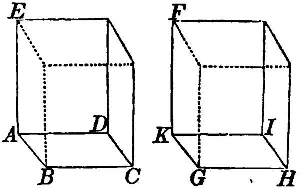


For, if we suppose the frustum of a right pyramid to be inscribed in the frustum of a cone, its convex surface will be equal to half the product of its slant height by the perimeters of its two bases. But if we increase the number of sides of the polygon indefinitely, the frustum of the pyramid will become the frustum of the cone: hence, the area of the frustum of the cone is equal to half the sum of the circumferences of its two bases multiplied by the slant height.

### THEOREM X.

*Two rectangular parallelopipedons, having equal altitudes and equal bases, are equal.*

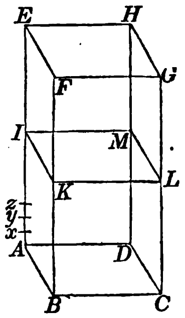
Let $E$—$ABCD$, and $F$—$KGHI$, be two rectangular parallelopipedons having equal bases, $AC$ and $KH$, and equal altitudes, $AE$ and $KF$: then will they be equal.



For, apply the base of the one parallelopipedon to that of the other, and since the bases are equal, they will coincide Again, since the edges are perpendicular to the bases, the edges of the one parallelopipedon will coincide with those of the other; and since the altitude $AE$ is equal to $KF$, the planes of the upper bases will coincide. Hence, the parallelopipedons will coincide, and consequently they are equal.

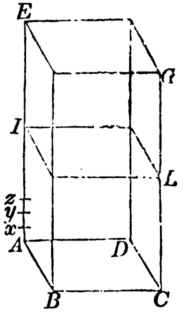
### THEOREM XI.

*Two rectangular parallelopipedons, which have the same base, are to each other as their altitudes.*



Let the parallelopipedons $AG$, $AL$, have the same base $BD$, then will they be to each other as their altitudes $AE$ $AI$.

Suppose the altitudes $AE$, $AI$, to be to each other as two whole numbers, as 15 is to 8, for example. Divide $AE$ into 15 equal parts, whereof $AI$ will contain 8; and through $x$. $y$, $z$, &c., the points of division, draw planes parallel to the base. These planes will cut the solid *AG* into 15 partial parallelopipedons, all equal to each other, because they have equal bases and equal altitudes—equal bases, since every section, *IL*, made parallel to the base *BD*, of a prism, is equal to that base; equal altitudes, because the altitudes are the equal divisions *Ax*, *xy*, *yz*, &c. But of these 15 equal parallelopipedons, 8 are contained in *AL*;



hence, *solid AG* : *solid AL* :: 15 : 8

or generally,

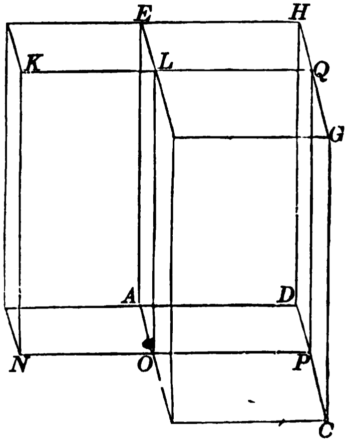
*solid AG* : *solid AL* :: *AE* : *AI*.

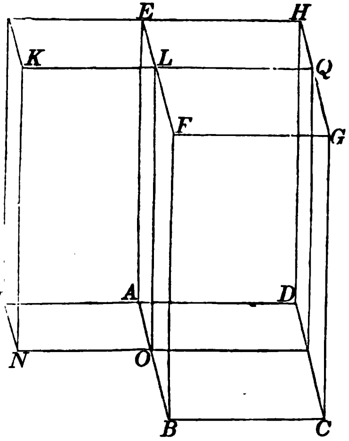
## THEOREM XII.

*Two regular parallelopipedons, having the same altitude, are to each other as their bases.*

Let the parallelopipedons *AG*, *AK*, have the same altitude *AE*; then will they be to each other as their bases *AC*, *AN*.

Having placed the two solids by the side of each other, as the figure represents, produce the plane *ONKL* until it meets the plane *DCGH* in *PQ*; you will thus





have a third parallelopipedon $AQ$, which may be compared with each of the parallelopipedons $AG$, $AK$. The two solids $AG$, $AQ$, having the same base $AEHD$, are to each other as their altitudes $AB$, $AO$; in like manner, the two solids $AQ$ $AK$, having the same base $AOLE$, are to each other as their altitudes $AD$, $AM$.

Hence, we have the two proportions,

$$\text{solid } AG : \text{solid } AQ :: AB : AO,$$

$$\text{solid } AQ : \text{solid } AK :: AD : AM.$$

Multiplying together the corresponding terms of these proportions, and omitting the common multiplier *solid* $AQ$, we have

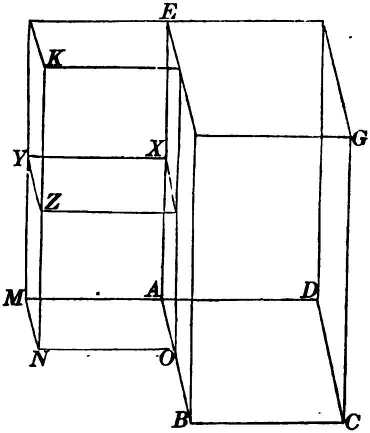
$$\text{solid } AG : \text{solid } AK :: AB \times AD : AO \times AM.$$

But $AB \times AD$ represents the base $ABCD$; and $AO \times AM$ represents the base $AMNO$: hence, two rectangular parallelopipedons of the same altitude are to each other as their bases.

### THEOREM XIII.

*Any two rectangular parallelopidedons are to each other as the products of their three dimensions.*

For, having placed the two solids $AG$, $AZ$, (see next figure) so that their surfaces have the common angle $BAE$, produce the planes necessary for completing the third parallelopipedon $AK$, having the same altitude with the parallelopipedon $AG$. By the last proposition we shall have the proportion, *solid* $AG$ : *solid* $AK$ :: $ABCD$ : $AMNO$



But the two parallelopipedons $AK$, $AZ$, having the same base $AMNO$, are to each other as their altitudes $AE$, $AX$; hence, we have

*solid* $AK$ : *solid* $AZ$ :: $AE$ : $AX$.

Multiplying together the corresponding terms of these proportions, and omitting in the result the common multiplier *solid* $AK$, we shall have

*solid* $AG$ : *solid* $AZ$ :: $ABCD \times AE$ : $AMNO \times AX$.

Instead of the bases $ABCD$ and $AMNO$, put $AB \times AD$ and $AO \times AM$, and we have

*solid* $AG$ : *solid* $AZ$ ·: $AB \times AD \times AE$ : $AO \times AM \times AX$.

Hence, any two rectangular parallelopipedons are to each other as the product of their three dimensions.

*Sch.* We are consequently authorized to assume, as the measure of a rectangular parallelopipedon, the product of its three dimensions.

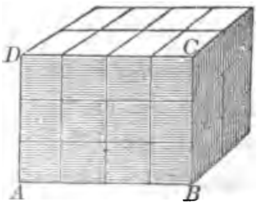
In order to comprehend the nature of this measurement, it is necessary to reflect, that the number of linear units in one dimension of the base multiplied by the number of linear units of the other dimension of the base, will give the number of superficial units in the base of the parallelopipedon (Bk. IV Th. vi. Sch). For each unit in height, there are evidently as many solid units as there are superficial units in the base. Therefore, the product of the number of superficial units in the base multiplied by the number of linear units in the altitude is the number of solid units in the parallelopipedon.

If the three dimensions of another parallelopipedon are valued according to the same linear unit, and multiplied together in the same manner, the two products will be to each other as the solids, and will serve to express their relative magnitude

Let us illustrate this by an example.

Let *ABCD* be the base of a parallelopipedon, and suppose $AB=4$ feet, and $BC=3$ feet. Then the number of square feet in the base *ABCD* will be equal to $3\times4=12$ square feet

Therefore, 12 equal cubes of 1 foot each, may be placed by the side of each other on the base. If the parallelopipedon be 1 foot in height, it will contain 12 cubic feet; were it 2 feet in height, it would contain two tiers of cubes, or 24 cubic feet; were it 3 feet in height, it would contain three tiers of cubes, or 36 cubic feet.



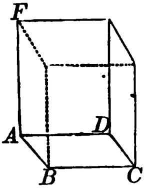
The magnitude of a solid, its volume or extent, forms what is called its *solidity;* and this word is exclusively employed to designate the measure of a solid; thus, we say the solidity of a rectangular parallelopipedon is equal to the product of its base by its altitude, or to the product of its three dimensions

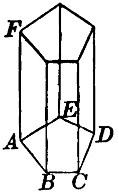
As the cube has all its three dimensions equal, if the side is 1, the solidity will be $1 \times 1 \times 1 = 1$; if the side is 2, the solidity will be $2 \times 2 \times 2 = 8$; if the side is 3, the solidity will be $3 \times 3 \times 3 = 27$; and so on: hence, if the sides of a series of cubes are to each other as the numbers 1, 2, 3, &c. the cubes themselves, or their solidities, will be as the numbers 1, 8, 27, &c. Hence it is, that in arithmetic, the *cube* of a number is the name given to a product which results from three factors, each equal to this number.

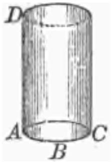
### THEOREM XIV.

*If a parallelopipedon, a prism, and a cylinder, have equivalent bases and equal altitudes, they will be equivalent.*

Let *F—ABCD*, be a parallelopipedon; *F—ABCDE*, a prism; and *D—ABC*, a cylinder, having equivalent bases and equal altitudes: then will they be equivalent.





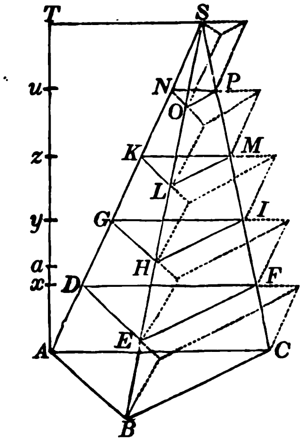


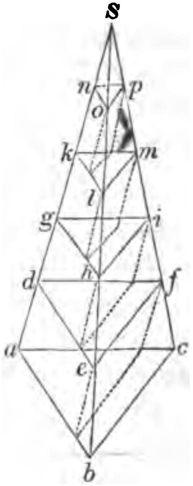
For, since their bases are equivalent they will contain the same number of units of surface (Bk. IV. Def. 9). Now, for each unit of height there will be one tier of equal cubes in each solid, and since the altitudes are equal, the number of tiers in each solid will be equal: hence, the solidities will be equal, and therefore the solids will be equivalent.

*Cor.* Hence, we conclude, that the solidity of a prism or cylinder is equal to the area of its base multiplied by its altitude.

## THEOREM XV.

*Two triangular pyramids, having equivalent bases and equal altitudes, are equivalent, or equal in solidity.*





Let their equivalent bases, *ABC*, *abc*, be situated in the same plane, and let *AT* be their common altitude. If they are not equivalent, let *S—abc* be the smaller; and suppose *Aa* to be the altitude of a prism, which, having *ABC* for its base, is equal to their difference.

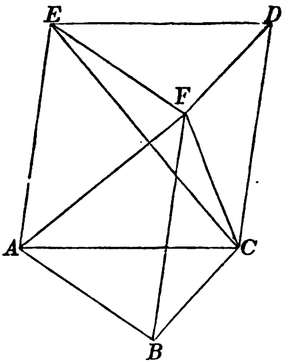
Divide the altitude *AT* into equal parts *Ax*, *xy*, *yz*, &c., each less than *Aa*, and let *k* be one of those parts: through the points of division pass planes parallel to the plane of the bases: the corresponding sections formed by these planes in the two pyramids will be respectively equivalent, namely *DEF* to *def*, *GHI* to *ghi*, &c. (Th. v. Cor.)

This being granted, upon the triangles *ABC*, *DEF*, *GHI*, &c., taken as bases, construct exterior prisms having for edges the parts *AD*, *DG*, *GK*, &c., of the edge *SA*; in like manner, on bases *def*, *ghi*, *klm*, &c., in the second pyramid, construct interior prisms, having for edges the corresponding parts of *Sa*. It is plain that the sum of the exterior prisms of the pyramid *S—ABC* will be greater than the pyramid; while the sum of the interior prisms of the pyramid *S—abc*, will be less than the pyramid. Hence, the difference between these sums will be greater than the difference between the pyramids.

Now, beginning with the bases *ABC*, *abc*, the second exterior prism *EFD—G* is equivalent to the first interior prism *efd—a*, because they have the same altitude *k*, and their bases *DEF*, *def*, are equivalent; for like reasons, the third exterior prism *HIG—K*, and the second interior prism *hig—d*, are equivalent; the fourth exterior and the third interior; and so on, to the last of each series. Hence, all the exterior prisms of the pyramid *S—ABC*, excepting the first prism *BCA—D*, have equivalent corresponding ones in the interior prisms of the pyramid *S—abc:* hence, the prism *BCA—D* is the difference between the sum of all the exterior prisms of the pyramid *S—ABC*, and of the interior prisms of the pyramid *S—abc* But this difference has already been proved to be greater than that of the two pyramids: which, by supposition, differ by the prism *a—ABC:* hence, the prism *BCA—D*, must be greater than the prism *a—ABC*. But in reality it is less, for they have the same base *ABC*, and the altitude *Ax*, of the first, is less than *Aa*, the altitude of the second. Hence, the supposed inequality between the two pyramids cannot exist; hence, the two pyramids; *S—ABC*, *S—abc*, having equal altitudes and equivalent bases, are themselves equivalent.

### THEOREM XVI.

*Every triangular pyramid is a third part of a triangular prism which has an equal base and the same altitude.*



Let *F—ABC* be a triangular pyramid, *ABC—DEF* a triangular prism of the same base and the same altitude: the pyramid will be equal to a third of the prism.

Cut off the pyramid *F—ABC* from the prism, by the plane *FAC;* there will remain the solid *F—ACDE,* which may be considered as a quadrangular pyramid, whose vertex is *F*, and whose base is the parallelogram *ACDE.* Draw the diagonal *CE,* and pass the plane *FCE*, which will cut the quadrangular pyramid into two triangular ones, *F–ACE*, *F–CDE.* These two triangular pyramids have for their common altitude the perpendicular let fall from *F* on the plane *ACDE;* and their bases are also equal, being halves of the parallelogram *AD:* hence, the pyramid *F–ACE*, and the pyramid *F–CDE,* are equivalent (Th. xv).

But the pyramid *F—CDE,* and the pyramid *F—ABC,* have equal bases, *ABC, DEF;* they have also the same altitude, namely, the distance between the parallel planes *ABC, DEF.* hence, the two pyramids are equivalent. Now, the pyramid *F—CDE* has already been proved equivalent to *F—ACE* ; hence, the three pyramids *F—ABC, F—CDE, F—ACE,* which compose the prism *ABC—DEF* are all equivalent

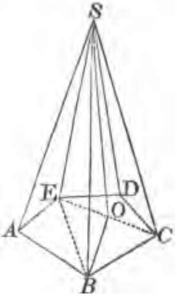
Hence, the pyramid $F—ABC$ is the third part of the prism $ABC—DEF$, which has an equal base and the same altitude.

*Cor.* The solidity of a triangular pyramid is equal to a third part of the product of its base by its altitude.

### THEOREM XVII.

*The solidity of every pyramid is equal to the base multiplied by a third of the altitude.*

Let $S—ABCDE$ be a pyramid.



Pass the planes $SEB$, $SEC$ through the diagonals $EB$, $EC$; the polygonal pyramid $S—ABCDE$ will be divided into several triangular pyramids all having the same altitude $SO$. But each of these pyramids is measured by multiplying its base $ABE$, $BCE$, or $CDE$, by the third part of its altitude $SO$ (Th. xvi. Cor.); hence the sum of these triangular pyramids, or the polygonal pyramid $S—ABCDE$, will be measured by the sum of the triangles $ABE$, $BCE$, $CDE$, or the polygon $ABCDE$, multiplied by one third of $SO$.

*Cor.* 1. Every pyramid is the third part of the prism which has the same base and the same altitude.

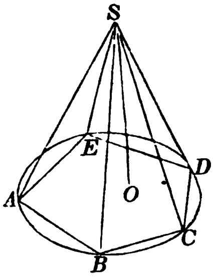
*Cor.* 2. Two pyramids having the same altitude, are to each other as their bases.

*Cor.* 3. Two pyramids having equivalent bases, are to each other as their altitudes.

*Cor.* 4. Pyramids are to each other as the products of their bases by their altitudes

### THEOREM XVIII.

*The solidity of a cone is equal to one third of the product of the base multiplied by the altitude.*



Let $ABCDE$ be the base, $S$ the vertex, and $SO$ the altitude of the cone: then will its solidity be equal to one third the product of its base by its altitude $SO$.

Inscribe in the base of the cone any regular polygon, $ABCDE$, and join the vertices $A$, $B$, $C$, &c., with the vertex $S$, of the cone; then will there be inscribed in the cone a right pyramid, having for its base the polygon $ABCDE$. The solidity of this pyramid is equal to one third of the base multiplied by the altitude (Th. xvii).

Let now, the number of sides of the polygon be indefinitely increased: the polygon will then become equal to the circle, and the pyramid and cone will coincide and become equal. But the solidity of the pyramid will still be equal to one third of the product of the base multiplied by the altitude, whatever be the number of sides of the polygon which forms its base; hence, the solidity of the cone is equal to one third of the product of its base multiplied by its altitude.

*Cor.* 1. A cone is the third part of a cylinder having the same base and the same altitude; whence it follows:

1st, That cones of equal altitudes are to each other as their bases.

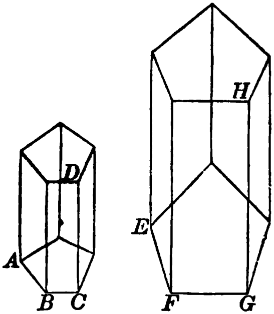
2nd, That cones of equal bases are to each other as their altitudes.

*Cor.* 2. The solidity of a cone is equivalent to the solidity of a pyramid having an equivalent base and the same altitude

### THEOREM XIX.

*Similar prisms are to each other as the cubes of their homologous edges.*

Let *ABC--D*, *EFG—H* be similar prisms: then we shall have



$$\text{solid } AD : \text{solid } EH :: \overline{AB}^3 : \overline{EF}^3;$$

or
$$\text{solid } AD : \text{solid } EH :: \overline{CD}^3 : \overline{HG}^3;$$

or, the solids will be to each other as the cubes of any other of their homologous edges.

For, the solids are to each other as the products of their bases and altitudes (Th. xiv. Cor.), that is,

$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: ABC\times CD : EFG\times GH.$$

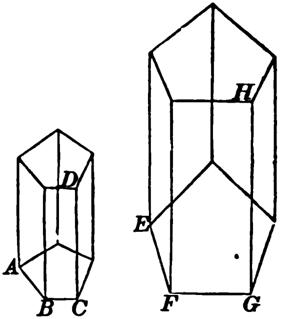
But the bases being similar polygons are to each other as the squares of their like sides (Bk. IV. Th. xxi); that is,

$$ABC : EFG :: \overline{AB}^2 : \overline{EF}^2,$$

therefore,

$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: \overline{AB}^2\times CD : \overline{EF}^2\times$$

But since the solids are similar, the parallelograms *BD* and *FH* are similar (Def. 3): hence, *CD* and *GH* are proportional to *BC* and *FG*, and consequently to *AB* and *EF*: hence, we have,



$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: \overline{AB}^2 \times AB : \overline{EF}^2 \times EF.$$

that is,

$$\text{solid } ABC\text{–}D : \text{solid } EFG\text{–}H :: \overline{AB}^3 : \overline{EF}^3;$$

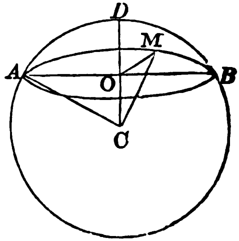
and in a similar manner it may be shown that the solids are to each other as the cubes of any other homologous edges.

*Cor.* Since cylinders are to each other as the product ot their bases and altitudes (Th. xiv. Cor.), it follows that similar cylinders are to each other as the cubes of the linear dimensions.

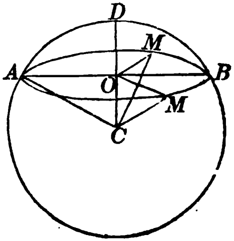
### THEOREM XX.

*Every section of a sphere, made by a plane, is a circle.*

Let *AMB* be a section, made by a plane, in the sphere whose centre is *C*.



From the centre *C* draw *CO*, perpendicular to the plane *AMB*, and also draw the lines *CA*, *CM*, &c., to the points of the curve *AMB*, which terminate the section [illegible] *OM*, &c.



Then, since *CO* is perdendicular to the plane *AMB*, the angles *COA*, *COM* &c., will be right angles, and since the radii of the sphere are all equal, the right angled triangles *CAO*, *COM*, &c., will have the hypothenuses equal, and the side *CO* common: hence, the remaining sides will be equal (Bk. I. Th. xix). Therefore, all lines drawn from *O* to any point of the curve *AMB* are equal: hence *AMB* is a circle.

*Cor.* 1. If the section passes through the centre of the sphere, its radius will be the radius of the sphere: hence, all great circles are equal.

*Cor.* 2. Two great circles always bisect each other; for their common intersection, passing through the centre, is a diameter.

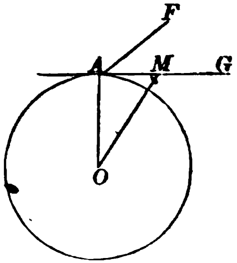
*Cor.* 3. Every great circle divides the sphere and its surface into two equal parts: for, if the two hemispheres were separated and afterwards placed on the common base, with their convexities turned the same way, the two surfaces would exactly coincide, no point of the one being nearer the centre than any point of the other.

*Cor.* 4. The centre of a small circle, and that of the sphere, are in the same straight line, perpendicular to the plane of the small circle.

*Cor.* 5. Small circles are the less the farther they l the centre of the sphere; for the greater $CO$ is, the less is the chord $AB$, the diameter of the small circle $AMB$.

### THEOREM XXI.

*Every plane perpendicular to a radius at its extremity is tangent to the sphere.*

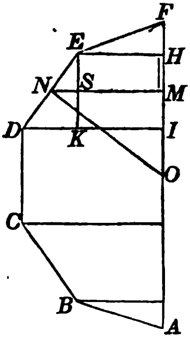


Let $FAG$ be a plane perpendicular to the radius $OA$, at its extremity $A$. Any point $M$, in this plane, being assumed, and $OM$, $AM$, being drawn, the angle $OAM$ will be a right angle, and hence, the distance $OM$ will be greater than $OA$. Hence, the point $M$ lies without the sphere; and as the same can be shown for every other point of the plane $FAG$, this plane can have no point but $A$ common to it and the surface of the sphere; hence it is a tangent plane (Def. 31).

*Sch.* In the same way it may be shown, that two spheres have but one point in common, and therefore touch each other, when the distance between their centres is equal to the sum, or the difference of their radii; in either case, the centres and the point of contact lie in the same straight line.

### THEOREM XXII.

*If a regular semi-polygon be revolved about a line passing through the centre and the vertices of two opposite angles, the surface described by its perimeter will be equal to the axis multiplied by the circumference of the inscribed circle.*



Suppose the regular semi-polygon *ABCDE* to be revolved about the line *AF* as an axis: then will the surface described by its perimeter be equal to *AF* multiplied by the circumference of the inscribed circle.

From *E* and *D*, the extremities of one of the equal sides, let fall the perpendiculars *EH*, *DI*, on the axis *AF*, and from the centre *O*, draw *ON* perpendicular to the side *DE*: *ON* will then be the radius of the inscribed circle (Bk. IV. Prob. x).

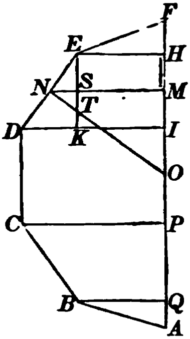
Let us first find the measure of the surface described by one of the equal sides, as *DE*.

From *N*, the middle point of *DE*, draw *NM* perpendicular to the axis *AF*, and through *E*, draw *EK*, parallel to it, meeting *MN* in *S*.

Then, since *EN* is half of *ED*, *NS* will be half of *DK* (Bk. IV. Th. xiii): and hence, *NM* is equal to half the sum of $EH+DI$.

But, since the circumferences of circles are to each other as their diameters (Bk. IV. Th. xxiv), or as their radii, the halves of the diameters, we shall have the circumference described by the point *N*, equal to half the sum of the circumferences described by the points *D* and *E*.

But in the revolution of the polygon the line *ED* describes the surface of the frustum of a cone, the measure of which is equal to *DE* multiplied into half the sum of the circumferences of the two bases (Th. ix); that is, equal to *DE* into the circumference described by the point *N*.



But, the triangle *ENS* is similar to *SNT* (Bk. IV. Th. xviii), and also to *EDK*, and since *TNS* is similar to *ONM*, it follows that *EDK* and *ONM* are similar; hence,

$$ED \; : \; EK \text{ or } HI \; :: \; ON \; : \; NM,$$

or $ED : HI :: \textit{circumference } ON : \textit{circumference } MN.$

consequently,

$$ED \times \textit{circumference } MN = HI \times \textit{circumference } ON,$$

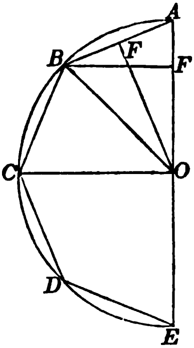
that is, *ED* multiplied into the circumference of the circle described with the radius *NM*, is equal to *HI* into the circumference of the circle described with the radius *ON*. But the former is equal to the surface described by the line *ED* in the revolution of the polygon about the axis *AF;* hence, the latter is equal to the same area; and since the same may be shown for each of the other sides, it is plain that the surface described by the entire perimeter is equal to

$$(FH+HI+IP+PQ+QA) \times \textit{cir'f. } ON = AF \times \textit{cir'f. } ON.$$

*Cor.* The surface described by any portion of the perimeter, as *EDC*, is equal to the distance between the two perpendiculars let fall from its extremities, on the axis, multiplied by the circumference of the inscribed circle. For, the surface described by *DE* is equal to $HI \times$ circumference *ON*. and the surface described by *DC* is equal to $IP \times$ circumference $ON$: hence, the surface described by $ED+DC$, is equal to $(HI+IP)\times$ circumference $ON$, or equal to $HP\times$ circumference $ON$.

### THEOREM XXIII.

*The surface of a sphere is equal to the product of its diameter by the circumference of a great circle.*



Let $ABCDE$ be a semicircle. Inscribe in it any regular semi-polygon, and from the centre $O$ draw $OF$ perpendicular to one of the sides.

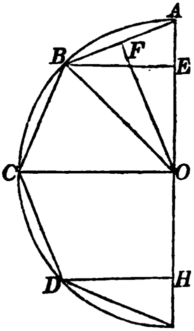
Let the semicircle and the semi-polygon be revolved about the axis $AE$: the semicircumference $ABCDE$ will describe the surface of a sphere (Def. 26); and the perimeter of the semi-polygon will describe a surface which has for its measure $AE\times$ circumference $OF$ (Th. xxii); and this will be true whatever be the number of sides of the polygon. But if the number of sides of the polygon be indefinitely increased, its perimeter will coincide with the circumference $ABCDE$, the perpendicular $OF$ will become equal to $OE$, and the surface described by the perimeter of the semi-polygon will then be the same as that described by the semicircumference $ABCDE$. Hence, the surface of the sphere is equal to $AE\times$ circumference $OE$.

*Cor.* Since the area of a great circle is equal to the product of its circumference by half the radius, or by one-fourth of the diameter (Bk. IV. Th. xxvii), it follows that the surface of a sphere is equal to four of its great circles.

### THEOREM XXIV.

*The surface of a zone is equal to its altitude multiplied by the circumference of a great circle.*

For, the surface described by any portion of the perimeter of the inscribed polygon, as $BC+CD$ is equal to $EH\times$ circumference $OF$ (Th. xxii. Cor). But when the number of sides of the polygon is indefinitely increased, $BC+CD$, becomes the arc $BCD$, $OF$ becomes equal to $OA$, and the surface described by $BC+CD$, becomes the surface of the zone described by the arc $BCD$: hence, the surface of the zone is equal to $EH\times$ circumference $OA$.



*Sch.* 1. When the zone has but one base, as the zone described by the arc $ABCD$, its surface will still be equal to the altitude $AE$ multiplied by the circumference of a great circle.

*Sch.* 2. Two zones taken in the same sphere, or in equal spheres, are to each other as their altitudes; and any zone is to the surface of the sphere as the altitude of the zone is to the diameter of the sphere.

### THEOREM XXV.

*The solidity of a sphere is equal to one third of the product of the surface multiplied by the radius.*

For, conceive a polyedron to be inscribed in the sphere.

This polyedron may be considered as formed of pyramids, each having for its vertex the centre of the sphere, and for its base one of the faces of the polyedron. Now, the solidity of each pyramid, will be equal to one third of the product of its base by its altitude (Th. xvii).

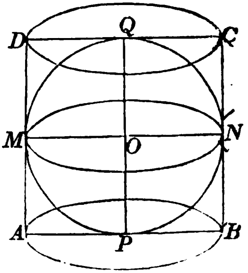
But if we suppose the faces of the polyedron to be continually diminished, and consequently, the number of the pyramids to be constantly increased, the polyedron will finally become the sphere, and the bases of all the pyramids will become the surface of the sphere. When this takes place, the solidities of the pyramids will still be equal to one third the product of the bases by the common altitude, which will then be equal to the radius of the sphere.

Hence, the solidity of a sphere is equal to one third of the product of the surface by the radius.

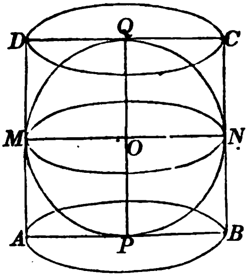
### THEOREM XXVI.

*The surface of a sphere is equal to the convex surface of the circumscribing cylinder; and the solidity of the sphere is two thirds the solidity of the circumscribing cylinder.*

Let *MPNQ* be a great circle of the sphere; *ABCD* the circumscribing square: if the semicircle *PMQ*, and the half square *PADQ*, are at the same time made to revolve about the diameter *PQ*, the semicircle will describe the sphere, while the half square will describe the cylinder circumscribed about that sphere.



The altitude *AD*, of the cylinder, is equal to the diameter



*PQ;* the base of the cylinder is equal to the great circle, since its diameter *AB* is equal *MN;* hence, the convex surface of the cylinder is equal to the circumference of the great circle multiplied by its diameter (Th. ii). This measure is the same as that of the surface of the sphere (Th. xxiii): hence, the surface of the sphere is equal to the convex surface of the circumscribing cylinder.

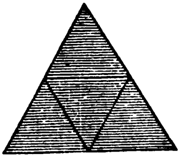
In the next place, since the base of the circumscribing cylinder is equal to a great circle, and its altitude to the diameter, the solidity of the cylinder will be equal to a great circle multiplied by a diameter (Th. xiv. Cor). But the solidity of the sphere is equal to its surface multiplied by a third of its radius; and since the surface is equal to four great circles (Th. xxiii. Cor.), the solidity is equal to four great circles multiplied by a third of the radius; in other words, to one great circle multiplied by four-thirds of the radius, or by two-thirds of the diameter; hence, the sphere is two-thirds of the circumscribing cylinder.

# APPENDIX

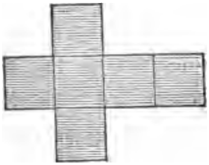
## OF THE FIVE REGULAR POLYEDRONS.

A *regular polyedron*, is one whose faces are all equal polygons, and whose polyedral angles are equal. There are five such solids.

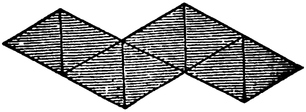
1. The *Tetraedron*, or equilateral pyramid, is a solid bounded by four equal triangles.



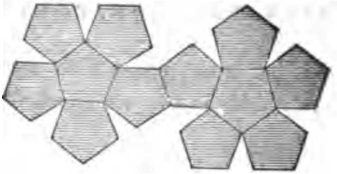
2. The *hexaedron* or *cube*, is a solid, bounded by six equal squares.



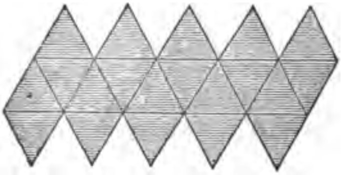
3. The *octaedron*, is a solid, bounded by eight equal equilateral triangles.



4. The *dodecaedron*, is a solid bounded by twelve equal pentagons.



5. The *icosaedron*, is a solid, bounded by twenty equal equilateral triangles.



6. The regular solids may easily be made of pasteboard. Draw the figures of the regular solids accurately on pasteboard, and then cut through the bounding lines: this will give figures of pasteboard similar to the diagrams. Then, cut the other lines half through the pasteboard, after which, turn up the parts, and glue them together, and you will form the bodies which have been described.

# ELEMENTS OF TRIGONOMETRY.

---

## INTRODUCTION.

### SECTION I.

### OF LOGARITHMS.

1. ***The logarithm of a number is the exponent of the power to which it is necessary to raise a fixed number, in order to produce the first number.***

This fixed number is called the *base* of the system, and may be any number except 1: in the common system 10 is assumed as the base.

2. If we form those powers of 10, which are denoted by entire exponents, we shall have

$$10^0=1 \qquad 10^1 = 10 \quad , \quad 10^3 = 1000$$

$$10^2 = 100 \quad , \quad 10^4 = 10000, \text{\&c. \&c.}$$

From the above table, it is plain, that 0, 1, 2, 3, 4, &c., are respectively the logarithms of 1, 10, 100, 1000, 10000, &c.; we also see that the logarithm of any number between 1 and 10 is greater than 0 and less than 1: thus

$$\text{Log } 2 = 0.301030$$

The logarithm of any number greater than 10, and less than 100, is greater than 1 and less than 2: thus

$$\text{Log } 50 = 1.698970$$

The logarithm of any number greater than 100, and less than 1000, is greater than 2 and less than 3: thus

$$\text{Log } 126 = 2.100371, \text{ \&c.}$$

If the above principles be extended to other numbers, it will appear, that the logarithm of any number, not an exact power of ten, is made up of two parts, *an entire* and a *decimal part.* The *entire part* is called the *characteristic of the logarithm,* and is always one less than the number of places of figures in the given number.

3. The principal use of logarithms, is to abridge numerical computations.

Let $M$ denote any number, and let its logarithm be denoted by $m$; also let $N$ denote a second number whose logarithm is $n$; then from the definition we shall have

$$10^m = M \ (1) \qquad 10^n = N \ (2)$$

Multiplying equations (1) and (2), member by member, we have

$$10^{m+n} = M \times N \quad \text{or, } m+n = \log M \times N: \quad \text{hence,}$$

*The sum of the logarithms of any two numbers is equal to the logarithm of their product.*

Dividing equation (1) by equation (2), member by member, we have

$$10^{m-n} = \frac{M}{N} \quad \text{or,} \quad m-n = \log \frac{M}{N}: \quad \text{hence,}$$

*The logarithm of the quotient of two numbers, is equal to the logarithm of the dividend diminished by the logarithm of the divisor.*

4. Since the logarithm of 10 is 1, *the logarithm of the product of any number by* 10, *will be greater by* 1 *than the logarithm of that number;* also, *the logarithm of any number divided by* 10, *will be less by* 1 *than the logarithm of that number.*

Similarly, it may be shown that the logarithm of any number multiplied by a hundred, is greater by 2 than the logarithm of that number, and the logarithm of any number divided by 100 is less by 2, than the logarithm of that number, and so on.

EXAMPLES.

| | | |
|---|---|---|
| log 327 | is | 2.514548 |
| log 32.7 | " | 1.514548 |
| log 3.27 | " | 0.514548 |
| log .327 | " | $\bar{1}$.514548 |
| log .0327 | " | $\bar{2}$.514548 |

From the above examples, we see, that in a number composed of an entire and decimal part, we may change the place of the decimal point without changing the decimal part of the logarithm; but *the characteristic is diminished by* 1 *for every place that the decimal point is removed to the left.*

In the logarithm of a decimal, the *characteristic* becomes negative, and is numerically 1 greater than the number of ciphers immediately after the decimal point. The negative sign extends only to the characteristic, and is written over it as in the examples given above.

TABLE OF LOGARITHMS.

5. A table of logarithms, is a table in which are written the logarithms of all numbers between 1 and some given number. The logarithms of all numbers between 1 and 10,000 are given

in the annexed table. Since rules have been given for determining the characteristics of logarithms by simple inspection, it has not been deemed necessary to write them in the table, the decimal part only being given. The characteristic, however, is given for all numbers less than 100.

The left hand column of each page of the table, is the column of numbers, and is designated by the letter N; the logarithms of these numbers are placed opposite them on the same horizontal line. The last column on each page, headed D, shows the difference between the logarithms of two consecutive numbers. This difference is found by subtracting the logarithm under the column headed 4, from the one in the column headed 5 in the same horizontal line, and is nearly a mean of the differences of any two consecutive logarithms on the line.

6. *To find from the table the logarithm of any number.*

If the number is less than 100, look on the first page of the table, in the column of numbers under N, until the number is found: the number opposite is the logarithm sought: Thus

$$\log 9 = 0.954243$$

7. *When the number is greater than* 100 *and less than* 10000.

Find in the column of numbers, the first three figures of the given number. Then pass across the page along a horizontal line until you come into the column under the fourth figure of the given number: at this place, there are four figures of the required logarithm, to which two figures taken from the column marked 0, are to be prefixed.

If the four figures already found stand opposite a row of six figures in the column marked 0, the two left hand figures of the six, are the two to be prefixed; but if they stand opposite a row of only four figures, you ascend the column till you find a row of six figures; the two left hand figures of this row are the two to be prefixed. If you prefix to the decimal part thus found, the characteristic, you will have the logarithm sought: Thus,

$$\log\ 8979 = 3.953228$$
$$\log .08979 = \bar{2}.953228$$

If however in passing back from the four figures found, to the 0 column, any dots be met with, the two figures to be prefixed must be taken from the horizontal line directly below: Thus,

$$\log\ 3098 = 3.491081$$
$$\log 30.98 = 1.491081$$

If the logarithm falls at a place where the dots occur, 0 must be written for each dot, and the two figures to be prefixed are as before taken from the line below: Thus,

$$\log\ 2188 = 3.340047$$
$$\log .2188 = \bar{1}.340047$$

8. *When the number exceeds* 10,000.

The characteristic is determined by the rules already given. To find the decimal part of the logarithm. Place a decimal point after the fourth figure from the left hand, converting the given number into a whole number and decimal. Find the logarithm of the entire part by the rule just given, then take from the right hand column of the page, under D, the number on the same horizontal line with the logarithm, and multiply it by the decimal part; add the product thus obtained to the logarithm already found, and the sum will be the logarithm sought.

If, in multiplying the number taken from the column D, the decimal part of the product exceeds .5 let 1 be added to the entire part; if it is less than .5 the decimal part of the product is neglected.

**EXAMPLE.**

To find log 672887.

The characteristic is 5.; placing a decimal point after the fourth figure from the left, we have 6728.87. The decimal part of the log 6728 is .827886 and the corresponding number in the column D is 65; then $65 \times .87 = 56.55$, and since the decimal part exceeds .5, we have 57 to be added to 827886, which gives .827943

$$\text{or} \quad \log 672887 = 5.827943$$

Similarly

$$\log .0672887 = \bar{2}.827943$$

The last rule has been deduced under the supposition that the difference of the numbers is proportional to the difference of their logarithms, which is sufficiently exact within the narrow limits considered.

In the above example, 65 is the difference between the logarithm of 672900 and the logarithm of 672800, that is, it is the difference between the logarithms of two numbers which differ by 100.

We have then the proportion 100 : 87 : : 65 : 56.55, the number to be added to the logarithm already found.

9. *To find from the table the number corresponding to a given logarithm.*

Search in the columns of logarithms for the decimal part of the given logarithm: if it cannot be found in the table, take out the number corresponding to the next less logarithm and set it aside. Subtract this less logarithm from the given logarithm, and annex to the remainder as many zeros as may be necessary, and divide this result by the corresponding number taken from the column marked D, continuing the division as long as desirable: annex the quotient to the number set aside. Point off, from the left hand, as many integer figures as there are units in the characteristic of the given logarithm increased by 1; the result is the required number.

If the characteristic is negative, the number will be entirely decimal, and the number of zeros to be placed immediately after the decimal point will be equal to the number of units in the characteristic diminished by 1.

This rule, like its converse, is founded on the supposition that the difference of the logarithms is proportional to the difference of their numbers within narrow limits.

### EXAMPLE.

Find the number corresponding to the logarithm 3.233568.

The decimal part of the given logarithm is .233568
The next less logarithm of the table is .233504 and its corresponding number 1712.
Their difference is - - - 64
Tabular difference 253)6400000(25
Hence the number sought 1712.25
The number corresponding to $\bar{3}.233568$ is .00171225

### MULTIPLICATION BY LOGARITHMS.

10. When it is required to multiply numbers by means of their logarithms, we first find from the table the logarithms of the numbers to be multiplied; we next add these logarithms together, and their sum is the logarithm of the product of the numbers (Art. 3).

The term *sum* is to be understood in its algebraic sense;

therefore, if any of the logarithms have negative characteristics, the difference between their sum and that of the positive characteristics, is to be taken; the sign of the remainder is that of the greater sum.

EXAMPLES.

1. Multiply 23.14 by 5.062.

$$\begin{aligned} \log 23.14 &= 1.364363 \\ \log 5.062 &= 0.704322 \\ \text{Product } 117.1347 \ \ldots\ldots &\ \ \underline{\overline{2.068685}} \end{aligned}$$

2. Multiply 3.902, 597.16 and 0.0314728 together.

$$\begin{aligned} \log 3.902 &= 0.591287 \\ \log 597.16 &= 2.776091 \\ \log 0.0314728 &= \bar{2}.497936 \\ \text{Product } 73.3354 \ \ldots\ldots &\ \ \underline{\overline{1.865314}} \end{aligned}$$

Here the $\bar{2}$ cancels the + 2, and the 1 carried from the decimal part is set down.

3. Multiply 3.586, 2.1046, 0.8372, and 0.0294, together.

$$\begin{aligned} \log 3.586 &= 0.554610 \\ \log 2.1046 &= 0.323170 \\ \log 0.8372 &= \bar{1}.922829 \\ \log 0.0294 &= \bar{2}.468347 \\ \text{Product } 0.1857615 \ \ldots &\ \ \underline{\overline{\bar{1}.268956}} \end{aligned}$$

In this example the 2, carried from the decimal part, cancels $\bar{2}$, and there remains $\bar{1}$ to be set down.

### DIVISION OF NUMBERS BY LOGARITHMS.

11. When it is required to divide numbers by means of their logarithms, we have only to recollect, that the subtraction of

logarithms corresponds to the division of their numbers (Art. 3). Hence, if we find the logarithm of the dividend, and from it subtract the logarithm of the divisor, the remainder will be the logarithm of the quotient.

This additional caution may be added. The difference of the logarithms, as here used, means the ***algebraic difference;*** so that, if the logarithm of the divisor have a negative characteristic its sign must be changed to positive, after diminishing it by the unit, if any, carried in the subtraction from the decimal part of the logarithm. Or, if the characteristic of the logarithm of the dividend is negative, it must be treated as a negative number.

EXAMPLES.

1. To divide 24163 by 4567.

log 24163 = 4.383151
log 4567 = 3.659631
Quotient 5.29078 . . . . 0.723520

2. To divide 0.06314 by .007241

log 0.06314 = $\bar{2}.800305$
log 0.007241 = $\bar{3}.859799$
Quotient . . . 8.7198 . . . . 0.940506

Here, 1 carried from the decimal part to the $\bar{3}$ changes it to $\bar{2}$, which being taken from $\bar{2}$, leaves 0 for the characteristic.

3. To divide 37.149 by 523.76

log 37.149 = 1.569947
log 523.76 = 2.719133
Quotient . . 0.0709274 . $\bar{2}.850814$

4. To divide 0.7438 by 12.9476

$$\log\ 0.7438 = \bar{1}.871456$$
$$\log\ 12.9476 = 1.112189$$

Quotient . . 0.057447 . . $\bar{2}.759267$

Here, the 1 taken from $\bar{1}$, gives $\bar{2}$ for a result, as set down.

### ARITHMETICAL COMPLEMENT.

12. The *Arithmetical complement* of a logarithm is the number which remains after subtracting the logarithm from 10.

Thus, . . $1-9.274687 = 0.725313$

Hence, 0.725313 is the arithmetical complement of 9.274687.

13. We will now show that, *the difference between two logarithms is truly found, by adding to the first logarithm the arithmetical complement of the logarithm to be subtracted, and then diminishing the sum by* 10.

Let $a =$ the first logarithm
$b =$ the logarithm to be subtracted
and $c = 10-b =$ the arithmetical complement of $b$.

Now the difference between the two logarithms will be expressed by $a-b$.

But, from the equation $c = 10-b$, we have

$$c-10 = -b$$

hence, if we place for $-b$ its value, we shall have

$$a-b = a+c-10$$

which agrees with the enunciation.

When we wish the arithmetical complement of a logarithm, we may write it directly from the table, *by subtracting the left* *hand figure from* 9, *then proceeding to the right, subtract each figure from* 9 *till we reach the last significant figure, which must be taken from* 10: *this will be the same as taking the logarithm from* 10.

EXAMPLES.

1. From 3.274107 take 2.104729.

| *By common method.* | | | *By arith. comp.* |
|---|---|---|---|
| | 3.274107 | | 3.274107 |
| | 2.104729 | its ar. comp. | 7.895271 |
| Diff. | 1.169378 | Sum | 1.169378 after subtracting 10. |

Hence, to perform division by means of the arithmetical complement we have the following

RULE.

*To the logarithm of the dividend add the arithmetical complement of the logarithm of the divisor: the sum after subtracting* 10, *will be the logarithm of the quotient.*

EXAMPLES.

1. Divide 327.5 by 22.07.

| | | | |
|---|---|---|---|
| log 327.5 | | . . . | 2.515211 |
| log 22.07 | | ar. comp. | 8.656198 |
| Quotient . . | 14.839 | . . . . | 1.171409 |

2. Divide 0.7438 by 12.9476.

| | | | |
|---|---|---|---|
| log 0.7438 | | . . . . . | $\bar{1}.871456$ |
| log 12.9476 | | ar. comp. | 8.887811 |
| Quotient . . | 0.057447 | . . . | $\bar{2}.759267$ |

In this example, the sum of the characteristics is 8 from which, taking 10, the remainder is $\bar{2}$.

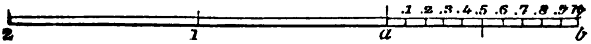
3. Divide 37.149 by 523.76.

| | | | |
|---|---|---|---|
| | log 37.149 | . . . . . . | 1.569947 |
| | log 523.76 | ar. comp. | 7.280867 |
| Quotient . . | 0.0709273 | . . . | $\bar{2}$.850814 |

## SECTION II.

### OF SCALES.

### SCALE OF EQUAL PARTS.



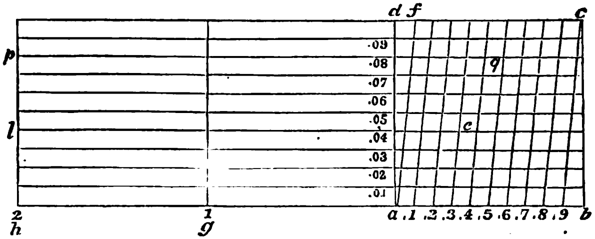
14. A scale of equal parts is formed by dividing a line of a given length into equal portions.

If, for example, the line *ab* of a given length, say one inch, be divided into any number of equal parts, as 10, the scale thus formed, is called a scale of ten parts to the inch. The line *ab*, which is divided, is called the *unit of the scale.* This unit is laid off several times on the left of the divided line, and its extremities marked, 1, 2, 3, &c.

The unit of scales of equal parts, is, in general, either an inch, or an exact part of an inch. If, for example, *ab* the unit of the scale, were half an inch, the scale would be one of 10 parts to half an inch, or of 20 parts to the inch.

If it were required to take from the scale a line equal to two inches and six-tenths, place one foot of the dividers at 2 on the left, and extend the other to .6, which marks the sixth of the small divisions: the dividers will then embrace the required distance.

DIAGONAL SCALE OF EQUAL PARTS.



15. This scale is thus constructed. Take *ab* for the unit of the scale, which may be one inch, $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{3}{4}$ of an inch, in length. On *ab* describe the square *abcd*. Divide the sides *ab* and *dc* each into ten equal parts. Draw *af* and the other nine parallels as in the figure.

Produce *ba* to the left, and lay off the unit of the scale any convenient number of times, and mark the points 1, 2, 3, &c. Then, divide the line *ad* into ten equal parts, and through the points of division draw parallels to *ab* as in the figure.

Now, the small divisions of the line *ab* are each one-tenth (.1) of *ab*; they are therefore .1 of *ad*, or .1 of *ag* or *gh*.

If we consider the triangle *adf*, we see that the base *df* is one-tenth of *ad*, the unit of the scale. Since the distance from *a* to the first horizontal line above *ab*, is one-tenth of the distance *ad*, it follows that the distance measured on that line between *ad* and *af* is one-tenth of *df:* but since one-tenth of a tenth is a hundredth, it follows that this distance is one-hundredth (.01) of the unit of the scale. A like distance measured on the second line will be two-hundredths (.02) of the unit of the scale; on the third, .03; on the fourth, .04, &c.

If it were required to take, in the dividers, the unit of the scale, and any number of tenths, place one foot of the dividers at 1, and extend the other to that figure between *a* and *b* which designates the tenths. If two or more units are required, the dividers must be placed on a point of division further to the left.

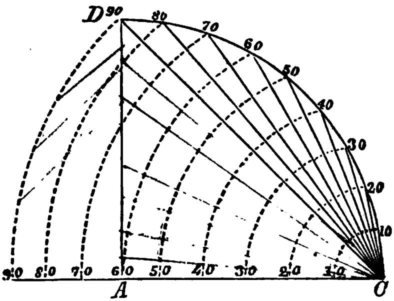
When units, tenths, and hundredths, are required, place one foot of the dividers where the vertical line through the point which designates the units, intersects the line which designates the hundredths: then, extend the dividers to that line between *ad* and *bc* which designates the tenths: the distance so determined will be the one required.

For example, to take off the distance 2.34, we place one foot of the dividers at *l*, and extend the other to *e:* and to take off the distance 2.58, we place one foot of the dividers at *p* and extend the other to *q*.

REMARK I. If a line is so long that the whole of it cannot be taken from the scale, it must be divided, and the parts of it taken from the scale in succession.

REMARK II. If a line be given upon the paper, its length can be found by taking it in the dividers and applying it to the scale.

SCALE OF CHORDS



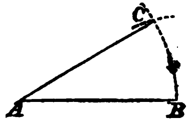
16. If, with any radius, as $AC$, we describe the quadrant $CD$, and then divide it into 90 equal parts, each part is called a degree.

Through $C$, and each point of division, let a chord be drawn, and let the lengths of these chords be accurately laid off on a scale: such a scale is called a *scale of chords.* In the figure, the chords are drawn for every ten degrees.

The scale of chords being once constructed, the radius of the circle from which the chords were obtained, is known; for, the chord marked 60 is always equal to the radius of the circle. A scale of chords is generally laid down on the scales which belong to cases of mathematical instruments, and is marked CHO.

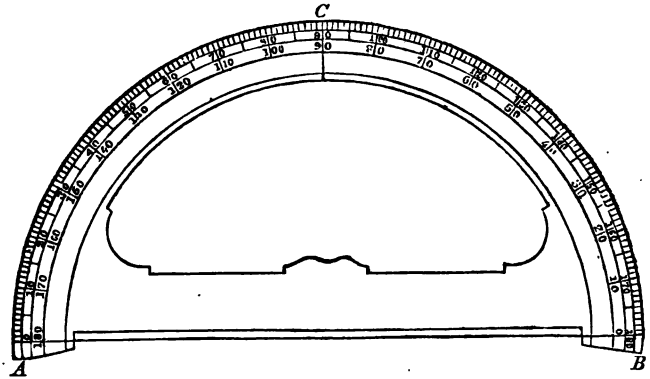
*To lay off, at a given point of a line, with the scale of chords, an angle equal to a given angle.*

Let $AB$ be the line, and $A$ the given point.



Take from the scale the chord of 60 degrees, and with this radius, and the point $A$ as a centre, describe the arc $BC$. Then take from the scale the chord of the given angle, say 30 degrees, and with this line as a radius, and *B* as a centre, describe an arc cutting *BC* in *C*. Through *A* and *C* draw the line *AC*, and *BAC* will be the required angle.

### SEMICIRCULAR PROTRACTOR.



17. This instrument is used to lay down, or protract angles. It may also be used to measure angles included between lines already drawn upon paper.

It consists of a brass semicircle *ABC* divided to half degrees. The degrees are numbered from 0 to 180, both ways; that is, from *A* to *B* and from *B* to *A*. The divisions, in the figure, are only made to degrees. There is a small notch at the middle of the diameter *AB*, which indicates the centre of the protractor.

### GUNTERS' SCALE.

18. This is a scale of two feet in length, on the faces of which a variety of scales is marked. The face on which the

divisions of inches are made, contains, however, all the scales necessary for laying down lines and angles. These are, the scale of equal parts, the diagonal scale of equal parts, and the scale of chords, all of which have been described.

## PLANE TRIGONOMETRY.

### DEFINITIONS AND EXPLANATION OF TABLES.

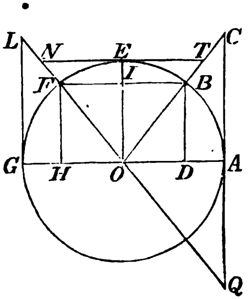
19. In every plane triangle there are six parts: three sides and three angles. These parts are so related to each other, that when one side and any two other parts are given, the remaining parts can be obtained, either by geometrical construction or by trigonometrical computation.

20. *Plane Trigonometry* explains the methods of computing the unknown parts of a plane triangle, when a sufficient number of the six parts is given.

21. For the purpose of trigonometrical calculation, the circumference of the circle is supposed to be divided into 360 equal parts, called degrees; each degree is supposed to be divided into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds.

Degrees, minutes, and seconds, are designated respectively, by the characters ° ′ ″. For example, *ten degrees, eighteen minutes, and fourteen seconds*, would be written 10° 18′ 14″

If two lines be drawn through the centre of the circle, at right angles to each other, they will divide the circumference into four equal parts, of 90° each. Every right angle then, as *EOA*, is measured by an arc of 90°; every acute angle, as *BOA*, by an arc less than 90°; and every obtuse angle, as *FOA*, by an arc greater than 90°.



22. The *complement* of an arc is what remains after subtracting the arc from 90°. Thus, the arc *EB* is the complement of *AB*. The sum of an arc and its complement is equal to 90°.

23. The *supplement* of an arc is what remains after subtracting the arc from 180°. Thus, *GF* is the supplement of the arc *AEF*. The sum of an arc and its supplement is equal to 180°.

24. The *sine* of an arc is the perpendicular let fall from one extremity of the arc on the diameter which passes through the other extremity. Thus, *BD* is the sine of the arc *AB*.

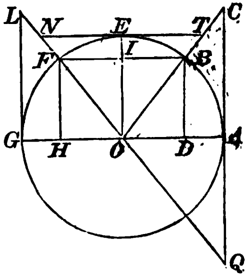
25. The *cosine* of an arc is the part of the diameter intercepted between the foot of the sine and centre. Thus, *OD* is the cosine of the arc *AB*.

26. The *tangent* of an arc is the line which touches it at one extremity, and is limited by a line drawn through the other extremity and the centre of the circle. Thus, *AC* is the tangent of the arc *AB*.

27. The *secant* of an arc is the line drawn from the centre of the circle through one extremity of the arc, and limited by the tangent passing through the other extremity. Thus, $OC$ is the secant of the arc $AB$.

28. The four lines, $BD$, $OD$, $AC$, $OC$, depend for their values on the arc $AB$ and the radius $OA$; they are thus designated:

sin $AB$ for $BD$
cos $AB$ for $OD$
tan $AB$ for $AC$
sec $AB$ for $OC$.



29. If $ABE$ be equal to a quadrant, or 90°, then $EB$ will be the complement of $AB$. Let the lines $ET$ and $IB$ be drawn perpendicular to $OE$. Then,

$ET$, the tangent of $EB$, is called the cotangent of $AB$;
$IB$, the sine of $EB$, is equal to the cosine of $AB$;
$OT$, the secant of $EB$, is called the cosecant of $AB$.

In general, if $A$ is any arc or angle, we have,

$$\cos A = \sin (90°-A)$$
$$\cot A = \tan (90°-A)$$
$$\operatorname{cosec} A = \sec (90°-A)$$

30. If we take an arc $ABEF$, greater than 90°, its sine will be $FH$; $OH$ will be its cosine; $AQ$ its tangent, and $OQ$ its secant. But $FH$ is the sine of the arc $GF$, which is the supplement of $AF$, and $OH$ is its cosine: hence, *the sine of*

*an arc is equal to the sine of its supplement; and the cosine of an arc is equal to the cosine of its supplement.**

Furthermore, $AQ$ is the tangent of the arc $AF$, and $OQ$ is its secant: $GL$ is the tangent, and $OL$ the secant of the supplemental arc $GF$. But since $AQ$ is equal to $GL$, and $OQ$ to $OL$, it follows that, *the tangent of an arc is equal to the tangent of its supplement; and the secant of an arc is equal to the secant of its supplement.**

Let us suppose, that in a circle of a given radius, the lengths of the sine, cosine, tangent, and cotangent, have been calculated for every minute or second of the quadrant, and arranged in a table; such a table is called a table of sines and tangents. If the radius of the circle is 1, the table is called a table of natural sines. A table of natural sines, therefore, shows the values of the sines, cosines, tangents and cotangents of all the arcs of a quadrant, divided to minutes or seconds.

If the sines, cosines, tangents, and secants are known for arcs less than 90°, those for arcs which are greater can be found from them. For if an arc is less than 90°, its supplement will be greater than 90°, and the values of these lines are the same for an arc and its supplement. Thus, if we know the sine of 20°, we also know the sine of its supplement 160°; for the two are equal to each other.

## TABLE OF LOGARITHMIC SINES.

31. In this table are arranged the logarithms of the numerical values of the sines, cosines, tangents, and cotangents of all

---

* These relations are between the numerical *values* of the trigonometrical lines; the algebraic signs, which they have in the different quadrants, are not considered.

the arcs of a quadrant, calculated to a radius of 10,000,000,000. The logarithm of this radius is 10. In the first and last horizontal lines of each page, are written the degrees whose sines, cosines, &c., are expressed on the page. The vertical columns on the left and right, are columns of minutes.

### CASE 1.

*To find, in the table, the logarithmic sine, cosine, tangent, or cotangent of any given arc or angle.*

32. If the angle is less than 45°, look for the degrees in the first horizontal line of the different pages: then descend along the column of minutes, on the left of the page, till you reach the number showing the minutes: then pass along the horizontal line till you come into the column designated, sine, cosine, tangent, or cotangent, as the case may be: the number so indicated is the logarithm sought. Thus, on page 37, for 19° 55′ we find,

| | |
|---|---|
| sine 19° 55′ | 9.532312 |
| cos 19° 55′ | 9.973215 |
| tan 19° 55′ | 9.559097 |
| cot 19° 55′ | 10.440903 |

33. If the angle is greater than 45°, search for the degrees along the bottom line of the different pages: then, ascend along the column of minutes on the right hand side of the page, till you reach the number expressing the minutes: then pass along the horizontal line into the column designated tang, cot, sine, or cosine, as the case may be: the number so pointed out is the logarithm required.

34. The column designated sine, at the top of the page, is

designated by cosine at the bottom; the one designated tang, by cotang, and the one designated cotang, by tang.

The angle found by taking the degrees at the top of the page and the minutes from the first vertical column on the left, is the complement of the angle found by taking the degrees at the bottom of the page, and the minutes traced up in the right hand column to the same horizontal line. Therefore, sine, at the top of the page, should correspond with cosine, at the bottom; cosine with sine, tang with cotang, and cotang with tang, as in the tables (Art. 11).

If the angle is greater than 90°, we have only to subtract it from 180°, and take the sine, cosine, tangent or cotangent of the remainder.

The column of the table next to the column of sines, and on the right of it, is designated by the letter *D*. This column is calculated in the following manner.

Opening the table at any page, as 42, the sine of 24° is found to be 9.609313; that of 24° 01′, 9.609597: their difference is 284; this being divided by 60, the number of seconds in a minute, gives 4.73, which is entered in the column *D*.

Now, supposing the increase of the logarithmic sine to be proportional to the increase of the arc, and it is nearly so for 60″, it follows, that 4.73 is the increase of the sine for 1″. Similarly, if the arc were 24° 20′ the increase of the sine for 1″, would be 4.65. .

The same remarks are applicable in respect of the column *D*, after the column cosine, and of the column *D*, between the tangents and cotangents. The column *D* between the columns tangents and cotangents, answers to both of these columns.

Now, if it were required to find the logarithmic sine of an arc expressed in degrees, minutes, and seconds, we have only to find the degrees and minutes as before; then, multiply the corresponding tabular difference by the seconds, and add the product to the number first found, for the sine of the given arc.

Thus, if we wish the sine of 40° 26′ 28″.

| | | |
|---|---|---|
| The sine 40° 26′ | | 9.811952 |
| Tabular difference | 2.47 | |
| Number of seconds | 28 | |
| Product | 69.16 to be added | 69.16 |
| Gives for the sine of 40° 26′ 28″ | | 9.812021. |

The decimal figures at the right are generally omitted in the final result; but when they exceed five-tenths, the figure on the left of the decimal point is increased by 1; this gives the nearest approximate result.

The tangent of an arc, in which there are seconds, is found in a manner entirely similar. In regard to the cosine and cotangent, it must be remembered, that they increase while the arcs decrease, and decrease as the arcs are increased; consequently, the proportional numbers found for the seconds, must be subtracted, not added.

### EXAMPLES.

1. To find the cosine of 3° 40′ 40″

| | | |
|---|---|---|
| The cosine of 3° 40′ | | 9.999110 |
| Tabular difference | .13 | |
| Number of seconds | 40 | |
| Product | 5.20 to be subtracted | 5.20 |
| Gives for the cosine of 3° 40′ 40″ | | 9.999105 |

2. Find the tangent of 37° 28′ 31″

*Ans.* 9.884592.

3. Find the cotangent of 87° 57′ 59″

*Ans.* 8.550356. #

### CASE II.

*To find the degrees, minutes and seconds, answering to any given logarithmic sine, cosine, tangent or cotangent.*

35. Search in the table, and in the proper column, and if the number be found, the degrees will be shown either at the top or bottom of the page, and the minutes in the side columns, either at the left or right.

But, if the number cannot be found in the table, take from the table the degrees and minutes answering to the nearest less logarithm, the logarithm itself, and also the corresponding tabular difference. Subtract the logarithm taken from the table from the given logarithm, annex two ciphers to the remainder, and then divide the remainder by the tabular difference: the quotient will be seconds, and is to be connected with the degrees and minutes before found; to be added for the sine and tangent, and subtracted for the cosine and cotangent.

### EXAMPLES.

1. Find the arc answering to the sine 9.880054
Sine 49° 20′, next less in the table 9.879963
Tabular difference . . . 1.81)91.00(50″

Hence, the arc 49° 20′ 50″ corresponds to the given sine 9.880054.

2. Find the arc whose cotangent is . 10.008688
cot 44° 26′, next less in the table . 10.008591
Tabular difference . . . 4.21)97.00(23″

Hence, $44° 26'-23'' = 44° 25' 37''$ is the arc answering to the given cotangent 10.008688.

3. Find the arc answering to tangent 9.979110.
*Ans.* $43° 37' 21''$.

4. Find the arc answering to cosine 9.944599.
*Ans.* $28° 19' 45''$.

36. We shall now demonstrate the principal theorems of Plane Trigonometry.

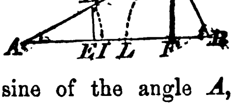
### THEOREM I.

*The sides of a plane triangle are proportional to the sines of their opposite angles.*

Let $ABC$ be a triangle; then will

$$CB : CA :: \sin A : \sin B.$$

For, with $A$ as a centre, and $AD$ equal to the less side $BC$, as a radius, describe the arc $DI$: and with $B$ as a centre and the equal radius $BC$, describe the arc $CL$: now $DE$ is the sine of the angle $A$, and $CF$ is the sine of $B$, to the same radius $AD$ or $BC$. But by similar triangles,



$$AD : DE :: AC : CF.$$

But $AD$ being equal to $BC$, we have

$$BC : \sin A :: AC : \sin B, \text{ or}$$

$$BC : AC :: \sin A : \sin B.$$

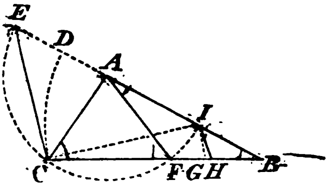
By comparing the sides $AB$, $AC$, in a similar manner, we should find, $AB : AC :: \sin C : \sin B$.

### THEOREM II.

*In any triangle, the sum of the two sides containing either angle, is to their difference, as the tangent of half the sum of the two other angles, to the tangent of half their difference.*

Let $ACB$ be a triangle: then will

$AB + AC : AB - AC : :$ tan $\frac{1}{2}(C + B)$ : tan $\frac{1}{2}(C - B)$.



With $A$ as a centre, and a radius $AC$ the less of the two given sides, let the semicircle $IFCE$ be described, meeting $AB$ in $I$, and $BA$ produced, in $E$. Then, $BE$ will be the sum of the sides, and $BI$ their difference. Draw $CI$ and $AF$.

Since $CAE$ is an outward angle of the triangle $ACB$, it is equal to the sum of the inward angles $C$ and $B$ (Bk. I, Th. xvi.) But the angle $CIE$ being at the circumference, is half the angle $CAE$ at the centre (Bk. II, Th. viii. Cor. 1); that is, half the sum of the angles $C$ and $B$, or equal to $\frac{1}{2}(C + B)$.

The angle $AFC = ACB$, is also equal to $ABC + BAF$; therefore, $BAF = ACB - ABC$.

But, $ICF = \frac{1}{2}(BAF) = \frac{1}{2}(ACB - ABC)$, or $\frac{1}{2}(C - B)$.

With $I$ and $C$ as centres, and the common radius $IC$, let the arcs $CD$ and $IG$ be described, and draw the lines $CE$ and $IH$ perpendicular to $IC$. The perpendicular $CE$ will pass through $E$, the extremity of the diameter $IE$, since the right angle $ICE$ must be inscribed in a semicircle.

But $CE$ is the tangent of $CIE = \frac{1}{2}(C + B)$; and $IH$ is the tangent of $ICB = \frac{1}{2}(C - B)$, to the common radius $CI$.

But since the lines $CE$ and $IH$ are parallel, the triangles $BHI$ and $BCE$ are similar, and give the proportion,

$$BE : BI :: CE : IH; \text{ or}$$

by placing for $BE$ and $BI$, $CE$ and $IH$, their values, we have

$$AB + AC : AB - AC :: \tan \tfrac{1}{2}(C+B) : \tan \tfrac{1}{2}(C-B).$$

## THEOREM III.

*In any plane triangle, if a line is drawn from the vertical angle perpendicular to the base, dividing it into two segments: then, the whole base, or sum of the segments, is to the sum of the two other sides, as the difference of those sides to the difference of the segments.*

Let $BAC$ be a triangle, and $AD$ perpendicular to the base; then will

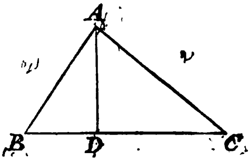
$$BC : CA + AB :: CA - AB : CD - DB$$

For, $\overline{AB}^2 = \overline{BD}^2 + \overline{AD}^2$

(Bk. IV, Th. xii);

and $\overline{AC}^2 = \overline{DC}^2 + \overline{AD}^2$

by subtraction $\overline{AC}^2 - \overline{AB}^2 = \overline{CD}^2 - \overline{BD}^2$.



But since the difference of the squares of two lines is equivalent to the rectangle contained by their sum and difference (Davies' Legendre, Bk. IV, Prop. x,) we have,

$$\overline{AC}^2 - \overline{AB}^2 = (AC + AB) \cdot (AC - AB)$$

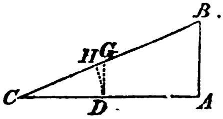
and $\overline{CD}^2 - \overline{DB}^2 = (CD + DB) \cdot (CD - DB)$

therefore, $(CD + DB) \cdot (CD - DB) = (AC + AB) \cdot (AC - AB)$

hence, $CD + DB : AC + AB :: AC - AB : CD - DB.$

### THEOREM IV.

*In any right-angled plane triangle, radius is to the tangent of either of the acute angles, as the side adjacent to the side opposite.*



Let $CAB$ be the proposed triangle, and denote the radius by $R$: then will

$$R : \tan C :: AC : AB.$$

For, with any radius as $CD$ describe the arc $DH$, and draw the tangent $DG$.

From the similar triangles $CDG$ and $CAB$ we have

$$CD : DG :: CA : AB;\text{ hence,}$$

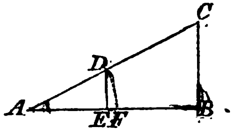
$$R : \tan C :: CA : AB.$$

By describing an arc with $B$ as a centre, we could show in the same manner that,

$$R : \tan B :: AB : AC.$$

### THEOREM V.

*In every right-angled plane triangle, radius is to the cosine of either of the acute angles, as the hypothenuse to the side adjacent.*



Let $ABC$ be a triangle, right-angled at $B$ then will

$$R. : \cos A :: AC : AB.$$

For, from the point $A$ as a centre, with any radius as $AD$, describe the arc $DF$, which will measure the angle $A$, and draw $DE$ perpendicular to $AB$: then will $AE$ be the cosine of $A$.

The triangles $ADE$ and $ACB$, being similar, we have

$$AD : AE :: AC : AB:\text{ that is,}$$

$$R : \cos A :: AC : AB.$$

**Remark.** The relations between the sides and angles of plane triangles, demonstrated in these five theorems, are sufficient to solve all the cases of Plane Trigonometry. Of the six parts which make up a plane triangle, three must be given, and at least one of these a side, before the others can be determined.

If the three angles are given, it is plain, that an indefinite number of similar triangles may be constructed, the angles of which shall be respectively equal to the angles that are given, and therefore, the sides could not be determined.

Assuming, with this restriction, any three parts of a triangle as given, one of the four following cases will always be presented.

I. When two angles and a side are given.
II. When two sides and an opposite angle are given.
III. When two sides and the included angle are given.
IV. When the three sides are given.

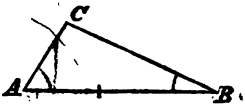
### CASE I.

When two angles and a side are given.

Add the given angles together and subtract their sum from 180 degrees. The remaining parts of the triangle can then be found by Theorem I.

#### EXAMPLES.

1. In a plane triangle $ABC$, there are given the angle $A = 58° \ 07'$, the angle $B = 22° \ 37'$, and the side $AB =$ 408 yards. Required the other parts.



### GEOMETRICALLY.

Draw an indefinite straight line $AB$, and from the scale of equal parts lay off $AB$ equal to 408. Then at $A$ lay off an angle equal to $58^\circ\ 07'$, and at $B$ an angle equal to $22^\circ\ 37'$, and draw the lines $AC$ and $BC$: then will $ABC$ be the triangle required.

The angle $C$ may be measured either with the protractor or the scale of chords (Arts. 16 and 17), and will be found equal to $99^\circ\ 16'$. The sides $AC$ and $BC$ may be measured by referring them to the scale of equal parts (Art. 2). We shall find $AC = 158.9$ and $BC = 351.$ yards.

### TRIGONOMETRICALLY BY LOGARITHMS.

To the angle . . $A = 58^\circ\ 07'$
Add the angle . $B = 22^\circ\ 37'$
Their sum $= 80^\circ\ 44'$
taken from . . . $180^\circ\ 00'$
leaves $C$ . . . $99^\circ\ 16'$ which, exceeding $90^\circ$,
we use its supplement $80^\circ\ 44'$.

180 00
80-44
99-16-
80 44

To find the side $BC$.

| | | | | |
|---|---|---|---|---|
| As sin $C$ | $99^\circ\ 16'$ | . | ar. comp. . | 0.005705 |
| : sin $A$ | $58^\circ\ 07'$ | . | . . . | 9.928972 |
| : : $AB$ | 408 | . | . . . | 2.610660 |
| .: $BC$ | 351.024 | (after rejecting 10) | | 2.545337 |

REMARK. The logarithm of the fourth term of a proportion is obtained by adding the logarithm of the second term to that of the third, and subtracting from their sum the logarithm of the first term. But to subtract the first term is the same as to add its arithmetical complement and reject 10 from the sum (Art. 13): hence, the arithmetical complement of the first term added to the logarithms of the second and third terms, minus ten, will give the logarithm of the fourth term.

To find side *AC*.

| | | | | |
|---|---|---|---|---|
| As sin | $C$ | 99° 16′ | ar. comp. . | 0.005705 |
| : sin | $B$ | 22° 37′ | . . . . | 9.584968 |
| : : | $AB$ | 408 | . . . . | 2.610660 |
| : | $AC$ | 158.976 | . . . . | 2.201333 |

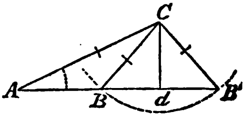
2. In a triangle $ABC$, there are given $A = 38° 25'$, $B = 57° 42'$, and $AB = 400$: required the remaining parts.

*Ans.* $C = 83° 53'$, $BC = 249.974$, $AC = 340.04$.

### CASE II.

When two sides and an opposite angle are given.

In a plane triangle $ABC$, there are given $AC = 216$, $CB = 117$, the angle $A = 22° 37'$, to find the other parts.



### GEOMETRICALLY.

Draw an indefinite right line $ABB'$: from any point as $A$, draw $AC$ making $BAC = 22° 37'$, and make $AC = 216$. With $C$ as a centre, and a radius equal to 117, the other given side, describe the arc $B'B$; draw $B'C$ and $BC$: then will either of the triangles $ABC$ or $AB'C$, answer all the conditions of the question.

### TRIGONOMETRICALLY.

To find the angle $B$.

| | | | |
|---|---|---|---|
| As $BC$ | 117 | . ar. comp. . . | 7.931814 |
| : $AC$ | 216 | . . . . . | 2.334454 |
| : : sin $A$ | 22° 37′ | . . . . . | 9.584968 |
| : sin $B'$ | 45° 13′ 55″, or $ABC$ 134° 46′ 05″ | | 9.851236 |

The ambiguity in this, and similar examples, arises in consequence of the first proportion being true for either of the angles $ABC$, or $AB'C$, which are supplements of each other, and therefore have the same sine (Art. 30). As long as the two triangles exist, the ambiguity will continue. But if the side $CB$, opposite the given angle, is greater than $AC$, the arc $BB'$ will cut the line $ABB'$, on the same side of the point $A$, in but one point, and then there will be only one triangle answering the conditions.

If the side $CB$ is equal to the perpendicular $Cd$, the arc $BB'$ will be tangent to $ABB'$, and in this case also there will be but one triangle. When $CB$ is less than the perpendicular $Cd$, the arc $BB'$ will not intersect the base $ABB'$, and in that case, no triangle can be formed, or it will be impossible to fulfil the conditions of the problem.

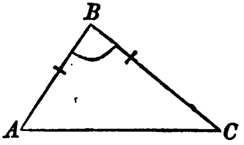
2. Given two sides of a triangle 50 and 40 respectively, and the angle opposite the latter equal to 32°: required the remaining parts of the triangle.

*Ans.* If the angle opposite the side 50 is acute, it is equal to 41° 28′ 59″; the third angle is then equal to 106° 31′ 01″, and the third side to 72.368. If the angle opposite the side

50 is obtuse, it is equal to 138° 31′ 01″, the third angle to 9° 28′ 59″, and the remaining side to 12.436.

### CASE III.

When the two sides and their included angle are given.



Let $ABC$ be a triangle; $AB$, $BC$, the given sides, and $B$ the given angle.

Since $B$ is known, we can find the sum of the two other angles: for

$$A + C = 180° - B \text{ and}$$
$$\tfrac{1}{2}(A + C) = \tfrac{1}{2}(180° - B)$$

We next find half the difference of the angles $A$ and $C$ by Theorem ii., viz.

$$BC + BA : BC - BA :: \tan \tfrac{1}{2}(A + C) : \tan \tfrac{1}{2}(A - C):$$

in which we consider $BC$ greater than $BA$, and therefore $A$ is greater than $C$; since the greater angle must be opposite the greater side.

Having found half the difference of $A$ and $C$, by adding it to the half sum, $\tfrac{1}{2}(A + C)$, we obtain the greater angle, and by subtracting it from half the sum, we obtain the less. That is

$$\tfrac{1}{2}(A + C) + \tfrac{1}{2}(A - C) = A, \text{ and}$$
$$\tfrac{1}{2}(A + C) - \tfrac{1}{2}(A - C) = C.$$

Having found the angles $A$ and $C$, the third side $AC$ may be found by the proportion.

$$\sin A : \sin B :: BC : AC.$$

### EXAMPLES.

1. In the triangle $ABC$, let $BC = 540$, $AB = 450$, and the included angle $B = 80°$: required the remaining parts.

### GEOMETRICALLY.

Draw an indefinite right line $BC$ and from any point, as $B$, lay off a distance $BC = 540$. At $B$ make the angle $CBA = 80°$: draw $BA$ and make the distance $BA = 450$: draw $AC$; then will $ABC$ be the required triangle.

### TRIGONOMETRICALLY.

$BC + BA = 540 + 450 = 990$; and $BC - BA = 540 - 450 = 90$.

$A + C = 180° - B = 180° - 80° = 100°$, and therefore,

$$\tfrac{1}{2}(A + C) = \tfrac{1}{2}(100°) = 50°$$

To find $\frac{1}{2}(A - C)$.

| | | | |
|---|---|---|---|
| As $BC + BA$ | 990 | ar. comp. | 7.004365 |
| : $BC - BA$ | 90 | | 1.954243 |
| :: $\tan \frac{1}{2}(A + C)$ | 50° | | 10.076187 |
| : $\tan \frac{1}{2}(A - C)$ | 6° 11′ | | 9.034795 |

Hence, $50° + 6° 11' = 56° 11' = A$; and $50° - 6° 11' = 43° 49' = C$.

To find the third side $AC$.

| | | | |
|---|---|---|---|
| As $\sin C$ | 43° 49′ | ar. comp. | 0.159672 |
| : $\sin B$ | 80° | | 9.993351 |
| :: $AB$ | 450 | | 2.653213 |
| : $AC$ | 640.082 | | 2.806236 |

2. Given two sides of a plane triangle, 1686 and 960, and their included angle 128° 04′: required the other parts.

*Ans.* Angles, 33° 34′ 39″; 18° 21′ 21″; side 2400.

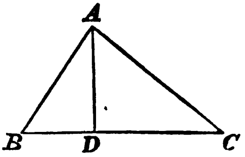
### CASE IV.

Having given the three sides of a plane triangle, to find the angles.

Let fall a perpendicular from the angle opposite the greater side, dividing the given triangle into two right-angled triangles: then find the difference of the segments of the base by Theorem iii. Half this difference being added to half the base, gives the greater segment; and, being subtracted from half the base, gives the less segment. Then, since the greater segment belongs to the right-angled triangle having the greatest hypothenuse, we have the sides and right angle of two right-angled triangles, to find the acute angles.

### EXAMPLES.

1. The sides of a plane triangle being given; viz. $BC = 40$, $AC = 34$ and $AB = 25$: required the angles.



### GEOMETRICALLY.

With the three given lines as sides construct a triangle as in Bk. II. Prob. xi. Then measure the angles of the triangle either with the protractor or scale of chords.

### TRIGONOMETRICALLY.

$$\text{As } BC : AC + AB :: AC - AB : CD - BD$$

$$\text{That is, } 40 : 59 :: 9 : \frac{59 \times 9}{40} = 13.275$$

$$\text{Then, } \frac{40 + 13.275}{2} = 26.6375 = CD$$

$$\text{And } \frac{40 - 13.275}{2} = 13.3625 = BD.$$

In the triangle *DAC*, to find the angle *DAC*.

| | | | | | |
|---|---|---|---|---|---|
| As | | *AC* | 34 | ar. comp. | 8.468521 |
| : | | *DC* | 26.6375 | | 1.425493 |
| : : | sin | *D* | 90° | | 10.000000 |
| : | sin | *DAC* | 51° 34′ 40″ | | 9.894014 |

In the triangle *BAD*, to find the angle *BAD*.

| | | | | | |
|---|---|---|---|---|---|
| As | | *AB* | 25 | ar. comp. | 8.602060 |
| : | | *BD* | 13.3625 | | 1.125887 |
| : : | sin | *D* | 90° | | 10.000000 |
| : | sin | *BAD* | 32° 18′ 35″ | | 9.727947 |

Hence $90° - DAC = 90° - 51° 34' 40'' = 38° 25' 20'' = C$
and $90° - BAD = 90° - 32° 18' 35'' = 57° 41' 25'' = B$
and $BAD + DAC = 51° 34' 40'' + 32° 18' 35'' = 83° 53' 15'' = A$.

2. In a triangle, in which the sides are 4, 5 and 6, what are the angles?

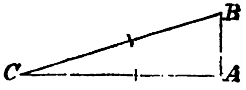
*Ans.* 41° 24′ 35″; 55° 46′ 16″; and 82° 49′ 09″.

### SOLUTION OF RIGHT-ANGLED TRIANGLES.

The unknown parts of a right-angled triangle may be found by either of the four last cases: or, if two of the sides are given, by means of the property that the square of the hypothenuse is equivalent to the sum of the squares of the two other sides. Or the parts may be found by Theorems iv. and v.

#### EXAMPLES.

1. In a right-angled triangle *BAC*, there are given the hypothenuse *BC* = 250, and the base *AC* = 240: required the other parts.



To find the angle $B$.

| | | | | |
|---|---|---|---|---|
| As | $BC$ | 250 | ar. comp. | 7.602060 |
| : | $AC$ | 240 | | 2.380211 |
| :: | $\sin A$ | 90° | | 10.000000 |
| : | $\sin B$ | 73° 44′ 23″ | | 9.982271 |

But $C = 90° - B = 90° - 73° 44' 23'' = 16° 15' 37''$:

Or $C$ may be found from the proportion.

| | | | | |
|---|---|---|---|---|
| As | $CB$ | 250 | ar. comp. | 7.602060 |
| : | $AC$ | 240 | | 2.380211 |
| :: | $R$ | | | 10.000000 |
| : | $\cos C$ | 16° 15′ 37″ | | 9.982271 |

To find side $AB$ by Theorem iv.

| | | | | |
|---|---|---|---|---|
| As | $R$ | | ar. comp. | 0.000000 |
| : | $\tan C$ | 16° 15′ 37″ | | 9.464889 |
| :: | $AC$ | 240 | | 2.380211 |
| ., | $AB$ | 70.0003 | | 1.845100 |

2. In a right-angled triangle $BAC$, there are given $AC = 384$, and $B = 53° 08'$: required the remaining parts.

*Ans.* $AB = 287.96$; $BC = 479.979$; $C = 36° 52'$

DEFINITIONS.

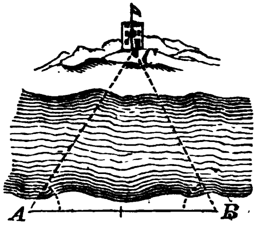
1. A *horizontal angle* is one whose sides are horizontal; its plane is also horizontal.

2. An angle of *elevation* or *depression*, has one horizontal side, and the other oblique, but lying directly above or below the first.

## APPLICATION TO HEIGHTS AND DISTANCES.

### PROBLEM I.

*To determine the horizontal distance to a point which is inaccessible by reason of an intervening river.*



Let $C$ be the point. Measure along the bank of the river a horizontal base line $AB$, and select the stations $A$ and $B$, in such a manner that each can be seen from the other, and the point $C$ from both of them. Then measure the horizontal angles $CAB$ and $CBA$, with an instrument adapted to that purpose.

Let us suppose that we have found $AB = 600$ yards, $CAB = 57° 35'$ and $CBA = 64° 51'$.

The angle $C = 180° - (A + B) = 57° 34'$.

To find the distance $BC$.

| | | | | |
|---|---|---|---|---|
| As | sin $C$ | $57° 34'$ | ar. comp. | 0.073649 |
| : | sin $A$ | $57° 35'$ | | 9.926431 |
| : : | $AB$ | 600 | | 2.778151 |
| : | $BC$ | 600.11 yards. | | 2.778231 |

To find the distance $AC$.

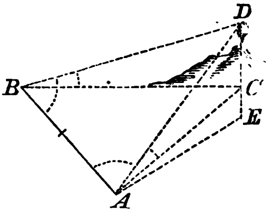
| | | | | |
|---|---|---|---|---|
| As | sin $C$ | $57° 34'$ | ar. comp. | 0.073649 |
| : | sin $B$ | $64° 51'$ | | 9.956744 |
| : : | $AB$ | 600 | | 2.778151 |
| : | $AC$ | 643.94 yards. | | 2.808544 |

### PROBLEM II.

### *To determine the altitude of an inaccessible object above a given horizontal plane.*

#### FIRST METHOD.

Suppose $D$ to be the inaccessible object, and $BC$ the horizontal plane from which the altitude is to be estimated: then, if we suppose $DC$ to be a vertical line, it will represent the required distance.



Measure any horizontal base line, as $BA$; and at the extremities $B$ and $A$, measure the horizontal angles $CBA$ and $CAB$. Measure also, the angle of elevation $DBC$.

Then in the triangle $CBA$ there will be known, two angles and the side $AB$; the side $BC$ can therefore be determined. Having found $BC$, we shall have, in the right-angled triangle $DBC$, the base $BC$ and the angle at the base, to find the perpendicular $DC$, which measures the altitude of the point $D$ above the horizontal plane $BC$.

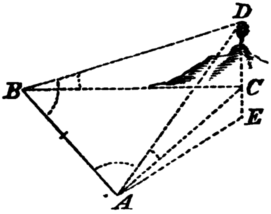
Let us suppose that we have found

$BA = 780$ yards, the horizontal angle $CBA = 41° \, 24'$, the horizontal angle $CAB = 96° \, 28'$, and the angle of elevation $DBC = 10° \, 43'$.

In the triangle $BCA$, to find the horizontal distance $BC$.

The angle $BCA = 180° - (41° \, 24' + 96° \, 28') = 42° \, 08' = C$.

| | | | | |
|---|---|---|---|---|
| As | sin $C$ | . 42° 08′ | ar. comp. . | 0.173369 |
| : | sin $A$ | . 96° 28′ | . . . . | 9.997228 |
| :: | $AB$ | . 780 | . . . . | 2.892095 |
| : | $BC$ | . 1155.29 | . . . . | 3.062692 |



In the right-angled triangle *DBC*, to find *DC*.

| | | | | |
|---|---|---|---|---|
| As | *R* | | ar. comp. . . | 0.000000 |
| : tan | *DBC* | 10° 43′ | . . . | 9.277043 |
| : : | *BC* | 1155.29 | . . . | 3.062692 |
| : | *DC* | 218.64 | . . . | 2.339735 |

Remark I. It might, at first, appear that the solution which we have given, requires that the points *B* and *A* should be in the same horizontal plane; but it is entirely independent of such a supposition.

For, the horizontal distance, which is represented by *BA*, is the same, whether the station *A* is on the same level with *B*, above it, or below it. The horizontal angles *CAB* and *CBA* are also the same, so long as the point *C* is in the vertical line *DC*. Therefore, if the horizontal line through *A* should cut the vertical line *DC*, at any point as *E*, above or below *C*, *AB* would still be the horizontal distance between *B* and *A*, and *AE* which is equal to *AC*, would be the horizontal distance between *A* and *C*.

If at *A*, we measure the angle of elevation of the point *D*, we shall know in the right-angled triangle *DAE*, the base *AE*, and the angle at the base; from which the perpendicular *DE* can be determined.

Let us suppose that we had measured the angle of elevation $DAE$, and found it equal to 20° 15′.

First: In the triangle $BAC$, to find $AC$ or its equal $AE$.

| | | | | |
|---|---|---|---|---|
| As | sin $C$ | 42° 08′ | ar. comp. | 0.173369 |
| : | sin $B$ | 41° 24′ | | 9.820406 |
| :: | $AB$ | 780 | | 2.892095 |
| : | $AC$ | 768.9 | | 2.885870 |

In the right-angled triangle $DAE$, to find $DE$.

| | | | | |
|---|---|---|---|---|
| As | $R$ | | ar. comp. | 0.000000 |
| : | tan $A$ | 20° 15′ | | 9.566932 |
| :: | $AE$ | 768.9 | | 2.885870 |
| : | $DE$ | 283.66 | | 2.452802 |

Now, since $DC$ is less than $DE$, it follows that the station $B$ is above the station $A$. That is,

$$DE - DC = 283.66 - 218.64 = 65.02 = EC,$$

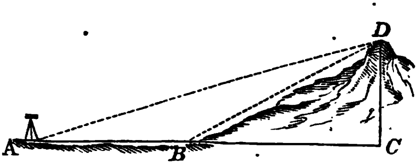
which expresses the vertical distance that the station $B$ is above the station $A$.

REMARK II. It should be remembered, that the vertical distance which is obtained by the calculation, is estimated from a horizontal line passing through the eye at the time of observation. Hence, the height of the instrument is to be added, in order to obtain the true result.

### SECOND METHOD.

When the nature of the ground will admit of it, measure a base line $AB$ in the direction of the object $D$. Then measure with the instrument the angles of elevation at $A$ and $B$.

Then, since the outward angle $DBC$ is equal to the sum



of the angles *A* and *ADB*, it follows, that the angle *ADB* is equal to the difference of the angles of elevation at *A* and *B*. Hence, we can find all the parts of the triangle *ADB*. Having found *DB*, and knowing the angle *DBC*, we can find the altitude *DC*.

This method supposes that the stations *A* and *B* are on the same horizontal plane; and therefore can only be used when the line *AB* is nearly horizontal.

Let us suppose that we have measured the base line, and the two angles of elevation, and

$$\text{found} \begin{cases} AB = 975 \text{ yards}, \\ A = 15^\circ\ 36', \\ DBC = 27^\circ\ 29'; \end{cases}$$

required the altitude *DC*.

First: $ADB = DBC - A = 27^\circ\ 29' - 15^\circ\ 36' = 11^\circ\ 53'$.

In the triangle *ADB*, to find *BD*.

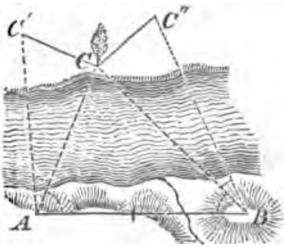
| | | | | |
|---|---|---|---|---|
| As | sin *D* | 11° 53′ | ar. comp. . | 0.686302 |
| : | sin *A* | 15° 36′ | . . . . | 9.429623 |
| : : | *AB* | 975 | . . . . | 2.989005 |
| : | *DB* | 1273.3 | . . . | 3.104930 |

In the triangle *DBC*, to find *DC*.

| | | | | |
|---|---|---|---|---|
| As | *R* | | ar. comp. . | 0.000000 |
| : | sin *B* | 27° 29′ | . . . | 9.664163 |
| : : | *DB* | 1273.3 | . . . | 3.104930 |
| : | *DC* | 587.61 | . . . | 2.769093 |

### PROBLEM III.

*To determine the perpendicular distance of an object below a given horizontal plane.*



Suppose $C$ to be directly over the given object, and $A$ the point through which the horizontal plane is supposed to pass.

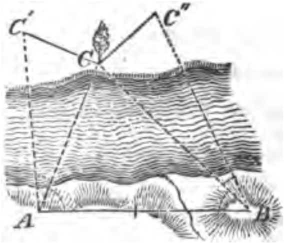
Measure a horizontal base line $AB$, and at the stations $A$ and $B$ conceive the two horizontal lines $AC$, $BC$, to be drawn. The oblique lines from $A$ and $B$ to the object will be the hypothenuses of two right-angled triangles, of which $AC$, $BC$, are the bases. The perpendiculars of these triangles will be the distances from the horizontal lines $AC$, $BC$, to the object. If we turn the triangles about their bases $AC$, $BC$, until they become horizontal, the object, in the first case, will fall at $C'$, and in the second at $C''$.

Measure the horizontal angles $CAB$, $CBA$, and also the angles of depression $C'AC$, $C''BC$.

Let us suppose that we have

$$\text{found}\begin{cases} AB = 672 \text{ yards} \\ BAC = 72^\circ\ 29' \\ ABC = 39^\circ\ 20' \\ C'AG = 27^\circ\ 49' \\ C''BC = 19^\circ\ 10' \end{cases}$$

First: In the triangle $ABC$, the horizontal angle $ACB = 180^\circ - (A + B) = 180^\circ - 111^\circ\ 49' = 68^\circ\ 11'$.



To find the horizontal distance $AC$.

| | | | |
|---|---|---|---|
| As | sin $C$ | 68° 11′ ar. comp. | 0.032275 |
| : | sin $B$ | 39° 20′ | 9.801973 |
| :: | $AB$ | 672 | 2.827369 |
| : | $AC$ | 458.79 | 2.661617 |

To find the horizontal distance $BC$.

| | | | |
|---|---|---|---|
| As | sin $C$ | 68° 11′ ar. comp. | 0.032275 |
| : | sin $A$ | 72° 29′ | 9.979380 |
| :: | $AB$ | 672 | 2.827369 |
| : | $BC$ | 690.28 | 2.839024 |

In the triangle $CAC'$, to find $CC'$.

| | | | |
|---|---|---|---|
| As | $R$ | ar. comp. | 0.000000 |
| : | tan $C'AC$ | 27° 49′ | 9.722315 |
| :: | $AC$ | 458.79 | 2.661617 |
| : | $CC'$ | 242.06 | 2.383932 |

In the triangle $CBC''$, to find $CC''$

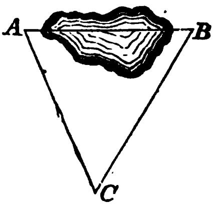
| | | | |
|---|---|---|---|
| As | $R$ | ar. comp. | 0.000000 |
| : | tan $C''BC$ | 19° 10′ | 9.541061 |
| :: | $BC$ | 690.28 | 2.839024 |
| : | $CC''$ | 239.93 | 2.380085 |

Hence also, $CC' - CC'' = 242.06 - 239.93 = 2.13$ yards which is the height of the station $A$ above station $B$.

PROBLEMS.

1. Wanting to know the distance between two inaccessible objects, which lie in a direct line from the bottom of a tower of 120 feet in height, the angles of depression are measured, and are found to be, of the nearer 57°, of the more remote 25° 30′: required the distance between them.

*Ans.* 173.656 feet.



2. In order to find the distance between two trees $A$ and $B$, which could not be directly measured because of a pool which occupied the intermediate space, the distances of a third point $C$ from each of them were measured, and also the included angle $ACB$: it was found that

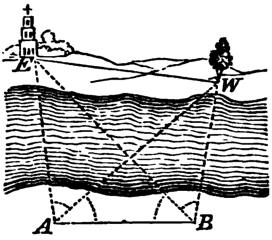
$$CB = 672 \text{ yards}$$
$$CA = 588 \text{ yards}$$
$$ACB = 55^\circ\ 40';$$

required the distance $AB$.

*Ans.* 592.967 yards.

3. Being on a horizontal plane, and wanting to ascertain the height of a tower, standing on the top of an inaccessible hill, there were measured, the angle of elevation of the top of the hill 40°, and of the top of the tower 51°; then measuring in a direct line 180 feet farther from the hill, the angle of elevation of the top of the tower was 33° 45′; required the height of the tower.

*Ans.* 83.998 feet.

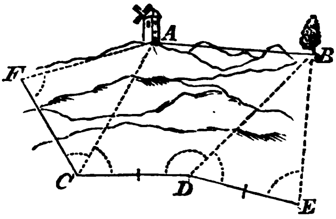


4. Wanting to know the horizontal distance between two inaccessible objects *E* and *W*, the following measurements were made,

$$\text{viz.}\left\{\begin{array}{l} AB = 536 \text{ yards} \\ BAW = 40^\circ\ 16' \\ WAE = 57^\circ\ 40' \\ ABE = 42^\circ\ 22' \\ EBW = 71^\circ\ 07' \end{array}\right.$$

required the distance *EW*.

*Ans.* 939.527 yards.



5. Wanting to know the horizontal distance between two inaccessible objects *A* and *B*, and not finding any station from which both of them could be seen, two points *C* and *D*, were chosen, at a distance from each other, equal to 200 yards; from the former of these points *A* could be seen, and from the latter *B*, and at each of the points *C* and *D* a staff was set up. From *C* a distance *CF* was measured, not in the direction *DC*, equal to 200 yards, and from *D* a distance *DE* equal to 200 yards, and the following angles taken,

$$\text{viz.}\left\{\begin{array}{ll} AFC = 83^\circ\ 00' & BDE = 54^\circ\ 30' \\ ACD = 53^\circ\ 30' & BDC = 156^\circ\ 25' \\ ACF = 54^\circ\ 31' & BED = 88^\circ\ 30' \end{array}\right.$$

*Ans.* $AB = 345.467$ yards.

# APPLICATIONS

# OF

# GEOMETRY.

## MENSURATION OF SURFACES.

### DEFINITIONS.

1 The area of any figure has already been defined to be the measure of its surface (Bk. IV. Def. 7). This measure is merely the number of squares which the figure contains.

A square whose side is one inch, one foot, or one yard, &c., is called the *measuring unit;* and the area or contents of a figure is expressed by the number of such squares which the figure contains.

2. In the questions involving decimals, the decimals are generally carried to four places, and then taken to the nearest figure. That is, if the fifth decimal figure is 5, or greater than 5, the fourth figure is increased by one.

3. Surveyors, in measuring land, generally use a chain called Gunter's chain. This chain is four rods, or 66 feet in length, and is divided into 100 links.

4. An *acre* is a surface equal in extent to 10 square chains; that is, equal to a rectangle of which one side is ten chains and the other side one chain.

One quarter of an acre, is called a *rood.*

Since the chain is 4 rods in length, 1 square chain contains 16 square rods; and therefore, an acre, which is 10 square chains, contains 160 square rods, and a rood contains 40 square rods. The square rods are called perches.

5. Land is generally computed in acres, roods, and perches which are respectively designated by the letters $A$, $R$, $P$.

When the linear dimensions of a survey are chains or links the area will be expressed in square chains or square links and it is necessary to form a rule for reducing this area to acres, roods, and perches. For this purpose, let us form the following

TABLE.

1 square chain$=100\times100=10000$ square links.

1 acre$=10$ square chains$=100000$ square links.

1 acre$=4$ roods$=160$ perches.

1 square mile$=6400$ square chains$=640$ acres.

6. Now, when the linear dimensions are links, the area will be expressed in square links, and may be reduced to acres by dividing by 100000, the number of square links in an acre: that is, by pointing off five decimal places from the right hand.

If the decimal part be then multiplied by 4, and five places of decimals pointed off from the right hand, the figures to the left hand will express the roods.

If the decimal part of this result be now multiplied by 40, and five places for decimals pointed off, as before, the figures to the left will express the perches.

If one of the dimensions be in links, and the other in chains, the chains may be reduced to links by annexing two ciphers, or, the multiplication may be made without annexing the ciphers, and the product reduced to acres and decimals of an acre, by pointing off three decimal places at the right hand.

When both dimensions are in chains, the product is reduced to acres by dividing by 10, or pointing off one decimal place.

From which we conclude: that,

I. *If links be multiplied by links, the product is reduced to acres by pointing off five decimal places from the right hand.*

II. *If chains be multiplied by links, the product is reduced to acres by pointing off three decimal places from the right hand.*

III. *If chains be multiplied by chains, the product is reduced to acres by pointing off one decimal place from the right hand.*

7. Since there are 16.5 feet in a rod, a square rod is equal to $16.5 \times 16.5 = 272.25$ square feet.

If the last number be multiplied by 160, we shall have

$272.25 \times 160 = 43560$ the square feet in an acre.

Since there are 9 square feet in a square yard, if the last number be divided by 9, we obtain

$4840 =$ the number of square yards in an acre.

### PROBLEM I.

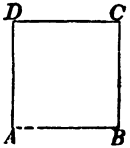
To find the area of a square, a rectangle, a rhombus, or a parallelogram.

### RULE.

*Multiply the base by the perpendicular height and the product will be the area* (Bk. IV. Th. viii).

### EXAMPLES.

1. Required the area of the square *ABCD*, each of whose sides is 36 feet.



We multiply *two* sides of he square together, and the product is the area in square feet.

*Operation.*

$36 \times 36 = 1296$ *sq. ft.*

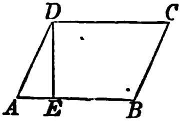
2. How many acres, roods, and perches, in a square whose side is 35.25 chains? *Ans.* 124 *A.* 1 *R.* 1 *P.*

3. What is the area of a square whose side is 8 feet 4 inches? *Ans.* 69 *ft.* 5′ 4″.

4. What is the contents of a square field whose side is 46 rods? *Ans.* 13 *A.* 0 *R.* 36 *P.*

5. What is the area of a square whose side is 4769 yards? *Ans.* 22743361 *sq. yds*

6. What is the area of the parallelogram *ABCD*, of which the base *AB* is 64 feet, and altitude *DE*, 36 feet?



We multiply the base 64, by the perpendicular height 36, and the product is the required area.

*Operation.*

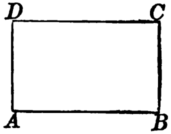
$64 \times 36 = 2304$ *sq. ft.*

7. What is the area of a parallelogram whose base is 12.25 yards, and altitude 8.5? *Ans.* 104,125 *sq. yds.*

8. What is the area of a parallelogram whose base is 8.75 chains, and altitude 6 chains? *Ans.* 5 *A.* 1 *R.* 0 *P.*

9. What is the area of a parallelogram whose base is 7 feet 9 inches, and altitude 3 feet 6 inches? *Ans.* 27 *sq. ft.* 1′ 6″.

10. To find the area of a rectangle *ABCD*, of which the base $AB=45$ yards, and the altitude $AD=15$ yards.



Here we simply multiply the base by the altitude, and the product is the area.

*Operation.*

$45\times15=675$ *sq. yds.*

11. What is the area of a rectangle whose base is 14 feet 6 inches, and breadth 4 feet 9 inches?

*Ans.* 68 *sq. ft.* 10′ 6″.

12. Find the area of a rectangular board whose length is 112 feet, and breadth 9 inches. *Ans.* 84 *sq. ft.*

13. Required the area of a rhombus whose base is 10.51 and breadth 4.28 chains. *Ans.* 4 *A.* 1 *R.* 39.7 *P*+.

14. Required the area of a rectangle whose base is 12 feet 6 inches, and altitude 9 feet 3 inches.

*Ans.* 115 *sq. ft.* 7′ 6″

PROBLEM II.

To find the area of a triangle, when the base and altitude are known.

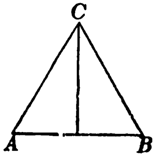
RULE.

I. *Multiply the base by the altitude, and half the product will be the area.*

II. *Multiply the base by half the altitude and the product will be the area* (Bk. IV. Th. ix).

EXAMPLES.

1. Required the area of the triangle *ABC*, whose base *AB* is 10,75 feet, and altitude 7,25 feet.



| We first multiply the base by the altitude, and then divide the product by 2. | *Operation.*<br>10,75 × 7,25 = 77,9375<br>and<br>77,9375 ÷ 2 = 38,96875<br>= area. |
|---|---|

2. What is the area of a triangle whose base is 18 feet 4 inches, and altitude 11 feet 10 inches?

*Ans.* 108 *sq. ft.* 5′ 8″.

3. What is the area of a triangle whose base is 12.25 chains, and altitude 8.5 chains? *Ans.* 5 *A.* 0 *R.* 33 *P.*

4. What is the area of a triangle whose base is 20 feet, and altitude 10.25 feet. *Ans.* 102.5 *sq. ft.*

5. Find the area of a triangle whose base is 625 and altitude 520 feet. *Ans.* 162500 *sq. ft.*

6. Find the number of square yards in a triangle whose base is 40 and altitude 30 feet. *Ans.* 66⅔ *sq. yds.*

7. What is the area of a triangle whose base is 72.7 yards, and altitude 36.5 yards? *Ans.* 1326,775 *sq. yds*

### PROBLEM III.

To find the area of a triangle when the three sides are known.

#### RULE,

I. *Add the three sides together and take half their sum.*

II. *From this half sum take each side separately.*

III. *Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.*

EXAMPLES.

1. Find the area of a triangle whose sides are 20, 30, and 40 rods.

| | | | |
|---|---|---|---|
| 20 | 45 | 45 | 45 |
| 30 | 20 | 30 | 40 |
| 40 | 25 *1st rem.* | 15 *2d rem.* | 5 *3d rem.* |
| 2)90 | | | |
| 45 half sum, | | | |

Then, to obtain the product, we have

$$45 \times 25 \times 15 \times 5 = 84375;$$

from which we find

$$\text{area} = \sqrt{84375} = 290{,}4737 \text{ perches.}$$

2. How many square yards of plastering are there in a triangle, whose sides are 30, 40, and 50 feet? *Ans.* 66$\frac{2}{3}$.

3. The sides of a triangular field are 49 chains, 50.25 chains, and 25.69: what is its area?

*Ans.* 61 *A.* 1 *R.* 39,68 *P.*

4. What is the area of an isosceles triangle, whose base is 20, and each of the equal sides 15? *Ans.* 111.803.

5. How many acres are there in a triangle whose three sides are 380, 420 and 765 yards. *Ans.* 9 *A.* 0 *R.* 38 *P.*

6. How many square yards in a triangle whose sides are 13, 14, and 15 feet. *Ans.* 9$\frac{1}{3}$.

7 What is the area of an equilateral triangle whose side is 25 feet? *Ans.* 270.6329 *sq. ft.*

8. What is the area of a triangle whose sides are 24, 36, and 48 yards? *Ans* 418.282 *sq. yds.*

### PROBLEM IV.

To find the hypothenuse of a right angled triangle when the base and perpendicular are known.

### RULE.

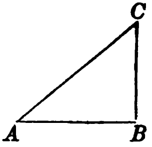
I. *Square each of the sides separately.*

II. *Add the squares together.*

III. *Extract the square root of the sum, which will be the hypothenuse of the triangle* (Bk. IV. Th. xii).

### EXAMPLES.

1. In the right angled triangle $ABC$, we have, $AB = 30$ feet, $BC = 40$ feet, to find $AC$.



We first square each side, and then take the sum, of which we extract the square root, which gives

$$AC = \sqrt{2500} = 50 \text{ feet.}$$

*Operation.*

$\overline{30}^2 = 900$

$\overline{40}^2 = 1600$

sum $= 2500$

2. The wall of a building, on the brink of a river, is 120 feet high, and the breadth of the river 70 yards: what is the length of a line which would reach from the top of the wall to the opposite edge of the river? *Ans.* 241.86 *ft.*

3. The side roofs of a house of which the eaves are of the same height, form a right angle at the top. Now, the length of the rafters on one side is 10 feet, and on the other 14 feet: what is the breadth of the house? *Ans.* 17.204 *ft.*

4. What would be the width of the house, in the last example, if the rafters on each side were 10 feet?

*Ans.* 14.142 *ft.*

5. What would be the width, if the rafters on each side were 14 feet? *Ans.* 19.7989 *ft.*

### PROBLEM V.

When the hypothenuse and one side of a right angled triangle are known, to find the other side.

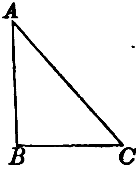
### RULE.

*Square the hypothenuse and also the other given side, and take their difference: extract the square root of this difference, and the result will be the required side* (Bk. IV. Th. xii. Cor.).

### EXAMPLES.

1. In the right angled triangle $ABC$, there are given

$AC=50$ feet, and $AB=40$ feet, required the side $BC$.



We first square the hypothenuse and the other side, after which we take the difference, and then extract the square root, which gives

*Operation.*

$$\begin{aligned} \overline{50}^2 &= 2500 \\ \overline{40}^2 &= 1600 \\ \text{Diff.} &= 900 \end{aligned}$$

$$BC=\sqrt{900}=30 \text{ feet.}$$

2 The height of a precipice on the brink of a river is 103 feet, and a line of 320 feet in length will just reach from the top of it to the opposite bank: required the breadth of the river. *Ans.* 302.9703 *ft.*

3. The hypothenuse of a triangle is 53 yards, and the perpendicular 45 yards: what is the base? *Ans.* 28 *yds.*

4. A ladder 60 feet in length, will reach to a window 40

feet from the ground on one side of the street, and by turning it over to the other side, it will reach a window 50 feet from the ground: required the breadth of the street.

*Ans.* 77.8875 *ft.*

### PROBLEM VI.

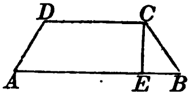
**To find the area of a trapezoid.**

#### RULE.

*Multiply the sum of the parallel sides by the perpendicular distance between them, and then divide the product by two: the quotient will be the area* (Bk. IV. Th. x).

#### EXAMPLES.

1. Required the area of the trapezoid *ABCD*, having given



$AB=321.51$ feet, $DC=214.24$ feet, and $CE=171.16$ feet

We first find the sum of the sides, and then multiply it by the perpendicular height, after which, we divide the product by 2, for the area.

*Operation.*

$321.51+214.24=535.75=$ sum of parallel sides.

Then,

$535.75\times171.16=91698.97$

and, $\frac{91698.97}{2}=45849.485$

$=$the area.

2. What is the area of a trapezoid, the parallel sides of which, are 12.41 and 8.22 chains and the perpendicular distance between them 5.15 chains?

*Ans.* 5 *A.* 1 *R.* 9.956 *P.*

3. Required the area of a trapezoid whose parallel sides

are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches.

*Ans.* 230 *sq. ft.* 5′ 7″.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards. *Ans.* 176.03125 *sq. yds.*

5. What is the area of a trapezoid whose parallel sides are 7.50 chains, and 12.25 chains, and the perpendicular height 15.40 chains? *Ans.* 15 *A.* 0 *R.* 33.2 *P*

### PROBLEM VII.

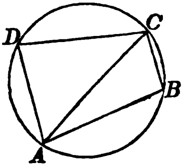
To find the area of a quadrilateral.

### RULE.

*Measure the four sides of the quadrilateral, and also one of the diagonals: the quadrilateral will thus be divided into two triangles, in both of which all the sides will be known. Then, find the areas of the triangles separately, and their sum will be the area of the quadrilateral.*

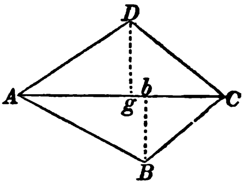
### EXAMPLES.

1. Suppose that we have measured the sides and diagonal $AC$, of the quadrilateral $ABCD$, and found



$AB=40.05$ chains; $CD=29.87$ chains,
$BC=26.27$ chains $AD=37.07$ chains,
and $AC=55$ chains:
required the area of the quadrilateral.

*Ans.* 101 *A.* 1 *R.* 15 *P*



REMARK.—Instead of measuring the four sides of the quadrilateral, we may let fall the perpendiculars $Bb$, $Dg$, on the diagonal $AC$. The area of the triangles may then be determined by measuring these perpendiculars and diagonal $AC$. The pendiculars are, $Dg=$ 18.95 chains, and $Bb=17.92$ chains.

2. Required the area of a quadrilateral whose diagonal is 80.5, and two perpendiculars 24.5, and 30.1 feet.

*Ans.* 2197.65 *sq. ft.*

3. What is the area of a quadrilateral whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? *Ans.* 6347 *sq. ft.* 3′.

4. How many square yards of paving in a quadrilateral whose diagonal is 65 feet, and the two perpendiculars 28, and $33\frac{1}{2}$ feet? *Ans.* $222\frac{1}{12}$ *sq. yds.*

5. Required the area of a quadrilateral whose diagonal is 42 feet, and the two perpendiculars 18, and 16 feet.

*Ans.* 714 *sq. ft.*

6. What is the area of a quadrilateral in which the diagonal is 320.75 chains, and the two perpendiculars 69.73 chains, and 130.27 chains? *Ans.* 3207 *A.* 2 *R.*

### PROBLEM VIII.

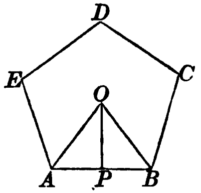
To find the area of a regular polygon.

### RULE.

*Multiply half the perimeter of the figure by the perpendicular let fall from the centre on one of the sides, and the product will be the area* (Bk. IV. Th. xxvi)

EXAMPLES.

1. Required the area of the regular pentagon *ABCDE*, each of whose sides *AB*, *BC*, &c., is 25 feet, and the perpendicular *OP*, 17.2 feet.



We first multiply one side by the number of sides and divide the product by 2: this gives half the perimeter which we multiply by the perpendicular for the area.

*Operation.*

$\frac{25\times5}{2}=62.5$=half the perimeter. Then,

$62.5\times17.2=1075$ *sq. ft.*=the area.

2. The side of a regular pentagon is 20 yards, and the perpendicular from the centre on one of the sides 13,76382; required the area.

*Ans.* 688.191 *sq. yds.*

3. The side of a regular hexagon is 14, and the perpendicular from the centre on one of the sides 12.1243556: required the area.

*Ans.* 509.2229352 *sq. ft.*

4. Required the area of a regular hexagon whose side is 14 6, and perpendicular from the centre 12.64 feet.

*Ans.* 553.632 *sq ft.*

5. Required the area of a heptagon whose side is 19,38 and perpendicular 20 feet.

*Ans.* 1356.6 *sq. ft.*

The following table shows the areas of the ten regular polygons when the side of each is equal to 1 : it also shows the length of the radius of the inscribed circle.

| Number of sides. | Names. | Areas. | Radius of inscribed circle. |
|---|---|---|---|
| 3 | Triangle, | 0.4330127 | 0.2886751 |
| 4 | Square, | 1.0000000 | 0.5000000 |
| 5 | Pentagon, | 1.7204774 | 0.6881910 |
| 6 | Hexagon, | 2.5980762 | 0.8660254 |
| 7 | Heptagon, | 3.6339124 | 1.0382617 |
| 8 | Octagon, | 4.8284271 | 1.2071068 |
| 9 | Nonagon, | 6.1818242 | 1.3737387 |
| 10 | Decagon, | 7.6942088 | 1.5388418 |
| 11 | Undecagon, | 9.3656404 | 1.2028437 |
| 12 | Dodecagon, | 11.1961524 | 1.8660254 |

Now, since the areas of similar polygons are to each other as the squares described on their homologous sides (Bk. IV Th. xx), we have

$1^2$ : tabular area : : any side squared : area.

Hence, to find the area of a regular polygon, we have the following

RULE.

I *Square the side of the polygon.*

II. *Multiply the square so found, by the tabular area set opposite the polygon of the same number of sides, and the product will be the area.*

EXAMPLES.

1. What is the area of a regular hexagon whose side is 20?

$\overline{20}^2=400$ and tabular area=2,5980762.

Hence,

$2.5980762 \times 400 = 1039.23048 =$ the area.

2. What is the area of a pentagon whose side is 25? *Ans.* 1075.298375.

3. What is the area of a heptagon whose side is 30 feet? *Ans.* 3270.52116.

4. What is the area of an octagon whose side is 10 feet? *Ans.* 482.84271 *sq. ft*

5. The side of a nonagon is 50: what is its area? *Ans.* 15454.5605

6. The side of an undecagon is 20: what is its area? *Ans.* 3746.25616.

7. The side of a dodecagon is 40: what is its area? *Ans.* 17913.84384.

### PROBLEM IX.

To find the area of a long and irregular figure, bounded on one side by a straight line.

### RULE.

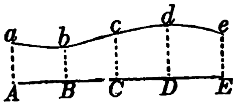
I. *Divide the right line or base into any number of equal parts, and measure the breadth of the figure at the points of division, and also at the extremities of the base.*

II. *Add together the intermediate breadths, and half the sum of the extreme ones.*

III. *Multiply this sum by the base line, and divide the product by the number of equal parts of the base.*

### EXAMPLES.

1. The breadths of an irregular figure, at five equidistant places, *A*, *B*, *C*, *D*, and *E*, being 8.20 chains, 7.40 chains,



9.20 chains, 10.20 chains, and 8.60 chains, and the whole length 40 chains: required the area.

8.20
8.60
2)16.80
8.40 mean of the extremes.
7.40
9.20
10.20
35.20 the sum.

35.20
40
4)1408.00
352.00 square chains.

*Ans.* 35 *A.* 32 *P.*

2. The length of an irregular piece of land being 21 chains and the breadths, at six equidistant points, being 4.35 chains 5.15 chains, 3.55 chains, 4.12 chains, 5.02 chains, and 6.10 chains: required the area. *Ans.* 9 *A.* 2 *R.* 30 *P.*

3. The length of an irregular figure is 84 yards, and the breadths at six equidistant places are 17.4; 20.6; 14.2; 16.5; 20.1; and 24.4: what is the area? *Ans.* 1550.64 *sq. yds.*

4. The length of an irregular field is 39 rods, and its breadths at five equidistant places, are 4.8; 5.2; 4.1; 7.3, and 7.2 rods: what is its area? *Ans.* 220.35 *sq. rods.*

5. The length of an irregular field is 50 yards, and its breadths at seven equidistant points, are 5.5; 6.2; 7.3; 6; 7.5; 7; and 8.8 yards: what is its area?

*Ans.* 342.916 *sq. yds.*

6. The length of an irregular figure being 37.6, and the breadths at nine equidistant places, 0; 4.4; 6.5; 7.6; 5.4; 8; 5.2; 6.5; and 6.1: what is the area? *Ans.* 219.255.

### PROBLEM X.

To find the circumference of a circle when the diameter is known.

### RULE

*Multiply the diameter by* 3.1416, *and the product will be the circumference.*

### EXAMPLES.

1. What is the circumference of a circle whose diameter is 17?

We simply multiply the number 3.1416 by the diameter, and the product is the circumference.

*Operation.*

3.1416×17=53.4072,

which is the circumference.

2. What is the circumference of a circle whose diameter is 40 feet? *Ans.* 125.664 *ft.*

3. What is the circumference of a circle whose diameter is 12 feet? *Ans.* 37.6992 *ft.*

4. What is the circumference of a circle whose diameter is 22 yards? *Ans.* 69.1152 *yds.*

5. What is the circumference of the earth—the mean diameter being about 7921 miles? *Ans.* 24884.6136 *mi.*

### PROBLEM XI.

To find the diameter of a circle when the circumference is known.

### RULE.

*Divide the circumference by the number* 3.1416, *and the quotient will be the diameter.*

### EXAMPLES.

1. The circumference of a circle is 69.1152 yards: what is the diameter?

We simply divide the circumference by 3.1416, and the quotient 22 is the diameter sought.

*Operation.*

$$\begin{array}{r} 3.1416)69\,1152(22 \\ \underline{62832\phantom{0}} \\ 62832 \\ \underline{62832} \end{array}$$

2. What is the diameter of a circle whose circumference is 11652.1944 feet? *Ans.* 3709.

3. What is the diameter of a circle whose circumference is 6850? *Ans.* 2180.4176.

4. What is the diameter of a circle whose circumference is 50? *Ans.* 15.915.

5. If the circumference of a circle is 25000.8528, what is the diameter? *Ans.* 7958.

### PROBLEM XII.

To find the length of a circular arc, when the number of degrees which it contains, and the radius of the circle are known.

### RULE.

*Multiply the number of degrees by the decimal* .01745, *and the product arising by the radius of the circle.*

### EXAMPLES.

1. What is the length of an arc of 30 degrees, in a circle whose radius is 9 feet.

We merely multiply the given decimal by the number of degrees, and by the radius.

*Operation.*

$.01745 \times 30 \times 9 = 4.7115$, which is the length of the arc

REMARK.—When the arc contains degrees and minutes, reduce the minutes to the decimals of a degree, which is done by dividing them by 60.

2. What is the length of an arc containing 12° 10′ or 12⅙,° the diameter of the circle being 20 yards?

*Ans.* 2.1231

3. What is the length of an arc of 10° 15′ or 10¼°, in a circle whose diameter is 68? *Ans.* 6.0813.

### PROBLEM XIII.

To find the length of the arc of a circle when the chord and radius are given.

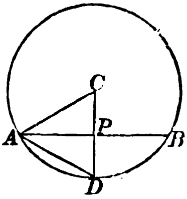
### RULE.

I. *Find the chord of half the arc.*

II. *From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by* 3, *and the quotient will be the length of the arc, nearly.*

### EXAMPLES.

1. The chord $AB=30$ feet, and the radius $AC=20$ feet: what is the length of the arc $ADB$?



First draw $CD$ perpendicular to the chord $AB$: it will bisect the chord at $P$, and the arc of the chord at $D$. Then $AP=15$ feet. Hence,

$$\overline{AC}^2-\overline{AP}^2=\overline{CP}^2: \text{ that is,}$$

$$400-225=175 \quad \text{and} \quad \sqrt{175}=13.228=CP.$$

Then $$CD-CP=20-13.228=6.772=DP.$$

Again, $$AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{225+45.859984}\cdot$$

hence, $$AD=16.4578=\text{chord of the half arc.}$$

Then, $$\frac{16.4578\times 8-30}{3}=33.8874=\text{arc } ADB.$$

2. What is the length of an arc the chord of which is 24 feet, and the radius of the circle 20 feet?

*Ans*. 25.7309 *ft*.

3. The chord of an arc is 16 and the diameter of the circle 20: what is the length of the arc? *Ans*. 18.5178.

4. The chord of an arc is 50, and the chord of half the arc is 27: what is the length of the arc? *Ans*. $55\frac{1}{3}$.

### PROBLEM XIV.

To find the area of a circle when the diameter and circumference are both known.

### RULE.

*Multiply the circumference by half the radius and the product will be the area* (Bk. IV. Th. xxvii).

### EXAMPLES.

1. What is the area of a circle whose diameter is 10, and circumference 31.416?

If the diameter be 10, the radius is 5, and half the radius is $2\frac{1}{2}$: hence, the circumference multiplied by $2\frac{1}{2}$ gives the area.

*Operation.*

$31.416 \times 2\frac{1}{2} = 78.54$; which is the area.

2. Find the area of a circle whose diameter is 7; and circumference 21.9912 yards. *Ans*. 38.4846 *yds*.

3. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet, and circumference 10.9956. *Ans*. 1.069016.

4. What is the area of a circle whose diameter is 100, and circumference 314.16? *Ans*. 7854.

5. What is the area of a circle whose diameter is 1, and circumference 3.1416. *Ans.* 0.7854.

6. What is the area of a circle whose diameter is 40, and circumference 131.9472? *Ans.* 1319.472.

### PROBLEM XV.

To find the area of a circle when the diameter only is known.

### RULE.

*Square the diameter, and then multiply by the decimal* .7854

### EXAMPLES.

What is the area of a circle whose diameter is 5?

We square the diameter, which gives us 25, and we then multiply this number and the decimal .7854 together.

*Operation.*

$$\begin{array}{r} .7854 \\ \overline{5}^2 = \quad 25 \\ \hline 39270 \\ 15708 \\ \hline \text{area} = 19.6350 \end{array}$$

2. What is the area of a circle whose diameter is 7? *Ans.* 38.4846.

3. What is the area of a circle whose diameter is 4,5? *Ans.* 15.90435.

4. What is the number of square yards in a circle whose diameter is $1\frac{1}{6}$ yards? *Ans.* 1.069016.

5. What is the area of a circle whose diameter is 8.75 feet? *Ans.* 60.1322 *sq. ft.*

### PROBLEM XVI.

To find the area of a circle when the circumference only is known.

### RULE.

*Multiply the square of the circumference by the decimal* .07958, *and the product will be the area very nearly.*

### EXAMPLES.

1. What is the area of a circle whose circumference is 3.1416?

We first square the circumference, and then multiply by the decimal .07958.

*Operation.*

$$\overline{3.1416}^2=9,86965056$$

$$,07958$$

$$\text{area}=.7854+$$

2. What is the area of a circle whose circumference is 91?

*Ans.* 659.00198.

3. Suppose a wheel turns twice in tracking 16½ feet, and that it turns just 200 times in going round a circular bowling-green: what is the area in acres, roods, and perches?

*Ans.* 4 *A.* 3 *R.* 35.8 *P.*

4. How many square feet are there in a circle whose circumference is 10.9956 yards? *Ans.* 86.5933.

5. How many perches are there in a circle whose circumference is 7 miles? *Ans.* 399300.608.

### PROBLEM XVII.

Having given a circle, to find a square which shall have an equal area.

### RULE.

I. *The diameter* × .8862 = *side of an equivalent square*

II. *The circumference* × .2821 = *side of an equivalent square*

### EXAMPLES.

1. The diameter of a circle is 100: what is the side of a square of equal area? *Ans.* 88.62.

2. The diameter of a circular fishpond is 20 feet, what would be the side of a square fishpond of an equal area? *Ans.* 17.724 *ft.*

3. A man has a circular meadow of which the diameter is 875 yards, and wishes to exchange it for a square one of equal size: what must be the side of the square? *Ans.* 775.425.

4. The circumference of a circle is 200: what is the side of a square of an equal area? *Ans.* 56.42.

5. The circumference of a round fishpond is 400 yards· what is the side of a square pond of equal area? *Ans.* 112.84.

6. The circumference of a circular bowling-green is 412 yards: what is the side of a square one of equal area? *Ans.* 116.2252 *yds.*

7. The circumference of a circular walk is 625: what is the side of a square containing the same area? *Ans.* 176.3125.

### PROBLEM XVIII.

Having given the diameter or circumference of a circle, to find the side of the inscribed square.

### RULE.

I. *The diameter* × .7071 = *side of the inscribed square.*

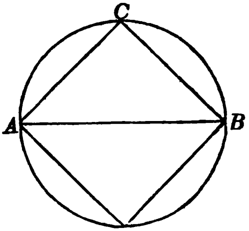
II. *The circumference* × .2251 = *side of the inscribed square.*

EXAMPLES.

1. The diameter $AB$ of a circle is 400: what is the value of $AC$, the side of the inscribed square?

Here,

$.7071 \times 400 = 282.8400 = AC.$



2. The diameter of a circle is 412 feet: what is the side of the inscribed square? *Ans.* 291.3252 *ft.*

3. If the diameter of a circle be 600 what is the side of the inscribed square? *Ans.* 424.26.

4. The circumference of a circle is 312 feet: what is the side of the inscribed square? *Ans.* 70.2312 *ft.*

5. The circumference of a circle is 819 yards: what is the side of the inscribed square? *Ans.* 184.3569 *yds.*

6. The circumference of a circle is 715: what is the side of the inscribed square? *Ans.* 160.9465.

7. The circumference of a circular walk is 625: what is the side of an inscribed square? *Ans.* 140.6875.

### PROBLEM XIX.

To find the area of a circular sector.

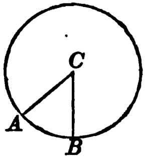
RULE.

I. *Find the length of the arc by Problem* XII.

II. *Multiply the arc by one half the radius, and the product will be the area.*

### EXAMPLES.

1. What is the area of the circular sector *ACB*, the arc *AB* containing 18°, and the radius *CA* being equal to 3 feet.



First, $.01745 \times 18 \times 3 = .94230 =$ length *AB*.
Then, $.94230 \times 1\frac{1}{2} = 1.41345 =$ area

2. What is the area of a sector of a circle in which the radius is 20 and the arc one of 22 degrees?

*Ans.* 76.7800.

3. Required the area of a sector whose radius is 25 and the arc of 147° 29′. *Ans.* 804.2448.

4. Required the area of a semicircle in which the radius is 13. *Ans.* 265.4143.

5. What is the area of a circular sector when the length of the arc is 650 feet and the radius 325?

*Ans.* 105625 *sq. ft.*

### PROBLEM XX.

To find the area of a segment of a circle.

### RULE.

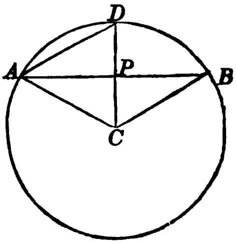
I. *Find the area of the sector having the same arc with the segment, by the last Problem.*

II. *Find the area of the triangle formed by the chord of the segment and the two radii through its extremities.*

III. *If the segment is greater than the semicircle, add the two areas together; but if it is less, subtract them, and the result in either case, will be the area required.*

EXAMPLES.

1. What is the area of the segment $ADB$, the chord $AB=24$ feet and $CA=20$ feet.



First, $CP=\sqrt{\overline{CA}^2-\overline{AP}^2}$
$=\sqrt{400-144}=16$

Then,
$PD=CD-CP=20-16=4.$

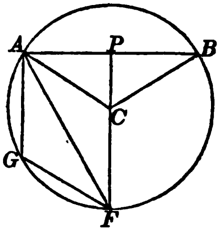
And, $AD=\sqrt{\overline{AP}^2+\overline{PD}^2}=\sqrt{144+16}=12,64911.$

then, $\text{arc } ADB=\dfrac{12,64911\times 8-24}{3}=25,7309.$

Arc $ADB=25,7309$
half radius $=10$
area sector $ADBC=257,3090$
area $CAB=192$
$65,309=$area of segment $ADB$

$AP=12$
$CP=16$
area $CAB=192$

2. Find the area of the segment $AFB$, knowing the following lines, viz: $AB=20.5$; $FP=17.17$; $AF=20$; $FG=11.5$; and $CA=11.64$.



$$\text{Arc } AGF=\frac{FG\times 8-AF}{3}=\frac{11.5\times 8-20}{3}=24:$$

and sector $AGFBC=24\times 11.64=279.36$:

but $CP=FP-AC=17.17-11.64=5.53$:

Then, area $ACB=\dfrac{AB\times CP}{2}=\dfrac{20.5\times 5.53}{2}=56.6825.$

Then, area of sector $AFBC=279.36$
do. of triangle $ABC=\underline{\ \ 56.6825}$
gives area of segment $AFB=\underline{336.0425}$

3 What is the area of a segment; the radius of the circle being 10, and the chord of the arc 12 yards?

*Ans.* 16.324 *sq. yds.*

4. Required the area of the segment of a circle whose chord is 16, and the diameter of the circle 20.

*Ans.* $44.590\dot{3}$.

5. What is the area of a segment whose arc is a quadrant, the diameter of the circle being 18? *Ans.* 63.6174.

6. The diameter of a circle is 100, and the chord of the segment 60: what is the area of the segment?

*Ans.* 408, nearly.

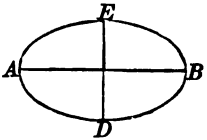
### PROBLEM XXI.

### To find the area of an ellipse.

*Multiply the two axes together, and their product by the decimal* ,7854, *and the result will be the required area.*

EXAMPLES.

1. Required the area of an ellipse, whose transverse axis $AB=70$ feet, and the conjugate axis $DE=50$ feet.



$AB\times DE=70\times 50=3500$:

Then, $.7854\times 3500=2748.9=$area.

2. Required the area of an ellipse whose axes are 24 and 18. *Ans.* 339.2928

3. What is the area of an ellipse whose axes are 80 and 60? *Ans*. 3769.92.

4. What is the area of an ellipse whose axes are 50 and 45? *Ans*. 1767.15.

### PROBLEM XXII.

To find the area of a circular ring: that is, the area included between the circumferences of two circles, having a common centre.

### RULE.

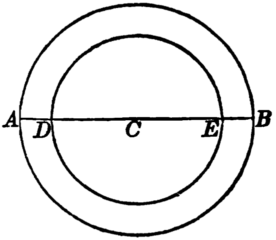
I. *Square the diameter of each ring, and subtract the square of the less from that of the greater.*

II. *Multiply the difference of the squares by the decimal* .7854, *and the product will be the area.*

### EXAMPLES.

1. In the concentric circles having the common centre $C$, we have

$AB=10$ yds., and $DE=6$ yards: what is the area of the space included between them?



$$\begin{aligned}\overline{BA}^2&=\overline{10}^2=100\\ \overline{DE}^2&=\ \overline{6}^2=\ 36\\ \hline \text{Difference}&=64\end{aligned}$$

Then, $63\times.7854=50.2656=$ area.

2. What is the area of the ring when the diameters of the circle are 20 and 10? *Ans*. 235.62.

3. If the diameters are 20 and 15, what will be the area included between the circumferences? *Ans.* 137.445.

4. If the diameters are 16 and 10, what will be the area included between the circumferences? *Ans.* 122.5224.

5. Two diameters are 21.75 and 9.5; required the area of the circular ring. *Ans.* 300.6609.

6. If the two diameters are 4 and 6, what is the area of the ring? *Ans.* 15.708

## MENSURATION OF SOLIDS.

### DEFINITIONS.

The mensuration of solids is divided into two parts.

1st, The mensuration of the surfaces of solids: and

2d, The mensuration of their solidities.

We have already seen that the unit of measure for plane surfaces, is a square whose side is the unit of length (Bk. IV Def. 7).

2. A curve line which is expressed by numbers is also referred to an unit of length, and its numerical value is the number of times which the line contains the unit.

If then, we suppose the linear unit to be reduced to a straight line, and a square constructed on this line, this square will be the unit of measure for curved surfaces.

3. The unit of solidity is a cube, whose edge is the unit in which the linear dimensions of the solid are expressed; and

the face of this cube is the superficial unit in which the surface of the solid is estimated (Bk. VI. Th. xiii. Sch).

4. The following is a table of solid measure.

1 cubic foot =1728 cubic inches.
1 cubic yard =27 cubic feet.
1 cubic rod =4492⅛ cubic feet.
1 ale gallon =282 cubic inches.
1 wine gallon=231 cubic inches.
1 bushel =2150,42 cubic inches.

### PROBLEM I.

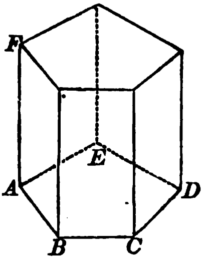
### To find the surface of a right prism.

RULE.

*Multiply the perimeter of the base by the altitude and the product will be the convex surface: and to this add the area of the bases, when the entire surface is required* (Bk. VI. Th. i).

EXAMPLES

1. Find the entire surface of the regular prism whose base is the regular polygon $ABCDE$ and altitude $AF$, when each side of the base is 20 feet and the altitude $AF$, 50 feet.



$AB+BC+CD+DE+EA=100$; and $AF=50$: then

$(AB+BC+CD+DE+EA)\times AF=$convex surface which becomes, $100 \times 50 = 5000$ square feet; which is the convex surface. For the area of the end, we have

$$\overline{AB}^2 \times \text{tabular number} = \text{area } ABCDE,$$

that is, $\overline{20}^2 \times$ tabular number, or $400 \times 1.720477 = 688.1908 =$ the area $ABCDE$.

| | | |
|---|---|---|
| Then, convex surface | =5000 | square feet. |
| lower base | 688.1908 | square feet. |
| upper base | 688.1908 | square feet. |
| Entire surface | 6376.3816 | |

2. What is the surface of a cube, the length of each side being 20 feet? *Ans.* 2400 *sq. ft.*

3. Find the entire surface of a triangular prism, whose base is an equilateral triangle, having each of its sides equal to 18 inches, and altitude 20 feet. *Ans.* 91.949 *sq. ft.*

4. What is the convex surface of a regular octagonal prism, the side of whose base is 15 and altitude 12 feet?

*Ans.* 1440 *sq. ft.*

5. What must be paid for lining a rectangular cistern with lead at 2*d* a pound, the thickness of the lead being such as to require 7*lb.* for each square foot of surface; the inner dimensions of the cistern being as follows: viz. the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches? *Ans.* £2 3*s.* $10\frac{5}{9}d$.

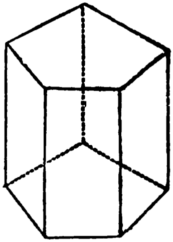
### PROBLEM II.

### To find the solidity of a prism.

#### RULE.

***Multiply the area of the base by the perpendicular height, and the product will be the solidity.***

### EXAMPLES.



1. What is the solidity of a regular pentagonal prism whose altitude is 20, and each side of the base 15 feet?

To find the area of the base we have by Problem VIII. page 178.

$\overline{15}^2=225$: and $225\times1.7204774=387.107415=$ the area of the base: hence,

$$387.107415\times20=7742.1483=\text{solidity}.$$

2. What is the solid contents of a cube whose side is 24 inches? *Ans.* 13824 *solid in.*

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches? *Ans.* $21\frac{1}{9}$ *solid ft.*

4. How many gallons of water, ale measure, will a cistern contain whose dimensions are the same as in the last example? *Ans.* $129\frac{17}{47}$

5. Required the solidity of a triangular prism whose altitude is 10 feet, and the three sides of its triangular base 3, 4, and 5 feet. *Ans.* 60 *solid ft.*

6. What is the solidity of a square prism whose height is $5\frac{1}{2}$ feet, and each side of the base $1\frac{1}{3}$ foot?

*Ans* $9\frac{7}{9}$ *solid ft.*

7. What is the solidity of a prism whose base is an equilateral triangle, each side of which is 4 feet, the height of the prism being 10 feet? *Ans.* 69.282 *solid ft.*

8. What is the number of cubic or solid feet in a regular pentagonal prism of which the altitude is 15 feet and each side of the base 3.75 feet? *Ans.* 362.913

### PROBLEM III.

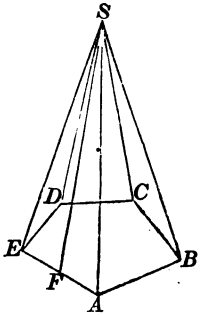
To find the surface of a regular pyramid.

### RULE.

*Multiply the perimeter of the base by half the slant height, and the product will be the convex surface: to this add the area of the base, if the entire surface is required* (Bk. VI. Th. vi).

### EXAMPLES.

1. In the regular pentagonal pyramid $S—ABCDE$, the slant height $SF$ is equal to 45, and each side of the base is 15 feet: required the convex surface, and also the entire surface.



$15 \times 5 = 75 =$ perimeter of the base,
$75 \times 22\frac{1}{2} = 1687.5$ square feet = area of convex surface.

And $\overline{15}^2 = 225$: then $225 \times 1.7204774 = 387.107415 =$ the area of the base.

Hence, convex surface $= 1687.5$
area of the base $= 387.107415$
Entire surface $= 2074.607415$ square feet.

2. What is the convex surface of a regular triangular pyramid, the slant height being 20 feet, and each side of the base 3 feet? *Ans.* 90 *sq. ft.*

3. What is the entire surface of a regular pyramid whose slant height is 15 feet, and the base a regular pentagon, of which each side is 25 feet? *Ans.* 2012.798 *sq. ft.*

### PROBLEM IV.

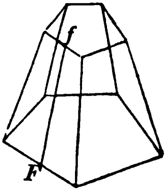
To find the convex surface of the frustum of a regular pyramid.

### RULE.

*Multiply half the sum of the perimeters of the two bases by the slant height of the frustum, and the product will be the convex surface* (Bk. VI. Th. vii).

### EXAMPLES.

1. In the frustum of the regular pentagonal pyramid each side of the lower base is 30, and each side of the upper base is 20 feet, and the slant height *fF* is equal to 15 feet. What is the convex surface of the frustum? *Ans.* 1875 *sq. ft.*



2. How many square feet are there in the convex surface of the frustum of a square pyramid, whose slant height is 10 feet, each side of the lower base 3 feet 4 inches, and each side of the upper base 2 feet 2 inches? *Ans.* 110.

3. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 55 feet, each side of the lower base 8 feet, and each side of the upper base 4 feet? *Ans.* 2310 *sq. ft.*

### PROBLEM V.

To find the solidity of a pyramid.

### RULE.

*Multiply the area of the base by the altitude and divide the product by* 3, *the quotient will be the solidity* (Bk. VI. Th. xvii).

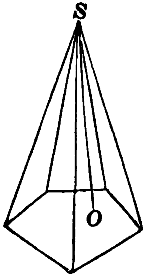
### EXAMPLES.

1 What is the solidity of a pyramid the area of whose base is 215 square feet and the altitude $SO=45$ feet?

First, $215\times45=9675$:

then, $9675\div3=3225$

which is the solidity expressed in solid feet.



2. Required the solidity of a square pyramid, each side of its base being 30 and its altitude 25. *Ans.* 7500 *solid ft.*

3. How many solid yards are there in a triangular pyramid whose altitude is 90 feet, and each side of its base 3 yards?
*Ans.* 38.97117.

4. How many solid feet in a triangular pyramid the altitude of which is 14 feet 6 inches, and the three sides of its base 5, 6 and 7 feet? *Ans.* 71.0352.

5. What is the solidity of a regular pentagonal pyramid, its altitude being 12 feet, and each side of its base 2 feet?
*Ans.* 27·5276 *solid ft*

6 How many solid feet in a regular hexagonal pyramid, whose altitude is 6.4 feet, and each side of the base 6 inches? *Ans.* 1.38564.

7. How many solid feet are contained in a hexagonal pyramid the height of which is 45 feet, and each side of the base 10 feet? *Ans.* 3897.1143.

8. The spire of a church is an octagonal pyramid, each side of the base being 5 feet 10 inches, and its perpendicular height 45 feet. Within is a cavity, or hollow part, each side of the base being 4 feet 11 inches, and its perpendicular height 41 feet: how many yards of stone does the spire contain? *Ans.* 32.197353

### PROBLEM VI.

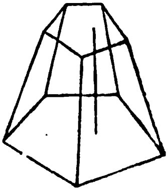
To find the solidity of the frustum of a pyramid.

### RULE.

*Add together the areas of the two bases of the frustum and a geometrical mean proportional between them; and then multiply the sum by the altitude, and take one-third the product for the solidity.*

### EXAMPLES.

1. What is the solidity of the frustum of a pentagonal pyramid the area of the lower base being 16 and of the upper base 9 square feet, the altitude being 7 feet?



First, $16 \times 9 = 144$: then, $\sqrt{144} = 12$, the mean

Then, area of lower base $= 16$
area of upper base $= 9$
mean of bases $= 12$
$37$
height $7$
$3)\ 259$
solidity $= 86\frac{1}{3}$ *solid ft.*

2. What is the number of solid feet in a piece of timber whose bases are squares, each side of the lower base being 15 inches, and each side of the upper base being 6 inches, the length being 24 feet? *Ans.* 19.5.

3. Required the solidity of a regular pentagonal frustum, whose altitude is 5 feet, each side of the lower base 18 inches, and each side of the upper base 6 inches.

*Ans.* 9.31925 *solid ft.*

4. What is the contents of a regular hexagonal frustum, whose height is 6 feet, the side of the greater end 18 inches, and of the less end 12 inches? *Ans.* 24.681724 *cubic ft.*

5. How many cubic feet in a square piece of timber, the areas of the two ends being 504 and 372 inches, and its length $31\frac{1}{2}$ feet? *Ans.* 95.447.

6. What is the solidity of a squared piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the smaller 12 inches?

*Ans.* 28.5 *cubic ft.*

7. What is the solidity of the frustum of a regular hexagonal pyramid, the side of the greater end being 3 feet, that of the less 2 feet, and the height 12 feet?

*Ans.* 197.453776 *solid ft.*

## MEASURES OF THE THREE ROUND BODIES.

### PROBLEM I.

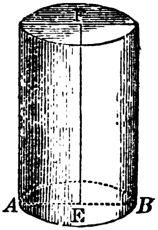
### To find the surface of a cylinder.

RULE.

*Multiply the circumference of the base by the altitude, and the product will be the convex surface; and to this, add the areas of the two bases, when the entire surface is required* (Bk. VI. Th. ii).

EXAMPLES.

1. What is the entire surface of the cylinder in which $AB$, the diameter of the base, is 12 feet, and the altitude $EF$ 30 feet?



First, to find the circumference of the base, (Prob. X. page 180): we have $3.1416 \times 12 = 37.6992 =$ circumference of the base.

Then, $37.6992 \times 30 = 1130.9760 =$ convex surface.

Also, $\overline{12}^2 = 144$: and $144 \times .7854 = 113.0976 =$ area of the base.

Then,

| | |
|---|---|
| convex surface | $=1130.9760$ |
| lower base | 113.0976 |
| upper base | 113.0976 |
| Entire area | $=1357.1712$ |

2. What is the convex surface of a cylinder, the diameter of whose base is 20, and the altitude 50 feet?

*Ans.* 3141.6 *sq. ft*

3. Required the entire surface of a cylinder, whose altitude is 20 feet, and the diameter of the base 2 feet.

*Ans.* 131.9472 *ft.*

4. What is the convex surface of a cylinder, the diameter of whose base is 30 inches, and altitude 5 feet?

*Ans.* 5654.88 *sq. in.*

5. Required the convex surface of a cylinder, whose altitude is 14 feet, and the circumference of the base 8 feet 4 inches. *Ans.* 116.6666, &c., *sq. ft.*

PROBLEM II.

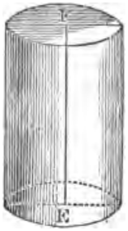
To find the solidity of a cylinder.

RULE.

*Multiply the area of the base by the altitude, and the product will be the solidity.*

EXAMPLES.

1. What is the solidity of a cylinder, the diameter of whose base is 40 feet, and altitude $EF$, 25 feet?



First, to find the area of the base, we have (Prob. xv. page 231).

$\overline{40}^2=1600$: then, $1600\times.7854=1256.64$. $=$area of the base.

Then, $1256.64\times25=31416$ solid feet, which is the solidity.

2. What is the solidity of a cylinder, the diameter of whose base is 30 feet, and altitude 50 feet?

*Ans.* 35343 *cubic ft.*

3. What is the solidity of a cylinder whose height is 5 feet, and the diameter of the end 2 feet? *Ans.* 15.708 *solid ft.*

4. What is the solidity of a cylinder whose height is 20 feet, and the circumference of the base 20 feet?

*Ans.* 636.64 *cubic ft.*

5. The circumference of the base of a cylinder is 20 feet, and the altitude 19.318 feet: what is the solidity?

*Ans.* 614.93 *cubic ft.*

6. What is the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet?

*Ans.* 2120.58 *cubic ft.*

7. Required the solidity of a cylinder whose altitude is 20 feet, and the circumference of whose base is 5 feet 6 inches?

*Ans.* 48.1459 *cubic ft.*

8. What is the solidity of a cylinder, the circumference of whose base is 38 feet, and altitude 25 feet?

*Ans.* 2872.838 *cubic ft.*

9. What is the solidity of a cylinder, the circumference of whose base is 40 feet, and altitude 30 feet?

10. The diameter of the base of a cylinder is 84 yards, and the altitude 21 feet: how many solid or cubic yards does it contain? *Ans.* 38792.4768.

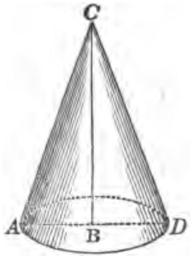
### PROBLEM III.

### To find the surface of a cone.

### RULE.

*Multiply the circumference of the base by the slant height, and divide the product by 2; the quotient will be the convex surface, to which add the area of the base, when the entire surface is required* (Bk. VI. Th. viii).

### EXAMPLES.



1. What is the convex surface of the cone whose vertex is *C*, the diameter *AD*, of its base being $8\frac{1}{2}$ feet, and the side *CA*, 50 feet.

First, $3.1416\times8\frac{1}{2}=26.7036=$ circumference of base.

Then $\dfrac{26.7036\times50}{2}=667.59=$ convex surface.

2. Required the entire surface of a cone whose side is 36 and the diameter of its base 18 feet.

*Ans.* 1272.348 *sq. ft.*

3. The diameter of the base is 3 feet, and the slant height 15 feet: what is the convex surface of the cone?

*Ans.* 70.686 *sq. ft.*

4. The diameter of the base of a cone is 4,5 feet, and the slant height 20 feet: what is the entire surface?

*Ans.* 157.27635 *sq. ft.*

5. The circumference of the base of a cone is 10.75, and the slant height is 18.25: what is the entire surface?

*Ans.* 107.29021 *sq. ft.*

### PROBLEM IV.

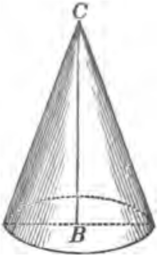
To find the solidity of a cone.

### RULE.

*Multiply the area of the base by the altitude; and divide the product by* 3, *the quotient will be the solidity* (Bk. VI. Th. xviii).

EXAMPLES.

1. What is the solidity of a cone, the area of whose base is 380 square feet, and altitude $CB$, 48 feet?



We simply multiply the area of the base by the altitude, and then divide the product by 3.

*Operation.*

$$\begin{array}{r} 380 \\ \underline{48} \\ 3040 \\ \underline{1520} \\ 3)\underline{18240} \\ \text{area}=6080 \end{array}$$

2. Required the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet.

*Ans.* 706.86 *cubic ft.*

3. Required the solidity of a cone whose altitude is 10½ feet, and the circumference of its base 9 feet?

*Ans.* 22.5609 *cubic ft.*

4. What is the solidity of a cone, the diameter of whose base is 18 inches, and altitude 15 feet?

*Ans.* 8.83575 *cubic ft.*

5. The circumference of the base of a cone is 40 feet, and the altitude 50 feet: what is the solidity?

*Ans.* 2122.1333 *solid ft.*

### PROBLEM V.

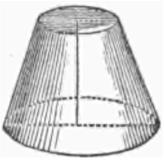
### To find the surface of the frustum of a cone.

#### RULE.

*Add together the circumferences of the two bases; and multiply the sum by half the slant height of the frustum; the product will be the convex surface, to which add the areas of the bases when the entire surface is required* (Bk. VI. Th. ix).

#### EXAMPLES.

1. What is the convex surface of the frustum of a cone, of which the slant height is $12\frac{1}{2}$ feet, and the circumferences of the bases 8,4 and 6 feet.



We merely take the sum of the circumferences of the bases, and multiply by half the slant height, or side.

*Operation.*

$$\begin{array}{rr} & 8.4 \\ & \underline{6} \\ & 14.4 \\ \text{half side} & \underline{6.25} \\ \text{area} = & 90 \; \textit{sq. ft.} \end{array}$$

2. What is the entire surface of the frustum of a cone, the side being 16 feet, and the radii of the bases 2 and 3 feet?

*Ans.* 292.1688 *sq. ft.*

3. What is the convex surface of the frustum of a cone, he circumference of the greater base being 30 feet, and of the less 10 feet; the slant height being 20 feet?

*Ans.* 400 *sq. ft.*

4. Required the entire surface of the frustum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet *Ans.* 439.824 *sq. ft.*

### PROBLEM VI.

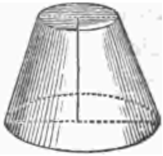
### To find the solidity of the frustum of a cone.

RULE.

I. *Add together the areas of the two ends and a geometrical mean between them.*

II. *Multiply this sum by one-third of the altitude and the product will be the solidity.*

EXAMPLES.



1. How many cubic feet in the frustum of a cone whose altitude is 26 feet, and the diameters of the bases 22 and 18 feet?

First, $\overline{22}^2 \times .7854 = 380.134 =$ area of lower base:

and $\overline{18}^2 \times .7854 = 254.47 =$ area of upper base.

Then, $\sqrt{380.134 \times 254.47} = 311.018 =$ mean.

Then, $(380.134 + 254,47 + 311.018) \times \frac{26}{3} = 8195.39$ which is the solidity.

2. How many cubic feet in a piece of round timber the diameter of the greater end being 18 inches, and that of the less 9 inches, and the length 14.25 feet? *Ans.* 14.68943.

3. What is the solidity of a frustum, the altitude being 18, the diameter of the lower base 8, and of the upper 4?

*Ans.* 527.7888.

4. If a cask, which is composed of two equal conic frustums joined together at their larger bases, have its bung diameter 28 inches, the head diameter 20 inches, and the length

40 inches, how many gallons of wine will it contain, there being 231 cubic inches in a gallon? *Ans.* 79.0613.

### PROBLEM VII.

To find the surface of a sphere.

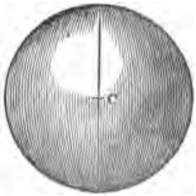
#### RULE.

*Multiply the circumference of a great circle by the diameter, and the product will be the surface* (Bk. VI. Th. xxiii).

#### EXAMPLES.

1. What is the surface of the sphere whose centre is C, the diameter being 7 feet?

*Ans.* 153.9384 *sq. ft.*



2. What is the surface of a sphere whose diameter is 24?
*Ans.* 1809.5616.

3. Required the surface of a sphere whose diameter is 7921 miles. *Ans.* 197111024 *sq. miles.*

4. What is the surface of a sphere the circumference of whose great circle is 78.54? *Ans.* 1963.5.

5. What is the surface of a sphere whose diameter is $1\frac{1}{3}$ feet? *Ans.* 5.58506 *sq. ft.*

### PROBLEM VIII.

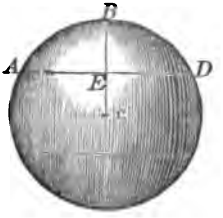
To find the convex surface of a spherical zone.

#### RULE.

*Multiply the height of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface* (Bk. VI. Th. xxiv).

### EXAMPLES.

1. What is the convex surface of the zone *ABD*, the height *BE* being 9 inches, and the diameter of the sphere 42 inches?



**First,** $42 \times 3.1416 = 131.9472 =$ circumference.
height $= 9$
surface $= \underline{\overline{1187.5248}}$ square inches.

2. The diameter of a sphere is $12\frac{1}{4}$ feet: what will be the surface of a zone whose altitude is 2 feet?

*Ans.* 78.54 *sq. ft.*

3. The diameter of a sphere is 21 inches: what is the surface of a zone whose height is $4\frac{1}{2}$ inches?

*Ans.* 296.8812 *sq. in.*

4. The diameter of a sphere is 25 feet and the height of the zone 4 feet: what is the surface of the zone?

*Ans.* 314.16 *sq. ft.*

5. The diameter of a sphere is 9, and the height of a zone 3 feet: what is the surface of the zone?

*Ans.* 84.8232.

### PROBLEM IX.

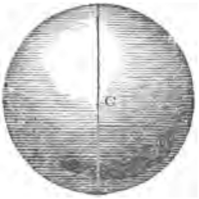
To find the solidity of a sphere.

#### RULE I.

*Multiply the surface by one-third of the radius and the product will be the solidity* (Bk. VI. Th. xxv).

### EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12 feet?



First, $3.1416 \times 12 = 37.6992 =$ circumference of sphere.

diameter = 12
surface = 452.3904
one-third radius = 2
Solidity = 904.7808 cubic feet.

2. The diameter of a sphere is 7957.8: what is its solidity?
*Ans.* 263863122758.4778.

3. The diameter of a sphere is 24 yards: what is its solid contents? *Ans.* 7238.2464 *cubic yds.*

4. The diameter of a sphere is 8: what is its solidity?
*Ans.* 268.0832.

5. The diameter of a sphere is 16: what is its solidity?
*Ans.* 2144.6656.

### RULE II.

*Cube the diameter and multiply the number thus found, by the decimal* .5236, *and the product will be the solidity.*

### EXAMPLES.

1. What is the solidity of a sphere whose diameter is 20?
*Ans.* 4188.8.

2. What is the solidity of a sphere whose diameter is 6?
*Ans.* 113.0976.

3. What is the solidity of a sphere whose diameter is 10?
*Ans.* 523.6

### PROBLEM X.

### To find the solidity of a spherical segment with one base

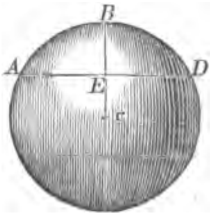
### RULE.

I. *To three times the square of the radius of the base, add the square of the height.*

II. *Multiply this sum by the height, and the product by the decimal* .5236, *the result will be the solidity of the segment.*

### EXAMPLES.

1. What is the solidity of the segment *ABD*, the height *BE* being 4 feet, and the diameter *AD* of the base being 14 feet?



First,

$$\overline{7}^2\times 3+\overline{4}^2=147+16=163:$$

Then, $163\times 4\times .5236=341.3872$ solid feet, which is the solidity of the segment.

2. What is the solidity of the segment of a sphere whose height is 4, and the radius of its base 8? *Ans.* 435.6352.

3. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

*Ans.* 572.5566.

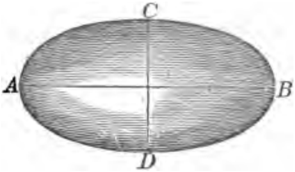
4. What is the solidity of a spherical segment, the diameter of the sphere being 8, and the height of the segment 2 feet? *Ans.* 41.888 *cubic ft.*

5. What is the solidity of a segment, when the diameter of the sphere is 20, and the altitude of the segment 9 feet?

*Ans.* 1781.2872 *cubic ft*

## OF THE SPHEROID.

A spheroid is a solid described by the revolution of an ellipse about either of its axes.



If an ellipse $ACBD$, be revolved about the transverse or longer axis $AB$, the solid described is called a *prolate spheroid:* and if it be revolved about the shorter axis $CD$, the solid described is called an *oblate spheroid.*

The earth is an oblate spheroid, the axis about which it revolves being about 34 miles shorter than the diameter perpendicular to it.

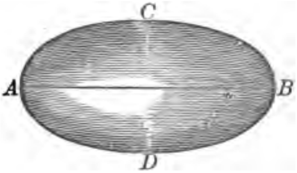
### PROBLEM XI.

To find the solidity of an ellipsoid

### RULE.

*Multiply the fixed axis by the square of the revolving axis and the product by the decimal* .5236, *the result will be the required solidity.*

### EXAMPLES.



1. In the prolate spheroid $ACBD$, the transverse axis $AB=90$, and the revolving axis $CD=70$ feet: what is the solidity?

Here, $AB=90$ feet: $\overline{CD}^2=\overline{70}^2=4900$: hence $AB\times\overline{CD}^2\times .5236=90\times 4900\times .5236=230907.6$ cubic feet, which is the solidity.

2. What is the solidity of a prolate spheriod, whose fixed axis is 100, and revolving axis 6 feet? *Ans*. 1884.96.

3. What is the solidity of an oblate spheroid, whose fixed axis is 60, and revolving axis 100? *Ans*. 314160.

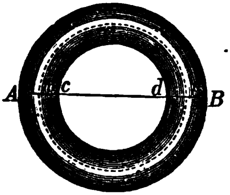
4. What is the solidity of a prolate spheroid, whose axes are 40 and 50? *Ans*. 41888.

5. What is the solidity of an oblate spheroid, whose axes are 20 and 10? *Ans*. 2094.4.

6. What is the solidity of a prolate spheroid, whose axes are 55 and 33? *Ans*. 31361.022.

7. What is the solidity of an oblate spheroid, whose axes are 85 and 75? *Ans*. ——

### OF CYLINDRICAL RINGS.



A cylindrical ring is formed by bending a cylinder until the two ends meet each other. Thus, if a cylinder be bent round until the axis takes the position *mon*, a solid will be formed, which is called a cylindrical ring.

The line *AB* is called the outer, and *cd* the inner diameter.

### PROBLEM XII.

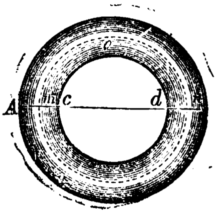
To find the convex surface of a cylindrical ring.

### RULE.

I. *To the thickness of the ring add the inner diameter.*

II. *Multiply this sum by the thickness, and the product by* 9.8696, *the result will be the area.*

EXAMPLES.



1. The thickness $Ac$, of a cylindrical ring is 3 inches, and the inner diameter $cd$, is 12 inches: what is the convex surface?

$Ac+cd=3+12=15$:

$15\times3\times9.8696=444.132$ square inches=the surface.

2. The thickness of a cylindrical ring is 4 inches, and the inner diameter 18 inches: what is the convex surface?

*Ans.* 868.52 *sq. in.*

3. The thickness of a cylindrical ring is 2 inches, and the inner diameter 18 inches: what is the convex surface?

*Ans.* 394.784 *sq. in.*

### PROBLEM XIII.

To find the solidity of a cylindrical ring.

RULE.

I *To the thickness of a ring add the inner diameter*

II. *Multiply this sum by the square of half the thickness, and the product by* 9.8696, *the result will be the required solidity.*

EXAMPLES.

1. What is the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches?

$8+3=11$: then, $11\times(\frac{3}{2})^2\times9.8696=244.2726$, which expresses the solidity in cubic inches.

2. The inner diameter of a cylindrical ring is 18 inches, d the thickness 4 inches: what is the solidity of the ring?

*Ans.* 868.5248 *cubic inches*

3. Required the solidity of a cylindrical ring whose thickness is 2 inches, and inner diameter 12 inches?

*Ans.* 138.1744 *cubic in*

4. What is the solidity of a cylindrical ring, whose thickness is 4 inches, and inner diameter 16 inches?

*Ans.* 789.568 *cubic in.*

5. What is the solidity of a cylindrical ring, whose thickness is 8 inches, and inner diameter 20 inches?

*Ans.* ——

6. What is the solidity of a cylindrical ring whose thickness is 5 inches, and inner diameter 18 inches?

*Ans.* ——

# A TABLE

OF

# LOGARITHMS OF NUMBERS

## From 1 to 10,000.

| N. | Log. | N. | Log. | N. | Log. | N. | Log. |
|---|---|---|---|---|---|---|---|
| 1 | 0·000000 | 26 | 1·414973 | 51 | 1·707570 | 76 | 1·880814 |
| 2 | 0·301030 | 27 | 1·431364 | 52 | 1·716003 | 77 | 1·886491 |
| 3 | 0·477121 | 28 | 1·447158 | 53 | 1·724276 | 78 | 1·892085 |
| 4 | 0·602060 | 29 | 1·462398 | 54 | 1·732394 | 79 | 1·897627 |
| 5 | 0·698970 | 30 | 1·477121 | 55 | 1·740363 | 80 | 1·903090 |
| 6 | 0·778151 | 31 | 1·491362 | 56 | 1·748188 | 81 | 1·908485 |
| 7 | 0·845098 | 32 | 1·505150 | 57 | 1·755875 | 82 | 1·913814 |
| 8 | 0·903090 | 33 | 1·518514 | 58 | 1·763428 | 83 | 1·919078 |
| 9 | 0·954243 | 34 | 1·531479 | 59 | 1·770852 | 84 | 1·924279 |
| 10 | 1·000000 | 35 | 1·544068 | 60 | 1·778151 | 85 | 1·929419 |
| 11 | 1·041393 | 36 | 1·556303 | 61 | 1·785330 | 86 | 1·934498 |
| 12 | 1·079181 | 37 | 1·568202 | 62 | 1·792392 | 87 | 1·939519 |
| 13 | 1·113943 | 38 | 1·579784 | 63 | 1·799341 | 88 | 1·944483 |
| 14 | 1·146128 | 39 | 1·591065 | 64 | 1·806181 | 89 | 1·949390 |
| 15 | 1·176091 | 40 | 1·602060 | 65 | 1·812913 | 90 | 1·954243 |
| 16 | 1·204120 | 41 | 1·612784 | 66 | 1·819544 | 91 | 1·959041 |
| 17 | 1·230449 | 42 | 1·623249 | 67 | 1·826075 | 92 | 1·963788 |
| 18 | 1·255273 | 43 | 1·633468 | 68 | 1·832509 | 93 | 1·968483 |
| 19 | 1·278754 | 44 | 1·643453 | 69 | 1·838849 | 94 | 1·973128 |
| 20 | 1·301030 | 45 | 1·653213 | 70 | 1·845098 | 95 | 1·977724 |
| 21 | 1·322219 | 46 | 1·662758 | 71 | 1·851258 | 96 | 1·982271 |
| 22 | 1·342423 | 47 | 1·672098 | 72 | 1·857333 | 97 | 1·986772 |
| 23 | 1·361728 | 48 | 1·681241 | 73 | 1·863323 | 98 | 1·991226 |
| 24 | 1·380211 | 49 | 1·690196 | 74 | 1·869232 | 99 | 1·995635 |
| 25 | 1·397940 | 50 | 1·698970 | 75 | 1·875061 | 100 | 2·000000 |

Remark.—In the following table, in the nine right-hand columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's, to catch the eye, and to indicate that from thence the two figures of the Logarithm to be taken from the second column, stand in the next line below.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 000000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 | 432 |
| 101 | 4321 | 4751 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 | 428 |
| 102 | 8600 | 9026 | 9451 | 9876 | •300 | •724 | 1147 | 1570 | 1993 | 2415 | 424 |
| 103 | 012837 | 3259 | 3680 | 4100 | 4521 | 4940 | 5360 | 5779 | 6197 | 6616 | 419 |
| 104 | 7033 | 7451 | 7868 | 8284 | 8700 | 9116 | 9532 | 9947 | •361 | •775 | 416 |
| 105 | 021189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 | 412 |
| 106 | 5306 | 5715 | 6125 | 6533 | 6942 | 7350 | 7757 | 8164 | 8571 | 8978 | 408 |
| 107 | 9384 | 9789 | •195 | •600 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 | 404 |
| 108 | 033424 | 3826 | 4227 | 4628 | 5029 | 5430 | 5830 | 6230 | 6629 | 7028 | 400 |
| 109 | 7426 | 7825 | 8223 | 8620 | 9017 | 9414 | 9811 | •207 | •602 | •998 | 396 |
| 110 | 041393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | 4148 | 4540 | 4932 | 393 |
| 111 | 5323 | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | 8053 | 8442 | 8830 | 389 |
| 112 | 9218 | 9606 | 9993 | •380 | •766 | 1153 | 1538 | 1924 | 2309 | 2694 | 386 |
| 113 | 053078 | 3463 | 3846 | 4230 | 4613 | 4996 | 5378 | 5760 | 6142 | 6524 | 382 |
| 114 | 6905 | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | 9563 | 9942 | •320 | 379 |
| 115 | 060698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 | 376 |
| 116 | 4458 | 4832 | 5206 | 5580 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 | 372 |
| 117 | 8186 | 8557 | 8928 | 9298 | 9668 | ••38 | •407 | •776 | 1145 | 1514 | 369 |
| 118 | 071882 | 2250 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 | 366 |
| 119 | 5547 | 5912 | 6276 | 6640 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 | 363 |
| 120 | 079181 | 9543 | 9904 | •266 | •626 | •987 | 1347 | 1707 | 2067 | 2426 | 360 |
| 121 | 082785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 | 357 |
| 122 | 6360 | 6716 | 7071 | 7426 | 7781 | 8136 | 8490 | 8845 | 9198 | 9552 | 355 |
| 123 | 9905 | •258 | •611 | •963 | 1315 | 1667 | 2018 | 2370 | 2721 | 3071 | 351 |
| 124 | 093422 | 3772 | 4122 | 4471 | 4820 | 5169 | 5518 | 5866 | 6215 | 6562 | 349 |
| 125 | 6910 | 7257 | 7604 | 7951 | 8298 | 8644 | 8990 | 9335 | 9681 | ••26 | 346 |
| 126 | 100371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 | 343 |
| 127 | 3804 | 4146 | 4487 | 4828 | 5169 | 5510 | 5851 | 6191 | 6531 | 6871 | 340 |
| 128 | 7210 | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | •253 | 338 |
| 129 | 110590 | 0926 | 1263 | 1599 | 1934 | 2270 | 2605 | 2940 | 3275 | 3609 | 335 |
| 130 | 113943 | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 6940 | 333 |
| 131 | 7271 | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | •245 | 330 |
| 132 | 120574 | 0903 | 1231 | 1560 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 | 328 |
| 133 | 3852 | 4178 | 4504 | 4830 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 | 325 |
| 134 | 7105 | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 9690 | ••12 | 323 |
| 135 | 130334 | 0655 | 0977 | 1298 | 1619 | 1939 | 2260 | 2580 | 2900 | 3219 | 321 |
| 136 | 3539 | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 | 318 |
| 137 | 6721 | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 | 315 |
| 138 | 9879 | •194 | •508 | •822 | 1136 | 1450 | 1763 | 2076 | 2389 | 2702 | 314 |
| 139 | 143015 | 3327 | 3639 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 | 311 |
| 140 | 146128 | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 | 309 |
| 141 | 9219 | 9527 | 9835 | •142 | •449 | •756 | 1063 | 1370 | 1676 | 1982 | 307 |
| 142 | 152288 | 2594 | 2900 | 3205 | 3510 | 3815 | 4120 | 4424 | 4728 | 5032 | 305 |
| 143 | 5336 | 5640 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 | 303 |
| 144 | 8362 | 8664 | 8965 | 9266 | 9567 | 9868 | •168 | •469 | •769 | 1068 | 301 |
| 145 | 161368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 3460 | 3758 | 4055 | 299 |
| 146 | 4353 | 4650 | 4947 | 5244 | 5541 | 5838 | 6134 | 6430 | 6726 | 7022 | 297 |
| 147 | 7317 | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 9380 | 9674 | 9968 | 295 |
| 148 | 170262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 | 293 |
| 149 | 3186 | 3478 | 3769 | 4060 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 | 291 |
| 150 | 176091 | 6381 | 6670 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 | 289 |
| 151 | 8977 | 9264 | 9552 | 9839 | •126 | •413 | •699 | •985 | 1272 | 1558 | 287 |
| 152 | 181844 | 2129 | 2415 | 2700 | 2985 | 3270 | 3555 | 3839 | 4123 | 4407 | 285 |
| 153 | 4691 | 4975 | 5259 | 5542 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 | 283 |
| 154 | 7521 | 7803 | 8084 | 8366 | 8647 | 8928 | 9209 | 9490 | 9771 | ••51 | 281 |
| 155 | 190332 | 0612 | 0892 | 1171 | 1451 | 1730 | 2010 | 2289 | 2567 | 2846 | 279 |
| 156 | 3125 | 3403 | 3681 | 3959 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 | 278 |
| 157 | 5899 | 6176 | 6453 | 6729 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 | 276 |
| 158 | 8657 | 8932 | 9206 | 9481 | 9755 | ••29 | •303 | •577 | •850 | 1124 | 274 |
| 159 | 201397 | 1670 | 1943 | 2216 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 | 272 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

1537

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 160 | 204120 | 4391 | 4663 | 4934 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 | 271 |
| 161 | 6826 | 7096 | 7365 | 7634 | 7904 | 8173 | 8441 | 8710 | 8979 | 9247 | 269 |
| 162 | 9515 | 9783 | ●●51 | ●319 | ●586 | ●853 | 1121 | 1388 | 1654 | 1921 | 267 |
| 163 | 212188 | 2454 | 2720 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 | 266 |
| 164 | 4844 | 5109 | 5373 | 5638 | 5902 | 6166 | 6430 | 6694 | 6957 | 7221 | 264 |
| 165 | 7484 | 7747 | 8010 | 8273 | 8536 | 8798 | 9060 | 9323 | 9585 | 9846 | 262 |
| 166 | 220108 | 0370 | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 | 261 |
| 167 | 2716 | 2976 | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 | 259 |
| 168 | 5309 | 5568 | 5826 | 6084 | 6342 | 6600 | 6858 | 7115 | 7372 | 7630 | 258 |
| 169 | 7887 | 8144 | 8400 | 8657 | 8913 | 9170 | 9426 | 9682 | 9938 | ●193 | 256 |
| 170 | 230449 | 0704 | 0960 | 1215 | 1470 | 1724 | 1979 | 2234 | 2488 | 2742 | 254 |
| 171 | 2996 | 3250 | 3504 | 3757 | 4011 | 4264 | 4517 | 4770 | 5023 | 5276 | 253 |
| 172 | 5528 | 5781 | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 | 252 |
| 173 | 8046 | 8297 | 8548 | 8799 | 9049 | 9299 | 9550 | 9800 | ●●50 | ●300 | 250 |
| 174 | 240549 | 0799 | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 2790 | 249 |
| 175 | 3038 | 3286 | 3534 | 3782 | 4030 | 4277 | 4525 | 4772 | 5019 | 5266 | 248 |
| 176 | 5513 | 5759 | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 | 246 |
| 177 | 7973 | 8219 | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | ●176 | 245 |
| 178 | 250420 | 0664 | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 2610 | 243 |
| 179 | 2853 | 3096 | 3338 | 3580 | 3822 | 4064 | 4306 | 4548 | 4790 | 5031 | 242 |
| 180 | 255273 | 5514 | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 | 241 |
| 181 | 7679 | 7918 | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 | 239 |
| 182 | 260071 | 0310 | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 | 238 |
| 183 | 2451 | 2688 | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 | 237 |
| 184 | 4818 | 5054 | 5290 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 | 235 |
| 185 | 7172 | 7406 | 7641 | 7875 | 8110 | 8344 | 8578 | 8812 | 9046 | 9279 | 234 |
| 186 | 9513 | 9746 | 9980 | ●213 | ●446 | ●679 | ●912 | 1144 | 1377 | 1609 | 233 |
| 187 | 271842 | 2074 | 2306 | 2538 | 2770 | 3001 | 3233 | 3464 | 3696 | 3927 | 232 |
| 188 | 4158 | 4389 | 4620 | 4850 | 5081 | 5311 | 5542 | 5772 | 6002 | 6232 | 230 |
| 189 | 6462 | 6692 | 6921 | 7151 | 7380 | 7609 | 7838 | 8067 | 8296 | 8525 | 229 |
| 190 | 278754 | 8982 | 9211 | 9439 | 9667 | 9895 | ●123 | ●351 | ●578 | ●806 | 228 |
| 191 | 281033 | 1261 | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 | 227 |
| 192 | 3301 | 3527 | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 | 226 |
| 193 | 5557 | 5782 | 6007 | 6232 | 6456 | 6681 | 6905 | 7130 | 7354 | 7578 | 225 |
| 194 | 7802 | 8026 | 8249 | 8473 | 8696 | 8920 | 9143 | 9366 | 9589 | 9812 | 223 |
| 195 | 290035 | 0257 | 0480 | 0702 | 0925 | 1147 | 1369 | 1591 | 1813 | 2034 | 222 |
| 196 | 2256 | 2478 | 2699 | 2920 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 | 221 |
| 197 | 4466 | 4687 | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6226 | 6446 | 220 |
| 198 | 6665 | 6884 | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8416 | 8635 | 219 |
| 199 | 8853 | 9071 | 9289 | 9507 | 9725 | 9943 | ●161 | ●378 | ●595 | ●813 | 218 |
| 200 | 301030 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 2980 | 217 |
| 201 | 3196 | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 216 |
| 202 | 5351 | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 215 |
| 203 | 7496 | 7710 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 213 |
| 204 | 9630 | 9843 | ●●56 | ●268 | ●481 | ●693 | ●906 | 1118 | 1330 | 1542 | 212 |
| 205 | 311754 | 1966 | 2177 | 2389 | 2600 | 2812 | 3023 | 3234 | 3445 | 3656 | 211 |
| 206 | 3867 | 4078 | 4289 | 4499 | 4710 | 4920 | 5130 | 5340 | 5551 | 5760 | 210 |
| 207 | 5970 | 6180 | 6390 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 209 |
| 208 | 8063 | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 9730 | 9938 | 208 |
| 209 | 320146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 207 |
| 210 | 322219 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 206 |
| 211 | 4282 | 4488 | 4694 | 4899 | 5105 | 5310 | 5516 | 5721 | 5926 | 6131 | 205 |
| 212 | 6336 | 6541 | 6745 | 6950 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 204 |
| 213 | 8380 | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | ●●●8 | ●211 | 203 |
| 214 | 330414 | 0617 | 0819 | 1022 | 1225 | 1427 | 1630 | 1832 | 2034 | 2236 | 202 |
| 215 | 2438 | 2640 | 2842 | 3044 | 3246 | 3447 | 3649 | 3850 | 4051 | 4253 | 202 |
| 216 | 4454 | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 6260 | 201 |
| 217 | 6460 | 6660 | 6860 | 7060 | 7260 | 7459 | 7659 | 7858 | 8058 | 8257 | 200 |
| 218 | 8456 | 8656 | 8855 | 9054 | 9253 | 9451 | 9650 | 9849 | ●●47 | ●246 | 199 |
| 219 | 340444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 1830 | 2028 | 2225 | 198 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 220 | 342423 | 2620 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 | 197 |
| 221 | 4392 | 4589 | 4785 | 4981 | 5178 | 5374 | 5570 | 5766 | 5962 | 6157 | 196 |
| 222 | 6353 | 6549 | 6744 | 6939 | 7135 | 7330 | 7525 | 7720 | 7915 | 8110 | 195 |
| 223 | 8305 | 8500 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 9860 | ••54 | 194 |
| 224 | 350248 | 0442 | 0636 | 0829 | 1023 | 1216 | 1410 | 1603 | 1796 | 1989 | 193 |
| 225 | 2183 | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 193 |
| 226 | 4108 | 4301 | 4493 | 4685 | 4876 | 5068 | 5260 | 5452 | 5643 | 5834 | 192 |
| 227 | 6026 | 6217 | 6408 | 6599 | 6790 | 6981 | 7172 | 7363 | 7554 | 7744 | 191 |
| 228 | 7935 | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 190 |
| 229 | 9835 | ••25 | •215 | •404 | •593 | •783 | •972 | 1161 | 1350 | 1539 | 189 |
| 230 | 361728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 188 |
| 231 | 3612 | 3800 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 188 |
| 232 | 5488 | 5675 | 5862 | 6049 | 6236 | 6423 | 6610 | 6796 | 6983 | 7169 | 187 |
| 233 | 7356 | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 9030 | 186 |
| 234 | 9216 | 9401 | 9587 | 9772 | 9958 | •143 | •328 | •513 | •698 | •883 | 185 |
| 235 | 371068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 2360 | 2544 | 2728 | 184 |
| 236 | 2912 | 3096 | 3280 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 | 184 |
| 237 | 4748 | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 | 183 |
| 238 | 6577 | 6759 | 6942 | 7124 | 7306 | 7488 | 7670 | 7852 | 8034 | 8216 | 182 |
| 239 | 8398 | 8580 | 8761 | 8943 | 9124 | 9306 | 9487 | 9668 | 9849 | ••30 | 181 |
| 240 | 380211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 | 181 |
| 241 | 2017 | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 | 180 |
| 242 | 3815 | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 5070 | 5249 | 5428 | 179 |
| 243 | 5606 | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 | 178 |
| 244 | 7390 | 7568 | 7746 | 7923 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 | 178 |
| 245 | 9166 | 9343 | 9520 | 9698 | 9875 | ••51 | •228 | •405 | •582 | •759 | 177 |
| 246 | 390935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 | 176 |
| 247 | 2697 | 2873 | 3048 | 3224 | 3400 | 3575 | 3751 | 3926 | 4101 | 4277 | 176 |
| 248 | 4452 | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 5850 | 6025 | 175 |
| 249 | 6199 | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 | 174 |
| 250 | 397940 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 | 173 |
| 251 | 9674 | 9847 | ••20 | •192 | •365 | •538 | •711 | •883 | 1056 | 1228 | 173 |
| 252 | 401401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 | 172 |
| 253 | 3121 | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 4320 | 4492 | 4663 | 171 |
| 254 | 4834 | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 6370 | 171 |
| 255 | 6540 | 6710 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 8070 | 170 |
| 256 | 8240 | 8410 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 | 169 |
| 257 | 9933 | •102 | •271 | •440 | •609 | •777 | •946 | 1114 | 1283 | 1451 | 169 |
| 258 | 411620 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 | 168 |
| 259 | 3300 | 3467 | 3635 | 3803 | 3970 | 4137 | 4305 | 4472 | 4639 | 4806 | 167 |
| 260 | 414973 | 5140 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 | 167 |
| 261 | 6641 | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 7970 | 8135 | 166 |
| 262 | 8301 | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 9460 | 9625 | 9791 | 165 |
| 263 | 9956 | •121 | •286 | •451 | •616 | •781 | •945 | 1110 | 1275 | 1439 | 165 |
| 264 | 421604 | 1788 | 1933 | 2097 | 2261 | 2426 | 2590 | 2754 | 2918 | 3082 | 164 |
| 265 | 3246 | 3410 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 | 164 |
| 266 | 4882 | 5045 | 5208 | 5371 | 5534 | 5697 | 5860 | 6023 | 6186 | 6349 | 163 |
| 267 | 6511 | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 | 162 |
| 268 | 8135 | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 | 162 |
| 269 | 9752 | 9914 | ••75 | •236 | •398 | •559 | •720 | •881 | 1042 | 1203 | 161 |
| 270 | 431364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 | 161 |
| 271 | 2969 | 3130 | 3290 | 3450 | 3610 | 3770 | 3930 | 4090 | 4249 | 4409 | 160 |
| 272 | 4569 | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 | 159 |
| 273 | 6163 | 6322 | 6481 | 6640 | 6798 | 6957 | 7116 | 7275 | 7433 | 7592 | 159 |
| 274 | 7751 | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 | 158 |
| 275 | 9333 | 9491 | 9648 | 9806 | 9964 | •122 | •279 | •437 | •594 | •752 | 158 |
| 276 | 440909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 | 157 |
| 277 | 2480 | 2637 | 2793 | 2950 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 | 157 |
| 278 | 4045 | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 | 156 |
| 279 | 5604 | 5760 | 5915 | 6071 | 6226 | 6382 | 6537 | 6692 | 6848 | 7003 | 155 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 280 | 447158 | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | 8242 | 8397 | 8552 | 155 |
| 281 | 8706 | 8861 | 9015 | 9170 | 9324 | 9478 | 9633 | 9787 | 9941 | ••95 | 154 |
| 282 | 450249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 | 154 |
| 283 | 1786 | 1940 | 2093 | 2247 | 2400 | 2553 | 2706 | 2859 | 3012 | 3165 | 153 |
| 284 | 3318 | 3471 | 3624 | 3777 | 3930 | 4082 | 4235 | 4387 | 4540 | 4692 | 153 |
| 285 | 4845 | 4997 | 5150 | 5302 | 5454 | 5606 | 5758 | 5910 | 6062 | 6214 | 152 |
| 286 | 6366 | 6518 | 6670 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 | 152 |
| 287 | 7882 | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 8940 | 9091 | 9242 | 151 |
| 288 | 9392 | 9543 | 9694 | 9845 | 9995 | •146 | •296 | •447 | •597 | •748 | 151 |
| 289 | 460898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 | 150 |
| 290 | 462398 | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 | 3744 | 150 |
| 291 | 3893 | 4042 | 4191 | 4340 | 4490 | 4639 | 4788 | 4936 | 5085 | 5234 | 149 |
| 292 | 5383 | 5532 | 5680 | 5829 | 5977 | 6126 | 6274 | 6423 | 6571 | 6719 | 149 |
| 293 | 6868 | 7016 | 7164 | 7312 | 7460 | 7608 | 7756 | 7904 | 8052 | 8200 | 148 |
| 294 | 8347 | 8495 | 8643 | 8790 | 8938 | 9085 | 9233 | 9380 | 9527 | 9675 | 148 |
| 295 | 9822 | 9969 | •116 | •263 | •410 | •557 | •704 | •851 | •998 | 1145 | 147 |
| 296 | 471292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 | 2610 | 146 |
| 297 | 2756 | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 | 4071 | 146 |
| 298 | 4216 | 4362 | 4508 | 4653 | 4799 | 4944 | 5090 | 5235 | 5381 | 5526 | 146 |
| 299 | 5671 | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 | 6976 | 145 |
| 300 | 477121 | 7266 | 7411 | 7555 | 7700 | 7844 | 7989 | 8133 | 8278 | 8422 | 145 |
| 301 | 8566 | 8711 | 8855 | 8999 | 9143 | 9287 | 9431 | 9575 | 9719 | 9863 | 144 |
| 302 | 480007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 | 1299 | 144 |
| 303 | 1443 | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 | 2731 | 143 |
| 304 | 2874 | 3016 | 3159 | 3302 | 3445 | 3587 | 3730 | 3872 | 4015 | 4157 | 143 |
| 305 | 4300 | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 | 5579 | 142 |
| 306 | 5721 | 5863 | 6005 | 6147 | 6289 | 6430 | 6572 | 6714 | 6855 | 6997 | 142 |
| 307 | 7138 | 7280 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 | 8410 | 141 |
| 308 | 8551 | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9677 | 9818 | 141 |
| 309 | 9958 | ••99 | •239 | •380 | •520 | •661 | •801 | •941 | 1081 | 1222 | 140 |
| 310 | 491362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 | 2621 | 140 |
| 311 | 2760 | 2900 | 3040 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 | 4015 | 139 |
| 312 | 4155 | 4294 | 4433 | 4572 | 4711 | 4850 | 4989 | 5128 | 5267 | 5406 | 139 |
| 313 | 5544 | 5683 | 5822 | 5960 | 6099 | 6238 | 6376 | 6515 | 6653 | 6791 | 139 |
| 314 | 6930 | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 | 8173 | 138 |
| 315 | 8311 | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 | 9550 | 138 |
| 316 | 9687 | 9824 | 9962 | ••99 | •236 | •374 | •511 | •648 | •785 | •922 | 137 |
| 317 | 501059 | 1196 | 1333 | 1470 | 1607 | 1744 | 1880 | 2017 | 2154 | 2291 | 137 |
| 318 | 2427 | 2564 | 2700 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 | 3655 | 136 |
| 319 | 3791 | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 4878 | 5014 | 136 |
| 320 | 505150 | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 | 6370 | 136 |
| 321 | 6505 | 6640 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 | 7721 | 135 |
| 322 | 7856 | 7991 | 8126 | 8260 | 8395 | 8530 | 8664 | 8799 | 8934 | 9068 | 135 |
| 323 | 9203 | 9337 | 9471 | 9606 | 9740 | 9874 | •••9 | •143 | •277 | •411 | 134 |
| 324 | 510545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 | 1750 | 134 |
| 325 | 1883 | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 | 3084 | 133 |
| 326 | 3218 | 3351 | 3484 | 3617 | 3750 | 3883 | 4016 | 4149 | 4282 | 4414 | 133 |
| 327 | 4548 | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 | 5741 | 133 |
| 328 | 5874 | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 6800 | 6932 | 7064 | 132 |
| 329 | 7196 | 7328 | 7460 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 | 8382 | 132 |
| 330 | 518514 | 8646 | 8777 | 8909 | 9040 | 9171 | 9303 | 9434 | 9566 | 9697 | 131 |
| 331 | 9828 | 9959 | ••90 | •221 | •353 | •484 | •615 | •745 | •876 | 1007 | 131 |
| 332 | 521138 | 1269 | 1400 | 1530 | 1661 | 1792 | 1922 | 2053 | 2183 | 2314 | 131 |
| 333 | 2444 | 2575 | 2705 | 2835 | 2966 | 3096 | 3226 | 3356 | 3486 | 3616 | 130 |
| 334 | 3746 | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 | 4915 | 130 |
| 335 | 5045 | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 6210 | 129 |
| 336 | 6339 | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501 | 129 |
| 337 | 7630 | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 8660 | 8788 | 129 |
| 338 | 8917 | 9045 | 9174 | 9302 | 9430 | 9559 | 9687 | 9815 | 9943 | ••72 | 128 |
| 339 | 530200 | 0328 | 0456 | 0584 | 0712 | 0840 | 0968 | 1096 | 1223 | 1351 | 128 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | T |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 340 | 531479 | 1607 | 1734 | 1862 | 1990 | 2117 | 2245 | 2372 | 2500 | 2627 | 128 |
| 341 | 2754 | 2882 | 3009 | 3136 | 3264 | 3391 | 3518 | 3645 | 3772 | 3899 | 127 |
| 342 | 4026 | 4153 | 4280 | 4407 | 4534 | 4661 | 4787 | 4914 | 5041 | 5167 | 127 |
| 343 | 5294 | 5421 | 5547 | 5674 | 5800 | 5927 | 6053 | 6180 | 6306 | 6432 | 126 |
| 344 | 6558 | 6685 | 6811 | 6937 | 7063 | 7189 | 7315 | 7441 | 7567 | 7693 | 126 |
| 345 | 7819 | 7945 | 8071 | 8197 | 8322 | 8448 | 8574 | 8699 | 8825 | 8951 | 126 |
| 346 | 9076 | 9202 | 9327 | 9452 | 9578 | 9703 | 9829 | 9954 | ●●79 | ●204 | 125 |
| 347 | 540329 | 0455 | 0580 | 0705 | 0830 | 0955 | 1080 | 1205 | 1330 | 1454 | 125 |
| 348 | 1579 | 1704 | 1829 | 1953 | 2078 | 2203 | 2327 | 2452 | 2576 | 2701 | 125 |
| 349 | 2825 | 2950 | 3074 | 3199 | 3323 | 3447 | 3571 | 3696 | 3820 | 3944 | 124 |
| 350 | 544068 | 4192 | 4316 | 4440 | 4564 | 4688 | 4812 | 4936 | 5060 | 5183 | 124 |
| 351 | 5307 | 5431 | 5555 | 5678 | 5802 | 5925 | 6049 | 6172 | 6296 | 6419 | 124 |
| 352 | 6543 | 6666 | 6789 | 6913 | 7036 | 7159 | 7282 | 7405 | 7529 | 7652 | 123 |
| 353 | 7775 | 7898 | 8021 | 8144 | 8267 | 8389 | 8512 | 8635 | 8758 | 8881 | 123 |
| 354 | 9003 | 9126 | 9249 | 9371 | 9494 | 9616 | 9739 | 9861 | 9984 | ●106 | 123 |
| 355 | 550228 | 0351 | 0473 | 0595 | 0717 | 0840 | 0962 | 1084 | 1206 | 1328 | 122 |
| 356 | 1450 | 1572 | 1694 | 1816 | 1938 | 2060 | 2181 | 2303 | 2425 | 2547 | 122 |
| 357 | 2668 | 2790 | 2911 | 3033 | 3155 | 3276 | 3398 | 3519 | 3640 | 3762 | 121 |
| 358 | 3883 | 4004 | 4126 | 4247 | 4368 | 4489 | 4610 | 4731 | 4852 | 4973 | 121 |
| 359 | 5094 | 5215 | 5336 | 5457 | 5578 | 5699 | 5820 | 5940 | 6061 | 6182 | 121 |
| 360 | 556303 | 6423 | 6544 | 6664 | 6785 | 6905 | 7026 | 7146 | 7267 | 7387 | 120 |
| 361 | 7507 | 7627 | 7748 | 7868 | 7988 | 8108 | 8228 | 8349 | 8469 | 8589 | 120 |
| 362 | 8709 | 8829 | 8948 | 9068 | 9188 | 9308 | 9428 | 9548 | 9667 | 9787 | 120 |
| 363 | 9907 | ●●26 | ●146 | ●265 | ●385 | ●504 | ●624 | ●743 | ●863 | ●982 | 119 |
| 364 | 561101 | 1221 | 1340 | 1459 | 1578 | 1698 | 1817 | 1936 | 2055 | 2174 | 119 |
| 365 | 2293 | 2412 | 2531 | 2650 | 2769 | 2887 | 3006 | 3125 | 3244 | 3362 | 119 |
| 366 | 3481 | 3600 | 3718 | 3837 | 3955 | 4074 | 4192 | 4311 | 4429 | 4548 | 119 |
| 367 | 4666 | 4784 | 4903 | 5021 | 5139 | 5257 | 5376 | 5494 | 5612 | 5730 | 118 |
| 368 | 5848 | 5966 | 6084 | 6202 | 6320 | 6437 | 6555 | 6673 | 6791 | 6909 | 118 |
| 369 | 7026 | 7144 | 7262 | 7379 | 7497 | 7614 | 7732 | 7849 | 7967 | 8084 | 118 |
| 370 | 568202 | 8319 | 8436 | 8554 | 8671 | 8788 | 8905 | 9023 | 9140 | 9257 | 117 |
| 371 | 9374 | 9491 | 9608 | 9725 | 9842 | 9959 | ●●76 | ●193 | ●309 | ●426 | 117 |
| 372 | 570543 | 0660 | 0776 | 0893 | 1010 | 1126 | 1243 | 1359 | 1476 | 1592 | 117 |
| 373 | 1709 | 1825 | 1942 | 2058 | 2174 | 2291 | 2407 | 2523 | 2639 | 2755 | 116 |
| 374 | 2872 | 2988 | 3104 | 3220 | 3336 | 3452 | 3568 | 3684 | 3800 | 3915 | 116 |
| 375 | 4031 | 4147 | 4263 | 4379 | 4494 | 4610 | 4726 | 4841 | 4957 | 5072 | 116 |
| 376 | 5188 | 5303 | 5419 | 5534 | 5650 | 5765 | 5880 | 5996 | 6111 | 6226 | 115 |
| 377 | 6341 | 6457 | 6572 | 6687 | 6802 | 6917 | 7032 | 7147 | 7262 | 7377 | 115 |
| 378 | 7492 | 7607 | 7722 | 7836 | 7951 | 8066 | 8181 | 8295 | 8410 | 8525 | 115 |
| 379 | 8639 | 8754 | 8868 | 8983 | 9097 | 9212 | 9326 | 9441 | 9555 | 9669 | 114 |
| 380 | 579784 | 9898 | ●●12 | ●126 | ●241 | ●355 | ●469 | ●583 | ●697 | ●811 | 114 |
| 381 | 580925 | 1039 | 1153 | 1267 | 1381 | 1495 | 1608 | 1722 | 1836 | 1950 | 114 |
| 382 | 2063 | 2177 | 2291 | 2404 | 2518 | 2631 | 2745 | 2858 | 2972 | 3085 | 114 |
| 383 | 3199 | 3312 | 3426 | 3539 | 3652 | 3765 | 3879 | 3992 | 4105 | 4218 | 113 |
| 384 | 4331 | 4444 | 4557 | 4670 | 4783 | 4896 | 5009 | 5122 | 5235 | 5348 | 113 |
| 385 | 5461 | 5574 | 5686 | 5799 | 5912 | 6024 | 6137 | 6250 | 6362 | 6475 | 113 |
| 386 | 6587 | 6700 | 6812 | 6925 | 7037 | 7149 | 7262 | 7374 | 7486 | 7599 | 112 |
| 387 | 7711 | 7823 | 7935 | 8047 | 8160 | 8272 | 8384 | 8496 | 8608 | 8720 | 112 |
| 388 | 8832 | 8944 | 9056 | 9167 | 9279 | 9391 | 9503 | 9615 | 9726 | 9838 | 112 |
| 389 | 9950 | ●●61 | ●173 | ●284 | ●396 | ●507 | ●619 | ●730 | ●842 | ●953 | 112 |
| 390 | 591065 | 1176 | 1287 | 1399 | 1510 | 1621 | 1732 | 1843 | 1955 | 2066 | 111 |
| 391 | 2177 | 2288 | 2399 | 2510 | 2621 | 2732 | 2843 | 2954 | 3064 | 3175 | 111 |
| 392 | 3286 | 3397 | 3508 | 3618 | 3729 | 3840 | 3950 | 4061 | 4171 | 4282 | 111 |
| 393 | 4393 | 4503 | 4614 | 4724 | 4834 | 4945 | 5055 | 5165 | 5276 | 5386 | 110 |
| 394 | 5496 | 5606 | 5717 | 5827 | 5937 | 6047 | 6157 | 6267 | 6377 | 6487 | 110 |
| 395 | 6597 | 6707 | 6817 | 6927 | 7037 | 7146 | 7256 | 7366 | 7476 | 7586 | 110 |
| 396 | 7695 | 7805 | 7914 | 8024 | 8134 | 8243 | 8353 | 8462 | 8572 | 8681 | 110 |
| 397 | 8791 | 8900 | 9009 | 9119 | 9228 | 9337 | 9446 | 9556 | 9665 | 9774 | 109 |
| 398 | 9883 | 9992 | ●101 | ●210 | ●319 | ●428 | ●537 | ●646 | ●755 | ●864 | 109 |
| 399 | 600973 | 1082 | 1191 | 1299 | 1408 | 1517 | 1625 | 1734 | 1843 | 1951 | 109 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 400 | 602060 | 2169 | 2277 | 2386 | 2494 | 2603 | 2711 | 2819 | 2928 | 3036 | 108 |
| 401 | 3144 | 3253 | 3361 | 3469 | 3577 | 3686 | 3794 | 3902 | 4010 | 4118 | 108 |
| 402 | 4226 | 4334 | 4442 | 4550 | 4658 | 4766 | 4874 | 4982 | 5089 | 5197 | 108 |
| 403 | 5305 | 5413 | 5521 | 5628 | 5736 | 5844 | 5951 | 6059 | 6166 | 6274 | 108 |
| 404 | 6381 | 6489 | 6596 | 6704 | 6811 | 6919 | 7026 | 7133 | 7241 | 7348 | 107 |
| 405 | 7455 | 7562 | 7669 | 7777 | 7884 | 7991 | 8098 | 8205 | 8312 | 8419 | 107 |
| 406 | 8526 | 8633 | 8740 | 8847 | 8954 | 9061 | 9167 | 9274 | 9381 | 9488 | 107 |
| 407 | 9594 | 9701 | 9808 | 9914 | ••21 | •128 | •234 | •341 | •447 | •554 | 107 |
| 408 | 610660 | 0767 | 0873 | 0979 | 1086 | 1192 | 1298 | 1405 | 1511 | 1617 | 106 |
| 409 | 1723 | 1829 | 1936 | 2042 | 2148 | 2254 | 2360 | 2466 | 2572 | 2678 | 106 |
| 410 | 612784 | 2890 | 2996 | 3102 | 3207 | 3313 | 3419 | 3525 | 3630 | 3736 | 106 |
| 411 | 3842 | 3947 | 4053 | 4159 | 4264 | 4370 | 4475 | 4581 | 4686 | 4792 | 106 |
| 412 | 4897 | 5003 | 5108 | 5213 | 5319 | 5424 | 5529 | 5634 | 5740 | 5845 | 105 |
| 413 | 5950 | 6055 | 6160 | 6265 | 6370 | 6476 | 6581 | 6686 | 6790 | 6895 | 105 |
| 414 | 7000 | 7105 | 7210 | 7315 | 7420 | 7525 | 7629 | 7734 | 7839 | 7943 | 105 |
| 415 | 8048 | 8153 | 8257 | 8362 | 8466 | 8571 | 8676 | 8780 | 8884 | 8989 | 105 |
| 416 | 9093 | 9198 | 9302 | 9406 | 9511 | 9615 | 9719 | 9824 | 9928 | ••32 | 104 |
| 417 | 620136 | 0240 | 0344 | 0448 | 0552 | 0656 | 0760 | 0864 | 0968 | 1072 | 104 |
| 418 | 1176 | 1280 | 1384 | 1488 | 1592 | 1695 | 1799 | 1903 | 2007 | 2110 | 104 |
| 419 | 2214 | 2318 | 2421 | 2525 | 2628 | 2732 | 2835 | 2939 | 3042 | 3146 | 104 |
| 420 | 623249 | 3353 | 3456 | 3559 | 3663 | 3766 | 3869 | 3973 | 4076 | 4179 | 103 |
| 421 | 4282 | 4385 | 4488 | 4591 | 4695 | 4798 | 4901 | 5004 | 5107 | 5210 | 103 |
| 422 | 5312 | 5415 | 5518 | 5621 | 5724 | 5827 | 5929 | 6032 | 6135 | 6238 | 103 |
| 423 | 6340 | 6443 | 6546 | 6648 | 6751 | 6853 | 6956 | 7058 | 7161 | 7263 | 103 |
| 424 | 7366 | 7468 | 7571 | 7673 | 7775 | 7878 | 7980 | 8082 | 8185 | 8287 | 102 |
| 425 | 8389 | 8491 | 8593 | 8695 | 8797 | 8900 | 9002 | 9104 | 9206 | 9308 | 102 |
| 426 | 9410 | 9512 | 9613 | 9715 | 9817 | 9919 | ••21 | •123 | •224 | •326 | 102 |
| 427 | 630428 | 0530 | 0631 | 0733 | 0835 | 0936 | 1038 | 1139 | 1241 | 1342 | 102 |
| 428 | 1444 | 1545 | 1647 | 1748 | 1849 | 1951 | 2052 | 2153 | 2255 | 2356 | 101 |
| 429 | 2457 | 2559 | 2660 | 2761 | 2862 | 2963 | 3064 | 3165 | 3266 | 3367 | 101 |
| 430 | 633468 | 3569 | 3670 | 3771 | 3872 | 3973 | 4074 | 4175 | 4276 | 4376 | 100 |
| 431 | 4477 | 4578 | 4679 | 4779 | 4880 | 4981 | 5081 | 5182 | 5283 | 5383 | 100 |
| 432 | 5484 | 5584 | 5685 | 5785 | 5886 | 5986 | 6087 | 6187 | 6287 | 6388 | 100 |
| 433 | 6488 | 6588 | 6688 | 6789 | 6889 | 6989 | 7089 | 7189 | 7290 | 7390 | 100 |
| 434 | 7490 | 7590 | 7690 | 7790 | 7890 | 7990 | 8090 | 8190 | 8290 | 8389 | 99 |
| 435 | 8489 | 8589 | 8689 | 8789 | 8888 | 8988 | 9088 | 9188 | 9287 | 9387 | 99 |
| 436 | 9486 | 9586 | 9686 | 9785 | 9885 | 9984 | ••84 | •183 | •283 | •382 | 99 |
| 437 | 640481 | 0581 | 0680 | 0779 | 0879 | 0978 | 1077 | 1177 | 1276 | 1375 | 99 |
| 438 | 1474 | 1573 | 1672 | 1771 | 1871 | 1970 | 2069 | 2168 | 2267 | 2366 | 99 |
| 439 | 2465 | 2563 | 2662 | 2761 | 2860 | 2959 | 3058 | 3156 | 3255 | 3354 | 99 |
| 440 | 643453 | 3551 | 3650 | 3749 | 3847 | 3946 | 4044 | 4143 | 4242 | 4340 | 98 |
| 441 | 4439 | 4537 | 4636 | 4734 | 4832 | 4931 | 5029 | 5127 | 5226 | 5324 | 98 |
| 442 | 5422 | 5521 | 5619 | 5717 | 5815 | 5913 | 6011 | 6110 | 6208 | 6306 | 98 |
| 443 | 6404 | 6502 | 6600 | 6698 | 6796 | 6894 | 6992 | 7089 | 7187 | 7285 | 98 |
| 444 | 7383 | 7481 | 7579 | 7676 | 7774 | 7872 | 7969 | 8067 | 8165 | 8262 | 98 |
| 445 | 8360 | 8458 | 8555 | 8653 | 8750 | 8848 | 8945 | 9043 | 9140 | 9237 | 97 |
| 446 | 9335 | 9432 | 9530 | 9627 | 9724 | 9821 | 9919 | ••16 | •113 | •210 | 97 |
| 447 | 650308 | 0405 | 0502 | 0599 | 0696 | 0793 | 0890 | 0987 | 1084 | 1181 | 97 |
| 448 | 1278 | 1375 | 1472 | 1569 | 1666 | 1762 | 1859 | 1956 | 2053 | 2150 | 97 |
| 449 | 2246 | 2343 | 2440 | 2536 | 2633 | 2730 | 2826 | 2923 | 3019 | 3116 | 97 |
| 450 | 653213 | 3309 | 3405 | 3502 | 3598 | 3695 | 3791 | 3888 | 3984 | 4080 | 96 |
| 451 | 4177 | 4273 | 4369 | 4465 | 4562 | 4658 | 4754 | 4850 | 4946 | 5042 | 96 |
| 452 | 5138 | 5235 | 5331 | 5427 | 5523 | 5619 | 5715 | 5810 | 5906 | 6002 | 96 |
| 453 | 6098 | 6194 | 6290 | 6386 | 6482 | 6577 | 6673 | 6769 | 6864 | 6960 | 96 |
| 454 | 7056 | 7152 | 7247 | 7343 | 7438 | 7534 | 7629 | 7725 | 7820 | 7916 | 96 |
| 455 | 8011 | 8107 | 8202 | 8298 | 8393 | 8488 | 8584 | 8679 | 8774 | 8870 | 95 |
| 456 | 8965 | 9060 | 9155 | 9250 | 9346 | 9441 | 9536 | 9631 | 9726 | 9821 | 95 |
| 457 | 9916 | ••11 | •106 | •201 | •296 | •391 | •486 | •581 | •676 | •771 | 95 |
| 458 | 660865 | 0960 | 1055 | 1150 | 1245 | 1339 | 1434 | 1529 | 1623 | 1718 | 95 |
| 459 | 1813 | 1907 | 2002 | 2096 | 2191 | 2286 | 2380 | 2475 | 2569 | 2663 | 95 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 460 | 662758 | 2852 | 2947 | 3041 | 3135 | 3230 | 3324 | 3418 | 3512 | 3607 | 94 |
| 461 | 3701 | 3795 | 3889 | 3983 | 4078 | 4172 | 4266 | 4360 | 4454 | 4548 | 94 |
| 462 | 4642 | 4736 | 4830 | 4924 | 5018 | 5112 | 5206 | 5299 | 5393 | 5487 | 94 |
| 463 | 5581 | 5675 | 5769 | 5862 | 5956 | 6050 | 6143 | 6237 | 6331 | 6424 | 94 |
| 464 | 6518 | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | 7266 | 7360 | 94 |
| 465 | 7453 | 7546 | 7640 | 7733 | 7826 | 7920 | 8013 | 8106 | 8199 | 8293 | 93 |
| 466 | 8386 | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | 9131 | 9224 | 93 |
| 467 | 9317 | 9410 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | ••60 | •153 | 93 |
| 468 | 670246 | 0339 | 0431 | 0524 | 0617 | 0710 | 0802 | 0895 | 0988 | 1080 | 93 |
| 469 | 1173 | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | 2005 | 93 |
| 470 | 672098 | 2190 | 2283 | 2375 | 2467 | 2560 | 2652 | 2744 | 2836 | 2929 | 92 |
| 471 | 3021 | 3113 | 3205 | 3297 | 3390 | 3482 | 3574 | 3666 | 3758 | 3850 | 92 |
| 472 | 3942 | 4034 | 4126 | 4218 | 4310 | 4402 | 4494 | 4586 | 4677 | 4769 | 92 |
| 473 | 4861 | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 | 92 |
| 474 | 5778 | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 | 92 |
| 475 | 6694 | 6785 | 6876 | 6968 | 7059 | 7151 | 7242 | 7333 | 7424 | 7516 | 91 |
| 476 | 7607 | 7698 | 7789 | 7881 | 7972 | 8063 | 8154 | 8245 | 8336 | 8427 | 91 |
| 477 | 8518 | 8609 | 8700 | 8791 | 8882 | 8973 | 9064 | 9155 | 9246 | 9337 | 91 |
| 478 | 9428 | 9519 | 9610 | 9700 | 9791 | 9882 | 9973 | ••63 | •154 | •245 | 91 |
| 479 | 680336 | 0426 | 0517 | 0607 | 0698 | 0789 | 0879 | 0970 | 1060 | 1151 | 91 |
| 480 | 681241 | 1332 | 1422 | 1513 | 1603 | 1693 | 1784 | 1874 | 1964 | 2055 | 90 |
| 481 | 2145 | 2235 | 2326 | 2416 | 2506 | 2596 | 2686 | 2777 | 2867 | 2957 | 90 |
| 482 | 3047 | 3137 | 3227 | 3317 | 3407 | 3497 | 3587 | 3677 | 3767 | 3857 | 90 |
| 483 | 3947 | 4037 | 4127 | 4217 | 4307 | 4396 | 4486 | 4576 | 4666 | 4756 | 90 |
| 484 | 4845 | 4935 | 5025 | 5114 | 5204 | 5294 | 5383 | 5473 | 5563 | 5652 | 90 |
| 485 | 5742 | 5831 | 5921 | 6010 | 6100 | 6189 | 6279 | 6368 | 6458 | 5547 | 89 |
| 486 | 6636 | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 | 89 |
| 487 | 7529 | 7618 | 7707 | 7796 | 7886 | 7975 | 8064 | 8153 | 8242 | 8331 | 89 |
| 488 | 8420 | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 | 89 |
| 489 | 9309 | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | ••19 | •107 | 89 |
| 490 | 690196 | 0285 | 0373 | 0462 | 0550 | 0639 | 0728 | 0816 | 0905 | •993 | 89 |
| 491 | 1081 | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 | 88 |
| 492 | 1965 | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 | 88 |
| 493 | 2847 | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 | 88 |
| 494 | 3727 | 3815 | 3903 | 3991 | 4078 | 4166 | 4254 | 4342 | 4430 | 4517 | 88 |
| 495 | 4605 | 4693 | 4781 | 4868 | 4956 | 5044 | 5131 | 5219 | 5307 | 5394 | 88 |
| 496 | 5482 | 5569 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 | 87 |
| 497 | 6356 | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 | 87 |
| 498 | 7229 | 7317 | 7404 | 7491 | 7578 | 7665 | 7752 | 7839 | 7926 | 8014 | 87 |
| 499 | 8101 | 8188 | 8275 | 8362 | 8449 | 8535 | 8622 | 8709 | 8796 | 8883 | 87 |
| 500 | 698970 | 9057 | 9144 | 9231 | 9317 | 9404 | 9491 | 9578 | 9664 | 9751 | 87 |
| 501 | 9838 | 9924 | ••11 | ••98 | •184 | •271 | •358 | •444 | •531 | •617 | 87 |
| 502 | 700704 | 0790 | 0877 | 0963 | 1050 | 1136 | 1222 | 1309 | 1395 | 1482 | 86 |
| 503 | 1568 | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 | 86 |
| 504 | 2431 | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 | 86 |
| 505 | 3291 | 3377 | 3463 | 3549 | 3635 | 3721 | 3807 | 3893 | 3979 | 4065 | 86 |
| 506 | 4151 | 4236 | 4322 | 4408 | 4494 | 4579 | 4665 | 4751 | 4837 | 4922 | 86 |
| 507 | 5008 | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 | 86 |
| 508 | 5864 | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 | 85 |
| 509 | 6718 | 6803 | 6888 | 6974 | 7059 | 7144 | 7229 | 7315 | 7400 | 7485 | 85 |
| 510 | 707570 | 7655 | 7740 | 7826 | 7911 | 7996 | 8081 | 8166 | 8251 | 8336 | 85 |
| 511 | 8421 | 8506 | 8591 | 8676 | 8761 | 8846 | 8931 | 9015 | 9100 | 9185 | 85 |
| 512 | 9270 | 9355 | 9440 | 9524 | 9609 | 9694 | 9779 | 9863 | 9948 | ••33 | 85 |
| 513 | 710117 | 0202 | 0287 | 0371 | 0456 | 0540 | 0625 | 0710 | 0794 | 0879 | 85 |
| 514 | 0963 | 1048 | 1132 | 1217 | 1301 | 1385 | 1470 | 1554 | 1639 | 1723 | 84 |
| 515 | 1807 | 1892 | 1976 | 2060 | 2144 | 2229 | 2313 | 2397 | 2481 | 2566 | 84 |
| 516 | 2650 | 2734 | 2818 | 2902 | 2986 | 3070 | 3154 | 3238 | 3323 | 3407 | 84 |
| 517 | 3491 | 3575 | 3659 | 3742 | 3826 | 3910 | 3994 | 4078 | 4162 | 4246 | 84 |
| 518 | 4330 | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5000 | 5084 | 84 |
| 519 | 5167 | 5251 | 5335 | 5418 | 5502 | 5586 | 5669 | 5753 | 5836 | 5920 | 84 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 520 | 716003 | 6087 | 6170 | 6254 | 6337 | 6421 | 6504 | 6588 | 6671 | 6754 | 83 |
| 521 | 6838 | 6921 | 7004 | 7088 | 7171 | 7254 | 7338 | 7421 | 7504 | 7587 | 83 |
| 522 | 7671 | 7754 | 7837 | 7920 | 8003 | 8086 | 8169 | 8253 | 8336 | 8419 | 83 |
| 523 | 8502 | 8585 | 8668 | 8751 | 8834 | 8917 | 9000 | 9083 | 9165 | 9248 | 83 |
| 524 | 9331 | 9414 | 9497 | 9580 | 9663 | 9745 | 9828 | 9911 | 9994 | ••77 | 83 |
| 525 | 720159 | 0242 | 0325 | 0407 | 0490 | 0573 | 0655 | 0738 | 0821 | 0903 | 83 |
| 526 | 0986 | 1068 | 1151 | 1233 | 1316 | 1398 | 1481 | 1563 | 1646 | 1728 | 82 |
| 527 | 1811 | 1893 | 1975 | 2058 | 2140 | 2222 | 2305 | 2387 | 2469 | 2552 | 82 |
| 528 | 2634 | 2716 | 2798 | 2881 | 2963 | 3045 | 3127 | 3209 | 3291 | 3374 | 82 |
| 529 | 3456 | 3538 | 3620 | 3702 | 3784 | 3866 | 3948 | 4030 | 4112 | 4194 | 82 |
| 530 | 724276 | 4358 | 4440 | 4522 | 4604 | 4685 | 4767 | 4849 | 4931 | 5013 | 82 |
| 531 | 5095 | 5176 | 5258 | 5340 | 5422 | 5503 | 5585 | 5667 | 5748 | 5830 | 82 |
| 532 | 5912 | 5993 | 6075 | 6156 | 6238 | 6320 | 6401 | 6483 | 6564 | 6646 | 82 |
| 533 | 6727 | 6809 | 6890 | 6972 | 7053 | 7134 | 7216 | 7297 | 7379 | 7460 | 81 |
| 534 | 7541 | 7623 | 7704 | 7785 | 7866 | 7948 | 8029 | 8110 | 8191 | 8273 | 81 |
| 535 | 8354 | 8435 | 8516 | 8597 | 8678 | 8759 | 8841 | 8922 | 9003 | 9084 | 81 |
| 536 | 9165 | 9246 | 9327 | 9408 | 9489 | 9570 | 9651 | 9732 | 9813 | 9893 | 81 |
| 537 | 9974 | ••55 | •136 | •217 | •298 | •378 | •459 | •540 | •621 | •702 | 81 |
| 538 | 730782 | 0863 | 0944 | 1024 | 1105 | 1186 | 1266 | 1347 | 1428 | 1508 | 81 |
| 539 | 1589 | 1669 | 1750 | 1830 | 1911 | 1991 | 2072 | 2152 | 2233 | 2313 | 81 |
| 540 | 732394 | 2474 | 2555 | 2635 | 2715 | 2796 | 2876 | 2956 | 3037 | 3117 | 80 |
| 541 | 3197 | 3278 | 3358 | 3438 | 3518 | 3598 | 3679 | 3759 | 3839 | 3919 | 80 |
| 542 | 3999 | 4079 | 4160 | 4240 | 4320 | 4400 | 4480 | 4560 | 4640 | 4720 | 80 |
| 543 | 4800 | 4880 | 4960 | 5040 | 5120 | 5200 | 5279 | 5359 | 5439 | 5519 | 80 |
| 544 | 5599 | 5679 | 5759 | 5838 | 5918 | 5998 | 6078 | 6157 | 6237 | 6317 | 80 |
| 545 | 6397 | 6476 | 6556 | 6635 | 6715 | 6795 | 6874 | 6954 | 7034 | 7113 | 80 |
| 546 | 7193 | 7272 | 7352 | 7431 | 7511 | 7590 | 7670 | 7749 | 7829 | 7908 | 79 |
| 547 | 7987 | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 | 79 |
| 548 | 8781 | 8860 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 | 79 |
| 549 | 9572 | 9651 | 9731 | 9810 | 9889 | 9968 | ••47 | •126 | •205 | •284 | 79 |
| 550 | 740363 | 0442 | 0521 | 0600 | 0678 | 0757 | 0836 | 0915 | 0994 | 1073 | 79 |
| 551 | 1152 | 1230 | 1309 | 1388 | 1467 | 1546 | 1624 | 1703 | 1782 | 1860 | 79 |
| 552 | 1939 | 2018 | 2096 | 2175 | 2254 | 2332 | 2411 | 2489 | 2568 | 2647 | 79 |
| 553 | 2725 | 2804 | 2882 | 2961 | 3039 | 3118 | 3196 | 3275 | 3353 | 3431 | 78 |
| 554 | 3510 | 3588 | 3667 | 3745 | 3823 | 3902 | 3980 | 4058 | 4136 | 4215 | 78 |
| 555 | 4293 | 4371 | 4449 | 4528 | 4606 | 4684 | 4762 | 4840 | 4919 | 4997 | 78 |
| 556 | 5075 | 5153 | 5231 | 5309 | 5387 | 5465 | 5543 | 5621 | 5699 | 5777 | 78 |
| 557 | 5855 | 5933 | 6011 | 6089 | 6167 | 6245 | 6323 | 6401 | 6479 | 6556 | 78 |
| 558 | 6634 | 6712 | 6790 | 6868 | 6945 | 7023 | 7101 | 7179 | 7256 | 7334 | 78 |
| 559 | 7412 | 7489 | 7567 | 7645 | 7722 | 7800 | 7878 | 7955 | 8033 | 8110 | 78 |
| 560 | 748188 | 8266 | 8343 | 8421 | 8498 | 8576 | 8653 | 8731 | 8808 | 8885 | 77 |
| 561 | 8963 | 9040 | 9118 | 9195 | 9272 | 9350 | 9427 | 9504 | 9582 | 9659 | 77 |
| 562 | 9736 | 9814 | 9891 | 9968 | ••45 | •123 | •200 | •277 | •354 | •431 | 77 |
| 563 | 750508 | 0586 | 0663 | 0740 | 0817 | 0894 | 0971 | 1048 | 1125 | 1202 | 77 |
| 564 | 1279 | 1356 | 1433 | 1510 | 1587 | 1664 | 1741 | 1818 | 1895 | 1972 | 77 |
| 565 | 2048 | 2125 | 2202 | 2279 | 2356 | 2433 | 2509 | 2586 | 2663 | 2740 | 77 |
| 566 | 2816 | 2893 | 2970 | 3047 | 3123 | 3200 | 3277 | 3353 | 3430 | 3506 | 77 |
| 567 | 3583 | 3660 | 3736 | 3813 | 3889 | 3966 | 4042 | 4119 | 4195 | 4272 | 77 |
| 568 | 4348 | 4425 | 4501 | 4578 | 4654 | 4730 | 4807 | 4883 | 4960 | 5036 | 76 |
| 569 | 5112 | 5189 | 5265 | 5341 | 5417 | 5494 | 5570 | 5646 | 5722 | 5799 | 76 |
| 570 | 755875 | 5951 | 6027 | 6103 | 6180 | 6256 | 6332 | 6408 | 6484 | 6560 | 76 |
| 571 | 6636 | 6712 | 6788 | 6864 | 6940 | 7016 | 7092 | 7168 | 7244 | 7320 | 76 |
| 572 | 7396 | 7472 | 7548 | 7624 | 7700 | 7775 | 7851 | 7927 | 8003 | 8079 | 76 |
| 573 | 8155 | 8230 | 8306 | 8382 | 8458 | 8533 | 8609 | 8685 | 8761 | 8836 | 76 |
| 574 | 8912 | 8988 | 9063 | 9139 | 9214 | 9290 | 9366 | 9441 | 9517 | 9592 | 76 |
| 575 | 9668 | 9743 | 9819 | 9894 | 9970 | ••45 | •121 | •196 | •272 | •347 | 75 |
| 576 | 760422 | 0498 | 0573 | 0649 | 0724 | 0799 | 0875 | 0950 | 1025 | 1101 | 75 |
| 577 | 1176 | 1251 | 1326 | 1402 | 1477 | 1552 | 1627 | 1702 | 1778 | 1853 | 75 |
| 578 | 1928 | 2003 | 2078 | 2153 | 2228 | 2303 | 2378 | 2453 | 2529 | 2604 | 75 |
| 579 | 2679 | 2754 | 2829 | 2904 | 2978 | 3053 | 3128 | 3203 | 3278 | 3353 | 75 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 580 | 763428 | 3503 | 3578 | 3653 | 3727 | 3802 | 3877 | 3952 | 4027 | 4101 | 75 |
| 581 | 4176 | 4251 | 4326 | 4400 | 4475 | 4550 | 4624 | 4699 | 4774 | 4848 | 75 |
| 582 | 4923 | 4998 | 5072 | 5147 | 5221 | 5296 | 5370 | 5445 | 5520 | 5594 | 75 |
| 583 | 5669 | 5743 | 5818 | 5892 | 5966 | 6041 | 6115 | 6190 | 6264 | 6338 | 74 |
| 584 | 6413 | 6487 | 6562 | 6636 | 6710 | 6785 | 6859 | 6933 | 7007 | 7082 | 74 |
| 585 | 7156 | 7230 | 7304 | 7379 | 7453 | 7527 | 7601 | 7675 | 7749 | 7823 | 74 |
| 586 | 7898 | 7972 | 8046 | 8120 | 8194 | 8268 | 8342 | 8416 | 8490 | 8564 | 74 |
| 587 | 8638 | 8712 | 8786 | 8860 | 8934 | 9008 | 9082 | 9156 | 9230 | 9303 | 74 |
| 588 | 9377 | 9451 | 9525 | 9599 | 9673 | 9746 | 9820 | 9894 | 9968 | ••42 | 74 |
| 589 | 770115 | 0189 | 0263 | 0336 | 0410 | 0484 | 0557 | 0631 | 0705 | 0778 | 74 |
| 590 | 770852 | 0926 | 0999 | 1073 | 1146 | 1220 | 1293 | 1367 | 1440 | 1514 | 74 |
| 591 | 1587 | 1661 | 1734 | 1808 | 1881 | 1955 | 2028 | 2102 | 2175 | 2248 | 73 |
| 592 | 2322 | 2395 | 2468 | 2542 | 2615 | 2688 | 2762 | 2835 | 2908 | 2981 | 73 |
| 593 | 3055 | 3128 | 3201 | 3274 | 3348 | 3421 | 3494 | 3567 | 3640 | 3713 | 73 |
| 594 | 3786 | 3860 | 3933 | 4006 | 4079 | 4152 | 4225 | 4298 | 4371 | 4444 | 73 |
| 595 | 4517 | 4590 | 4663 | 4736 | 4809 | 4882 | 4955 | 5028 | 5100 | 5173 | 73 |
| 596 | 5246 | 5319 | 5392 | 5465 | 5538 | 5610 | 5683 | 5756 | 5829 | 5902 | 73 |
| 597 | 5974 | 6047 | 6120 | 6193 | 6265 | 6338 | 6411 | 6483 | 6556 | 6629 | 73 |
| 598 | 6701 | 6774 | 6846 | 6919 | 6992 | 7064 | 7137 | 7209 | 7282 | 7354 | 73 |
| 599 | 7427 | 7499 | 7572 | 7644 | 7717 | 7789 | 7862 | 7934 | 8006 | 8079 | 72 |
| 600 | 778151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585 | 8658 | 8730 | 8802 | 72 |
| 601 | 8874 | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 9380 | 9452 | 9524 | 72 |
| 602 | 9596 | 9669 | 9741 | 9813 | 9885 | 9957 | ••29 | •101 | •173 | •245 | 72 |
| 603 | 780317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749 | 0821 | 0893 | 0965 | 72 |
| 604 | 1037 | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 1540 | 1612 | 1684 | 72 |
| 605 | 1755 | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 | 72 |
| 606 | 2473 | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 | 72 |
| 607 | 3189 | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 | 71 |
| 608 | 3904 | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 | 71 |
| 609 | 4617 | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 | 71 |
| 610 | 785330 | 5401 | 5472 | 5543 | 5615 | 5686 | 5757 | 5828 | 5899 | 5970 | 71 |
| 611 | 6041 | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 | 71 |
| 612 | 6751 | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 | 71 |
| 613 | 7460 | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 | 71 |
| 614 | 8168 | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 | 71 |
| 615 | 8875 | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9510 | 71 |
| 616 | 9581 | 9651 | 9722 | 9792 | 9863 | 9933 | •••4 | ••74 | •144 | •215 | 70 |
| 617 | 790285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707 | 0778 | 0848 | 0918 | 70 |
| 618 | 0988 | 1059 | 1129 | 1199 | 1269 | 1340 | 1410 | 1480 | 1550 | 1620 | 70 |
| 619 | 1691 | 1761 | 1831 | 1901 | 1971 | 2041 | 2111 | 2181 | 2252 | 2322 | 70 |
| 620 | 792392 | 2462 | 2532 | 2602 | 2672 | 2742 | 2812 | 2882 | 2952 | 3022 | 70 |
| 621 | 3092 | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 | 70 |
| 622 | 3790 | 3860 | 3930 | 4000 | 4070 | 4139 | 4209 | 4279 | 4349 | 4418 | 70 |
| 623 | 4488 | 4558 | 4627 | 4697 | 4767 | 4836 | 4906 | 4976 | 5045 | 5115 | 70 |
| 624 | 5185 | 5254 | 5324 | 5393 | 5463 | 5532 | 5602 | 5672 | 5741 | 5811 | 70 |
| 625 | 5880 | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 | 69 |
| 626 | 6574 | 6644 | 6713 | 6782 | 6852 | 6921 | 6990 | 7060 | 7129 | 7198 | 69 |
| 627 | 7268 | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 | 69 |
| 628 | 7960 | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 | 69 |
| 629 | 8651 | 8720 | 8789 | 8858 | 8927 | 8996 | 9065 | 9134 | 9203 | 9272 | 69 |
| 630 | 799341 | 9409 | 9478 | 9547 | 9616 | 9685 | 9754 | 9823 | 9892 | 9961 | 69 |
| 631 | 800029 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442 | 0511 | 0580 | 0648 | 69 |
| 632 | 0717 | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 | 69 |
| 633 | 1404 | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 | 69 |
| 634 | 2089 | 2158 | 2226 | 2295 | 2363 | 2432 | 2500 | 2568 | 2637 | 2705 | 69 |
| 635 | 2774 | 2842 | 2910 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 | 68 |
| 636 | 3457 | 3525 | 3594 | 3662 | 3730 | 3798 | 3867 | 3935 | 4003 | 4071 | 68 |
| 637 | 4139 | 4208 | 4276 | 4344 | 4412 | 4480 | 4548 | 4616 | 4685 | 4753 | 68 |
| 638 | 4821 | 4889 | 4957 | 5025 | 5093 | 5161 | 5229 | 5297 | 5365 | 5433 | 68 |
| 639 | 5501 | 5569 | 5637 | 5705 | 5773 | 5841 | 5908 | 5976 | 6044 | 6112 | 68 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 640 | 806180 | 6248 | 6316 | 6384 | 6451 | 6519 | 6587 | 6655 | 6723 | 6790 | 68 |
| 641 | 6858 | 6926 | 6994 | 7061 | 7129 | 7197 | 7264 | 7332 | 7400 | 7467 | 68 |
| 642 | 7535 | 7603 | 7670 | 7738 | 7806 | 7873 | 7941 | 8008 | 8076 | 8143 | 68 |
| 643 | 8211 | 8279 | 8346 | 8414 | 8481 | 8549 | 8616 | 8684 | 8751 | 8818 | 67 |
| 644 | 8886 | 8953 | 9021 | 9088 | 9156 | 9223 | 9290 | 9358 | 9425 | 9492 | 67 |
| 645 | 9560 | 9627 | 9694 | 9762 | 9829 | 9896 | 9964 | ••31 | ••98 | •165 | 67 |
| 646 | 810233 | 0300 | 0367 | 0434 | 0501 | 0569 | 0636 | 0703 | 0770 | 0837 | 67 |
| 647 | 0904 | 0971 | 1039 | 1106 | 1173 | 1240 | 1307 | 1374 | 1441 | 1508 | 67 |
| 648 | 1575 | 1642 | 1709 | 1776 | 1843 | 1910 | 1977 | 2044 | 2111 | 2178 | 67 |
| 649 | 2245 | 2312 | 2379 | 2445 | 2512 | 2579 | 2646 | 2713 | 2780 | 2847 | 67 |
| 650 | 812913 | 2980 | 3047 | 3114 | 3181 | 3247 | 3314 | 3381 | 3448 | 3514 | 67 |
| 651 | 3581 | 3648 | 3714 | 3781 | 3848 | 3914 | 3981 | 4048 | 4114 | 4181 | 67 |
| 652 | 4248 | 4314 | 4381 | 4447 | 4514 | 4581 | 4647 | 4714 | 4780 | 4847 | 67 |
| 653 | 4913 | 4980 | 5046 | 5113 | 5179 | 5246 | 5312 | 5378 | 5445 | 5511 | 66 |
| 654 | 5578 | 5644 | 5711 | 5777 | 5843 | 5910 | 5976 | 6042 | 6109 | 6175 | 66 |
| 655 | 6241 | 6308 | 6374 | 6440 | 6506 | 6573 | 6639 | 6705 | 6771 | 6838 | 66 |
| 656 | 6904 | 6970 | 7036 | 7102 | 7169 | 7235 | 7301 | 7367 | 7433 | 7499 | 66 |
| 657 | 7565 | 7631 | 7698 | 7764 | 7830 | 7896 | 7962 | 8028 | 8094 | 8160 | 66 |
| 658 | 8226 | 8292 | 8358 | 8424 | 8490 | 8556 | 8622 | 8688 | 8754 | 8820 | 66 |
| 659 | 8885 | 8951 | 9017 | 9083 | 9149 | 9215 | 9281 | 9346 | 9412 | 9478 | 66 |
| 660 | 819544 | 9610 | 9676 | 9741 | 9807 | 9873 | 9939 | •••4 | ••70 | •136 | 66 |
| 661 | 820201 | 0267 | 0333 | 0399 | 0464 | 0530 | 0595 | 0661 | 0727 | 0792 | 66 |
| 662 | 0858 | 0924 | 0989 | 1055 | 1120 | 1186 | 1251 | 1317 | 1382 | 1448 | 66 |
| 663 | 1514 | 1579 | 1645 | 1710 | 1775 | 1841 | 1906 | 1972 | 2037 | 2103 | 65 |
| 664 | 2168 | 2233 | 2299 | 2364 | 2430 | 2495 | 2560 | 2626 | 2691 | 2756 | 65 |
| 665 | 2822 | 2887 | 2952 | 3018 | 3083 | 3148 | 3213 | 3279 | 3344 | 3409 | 65 |
| 666 | 3474 | 3539 | 3605 | 3670 | 3735 | 3800 | 3865 | 3930 | 3996 | 4061 | 65 |
| 667 | 4126 | 4191 | 4256 | 4321 | 4386 | 4451 | 4516 | 4581 | 4646 | 4711 | 65 |
| 668 | 4776 | 4841 | 4906 | 4971 | 5036 | 5101 | 5166 | 5231 | 5296 | 5361 | 65 |
| 669 | 5426 | 5491 | 5556 | 5621 | 5686 | 5751 | 5815 | 5880 | 5945 | 6010 | 65 |
| 670 | 826075 | 6140 | 6204 | 6269 | 6334 | 6399 | 6464 | 6528 | 6593 | 6658 | 65 |
| 671 | 6723 | 6787 | 6852 | 6917 | 6981 | 7046 | 7111 | 7175 | 7240 | 7305 | 65 |
| 672 | 7369 | 7434 | 7499 | 7563 | 7628 | 7692 | 7757 | 7821 | 7886 | 7951 | 65 |
| 673 | 8015 | 8080 | 8144 | 8209 | 8273 | 8338 | 8402 | 8467 | 8531 | 8595 | 64 |
| 674 | 8660 | 8724 | 8789 | 8853 | 8918 | 8982 | 9046 | 9111 | 9175 | 9239 | 64 |
| 675 | 9304 | 9368 | 9432 | 9497 | 9561 | 9625 | 9690 | 9754 | 9818 | 9882 | 64 |
| 676 | 9947 | ••11 | ••75 | •139 | •204 | •268 | •332 | •396 | •460 | •525 | 64 |
| 677 | 830589 | 0653 | 0717 | 0781 | 0845 | 0909 | 0973 | 1037 | 1102 | 1166 | 64 |
| 678 | 1230 | 1294 | 1358 | 1422 | 1486 | 1550 | 1614 | 1678 | 1742 | 1806 | 64 |
| 679 | 1870 | 1934 | 1998 | 2062 | 2126 | 2189 | 2253 | 2317 | 2381 | 2445 | 64 |
| 680 | 832509 | 2573 | 2637 | 2700 | 2764 | 2828 | 2892 | 2956 | 3020 | 3083 | 64 |
| 681 | 3147 | 3211 | 3275 | 3338 | 3402 | 3466 | 3530 | 3593 | 3657 | 3721 | 64 |
| 682 | 3784 | 3848 | 3912 | 3975 | 4039 | 4103 | 4166 | 4230 | 4294 | 4357 | 64 |
| 683 | 4421 | 4484 | 4548 | 4611 | 4675 | 4739 | 4802 | 4866 | 4929 | 4993 | 64 |
| 684 | 5056 | 5120 | 5183 | 5247 | 5310 | 5373 | 5437 | 5500 | 5564 | 5627 | 63 |
| 685 | 5691 | 5754 | 5817 | 5881 | 5944 | 6007 | 6071 | 6134 | 6197 | 6261 | 63 |
| 686 | 6324 | 6387 | 6451 | 6514 | 6577 | 6641 | 6704 | 6767 | 6830 | 6894 | 63 |
| 687 | 6957 | 7020 | 7083 | 7146 | 7210 | 7273 | 7336 | 7399 | 7462 | 7525 | 63 |
| 688 | 7588 | 7652 | 7715 | 7778 | 7841 | 7904 | 7967 | 8030 | 8093 | 8156 | 63 |
| 689 | 8219 | 8282 | 8345 | 8408 | 8471 | 8534 | 8597 | 8660 | 8723 | 8786 | 63 |
| 690 | 838849 | 8912 | 8975 | 9038 | 9101 | 9164 | 9227 | 9289 | 9352 | 9415 | 63 |
| 691 | 9478 | 9541 | 9604 | 9667 | 9729 | 9792 | 9855 | 9918 | 9981 | ••43 | 63 |
| 692 | 840106 | 0169 | 0232 | 0294 | 0357 | 0420 | 0482 | 0545 | 0608 | 0671 | 63 |
| 693 | 0733 | 0796 | 0859 | 0921 | 0984 | 1046 | 1109 | 1172 | 1234 | 1297 | 63 |
| 694 | 1359 | 1422 | 1485 | 1547 | 1610 | 1672 | 1735 | 1797 | 1860 | 1922 | 63 |
| 695 | 1985 | 2047 | 2110 | 2172 | 2235 | 2297 | 2360 | 2422 | 2484 | 2547 | 62 |
| 696 | 2609 | 2672 | 2734 | 2796 | 2859 | 2921 | 2983 | 3046 | 3108 | 3170 | 62 |
| 697 | 3233 | 3295 | 3357 | 3420 | 3482 | 3544 | 3606 | 3669 | 3731 | 3793 | 62 |
| 698 | 3855 | 3918 | 3980 | 4042 | 4104 | 4166 | 4229 | 4291 | 4353 | 4415 | 62 |
| 699 | 4477 | 4539 | 4601 | 4664 | 4726 | 4788 | 4850 | 4912 | 4974 | 5036 | 62 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 700 | 845098 | 5160 | 5222 | 5284 | 5346 | 5408 | 5470 | 5532 | 5594 | 5656 | 62 |
| 701 | 5718 | 5780 | 5842 | 5904 | 5966 | 6028 | 6090 | 6151 | 6213 | 6275 | 62 |
| 702 | 6337 | 6399 | 6461 | 6523 | 6585 | 6646 | 6708 | 6770 | 6832 | 6894 | 62 |
| 703 | 6955 | 7017 | 7079 | 7141 | 7202 | 7264 | 7326 | 7388 | 7449 | 7511 | 62 |
| 704 | 7573 | 7634 | 7696 | 7758 | 7819 | 7881 | 7943 | 8004 | 8066 | 8128 | 62 |
| 705 | 8189 | 8251 | 8312 | 8374 | 8435 | 8497 | 8559 | 8620 | 8682 | 8743 | 62 |
| 706 | 8805 | 8866 | 8928 | 8989 | 9051 | 9112 | 9174 | 9235 | 9297 | 9358 | 61 |
| 707 | 9419 | 9481 | 9542 | 9604 | 9665 | 9726 | 9788 | 9849 | 9911 | 9972 | 61 |
| 708 | 850033 | 0095 | 0156 | 0217 | 0279 | 0340 | 0401 | 0462 | 0524 | 0585 | 61 |
| 709 | 0646 | 0707 | 0769 | 0830 | 0891 | 0952 | 1014 | 1075 | 1136 | 1197 | 61 |
| 710 | 851258 | 1320 | 1381 | 1442 | 1503 | 1564 | 1625 | 1686 | 1747 | 1809 | 61 |
| 711 | 1870 | 1931 | 1992 | 2053 | 2114 | 2175 | 2236 | 2297 | 2358 | 2419 | 61 |
| 712 | 2480 | 2541 | 2602 | 2663 | 2724 | 2785 | 2846 | 2907 | 2968 | 3029 | 61 |
| 713 | 3090 | 3150 | 3211 | 3272 | 3333 | 3394 | 3455 | 3516 | 3577 | 3637 | 61 |
| 714 | 3698 | 3759 | 3820 | 3881 | 3941 | 4002 | 4063 | 4124 | 4185 | 4245 | 61 |
| 715 | 4306 | 4367 | 4428 | 4488 | 4549 | 4610 | 4670 | 4731 | 4792 | 4852 | 61 |
| 716 | 4913 | 4974 | 5034 | 5095 | 5156 | 5216 | 5277 | 5337 | 5398 | 5459 | 61 |
| 717 | 5519 | 5580 | 5640 | 5701 | 5761 | 5822 | 5882 | 5943 | 6003 | 6064 | 61 |
| 718 | 6124 | 6185 | 6245 | 6306 | 6366 | 6427 | 6487 | 6548 | 6608 | 6668 | 60 |
| 719 | 6729 | 6789 | 6850 | 6910 | 6970 | 7031 | 7091 | 7152 | 7212 | 7272 | 60 |
| 720 | 857332 | 7393 | 7453 | 7513 | 7574 | 7634 | 7694 | 7755 | 7815 | 7875 | 60 |
| 721 | 7935 | 7995 | 8056 | 8116 | 8176 | 8236 | 8297 | 8357 | 8417 | 8477 | 60 |
| 722 | 8537 | 8597 | 8657 | 8718 | 8778 | 8838 | 8898 | 8958 | 9018 | 9078 | 60 |
| 723 | 9138 | 9198 | 9258 | 9318 | 9379 | 9439 | 9499 | 9559 | 9619 | 9679 | 60 |
| 724 | 9739 | 9799 | 9859 | 9918 | 9978 | ••38 | ••98 | •158 | •218 | •278 | 60 |
| 725 | 860338 | 0398 | 0458 | 0518 | 0578 | 0637 | 0697 | 0757 | 0817 | 0877 | 60 |
| 726 | 0937 | 0996 | 1056 | 1116 | 1176 | 1236 | 1295 | 1355 | 1415 | 1475 | 60 |
| 727 | 1534 | 1594 | 1654 | 1714 | 1773 | 1833 | 1893 | 1952 | 2012 | 2072 | 60 |
| 728 | 2131 | 2191 | 2251 | 2310 | 2370 | 2430 | 2489 | 2549 | 2608 | 2668 | 60 |
| 729 | 2728 | 2787 | 2847 | 2906 | 2966 | 3025 | 3085 | 3144 | 3204 | 3263 | 60 |
| 730 | 803323 | 3382 | 3442 | 3501 | 3561 | 3620 | 3680 | 3739 | 3799 | 3858 | 59 |
| 731 | 3917 | 3977 | 4036 | 4096 | 4155 | 4214 | 4274 | 4333 | 4392 | 4452 | 59 |
| 732 | 4511 | 4570 | 4630 | 4689 | 4748 | 4808 | 4867 | 4926 | 4985 | 5045 | 59 |
| 733 | 5104 | 5163 | 5222 | 5282 | 5341 | 5400 | 5459 | 5519 | 5578 | 5637 | 59 |
| 734 | 5696 | 5755 | 5814 | 5874 | 5933 | 5992 | 6051 | 6110 | 6169 | 6228 | 59 |
| 735 | 6287 | 6346 | 6405 | 6465 | 6524 | 6583 | 6642 | 6701 | 6760 | 6819 | 59 |
| 736 | 6878 | 6937 | 6996 | 7055 | 7114 | 7173 | 7232 | 7291 | 7350 | 7409 | 59 |
| 737 | 7467 | 7526 | 7585 | 7644 | 7703 | 7762 | 7821 | 7880 | 7939 | 7998 | 59 |
| 738 | 8056 | 8115 | 8174 | 8233 | 8292 | 8350 | 8409 | 8468 | 8527 | 8586 | 59 |
| 739 | 8644 | 8703 | 8762 | 8821 | 8879 | 8938 | 8997 | 9056 | 9114 | 9173 | 59 |
| 740 | 869232 | 9290 | 9349 | 9408 | 9466 | 9525 | 9584 | 9642 | 9701 | 9760 | 59 |
| 741 | 9818 | 9877 | 9935 | 9994 | ••53 | •111 | •170 | •228 | •287 | •345 | 59 |
| 742 | 870404 | 0462 | 0521 | 0579 | 0638 | 0696 | 0755 | 0813 | 0872 | 0930 | 58 |
| 743 | 0989 | 1047 | 1106 | 1164 | 1223 | 1281 | 1339 | 1398 | 1456 | 1515 | 58 |
| 744 | 1573 | 1631 | 1690 | 1748 | 1806 | 1865 | 1923 | 1981 | 2040 | 2098 | 58 |
| 745 | 2156 | 2215 | 2273 | 2331 | 2389 | 2448 | 2506 | 2564 | 2622 | 2681 | 58 |
| 746 | 2739 | 2797 | 2855 | 2913 | 2972 | 3030 | 3088 | 3146 | 3204 | 3262 | 58 |
| 747 | 3321 | 3379 | 3437 | 3495 | 3553 | 3611 | 3669 | 2727 | 3785 | 3844 | 58 |
| 748 | 3902 | 3960 | 4018 | 4076 | 4134 | 4192 | 4250 | 4308 | 4366 | 4424 | 58 |
| 749 | 4482 | 4540 | 4598 | 4656 | 4714 | 4772 | 4830 | 4888 | 4945 | 5003 | 58 |
| 750 | 875061 | 5119 | 5177 | 5235 | 5293 | 5351 | 5409 | 5466 | 5524 | 5582 | 58 |
| 751 | 5640 | 5698 | 5756 | 5813 | 5871 | 5929 | 5987 | 6045 | 6102 | 6160 | 58 |
| 752 | 6218 | 6276 | 6333 | 6391 | 6449 | 6507 | 6564 | 6622 | 6680 | 6737 | 58 |
| 753 | 6795 | 6853 | 6910 | 6968 | 7026 | 7083 | 7141 | 7199 | 7256 | 7314 | 58 |
| 754 | 7371 | 7429 | 7487 | 7544 | 7602 | 7659 | 7717 | 7774 | 7832 | 7889 | 58 |
| 755 | 7947 | 8004 | 8062 | 8119 | 8177 | 8234 | 8292 | 8349 | 8407 | 8464 | 57 |
| 756 | 8522 | 8579 | 8637 | 8694 | 8752 | 8809 | 8866 | 8924 | 8981 | 9039 | 57 |
| 757 | 9096 | 9153 | 9211 | 9268 | 9325 | 9383 | 9440 | 9497 | 9555 | 9612 | 57 |
| 758 | 9669 | 9726 | 9784 | 9841 | 9898 | 9956 | ••13 | ••70 | •127 | •185 | 57 |
| 759 | 880242 | 0299 | 0356 | 0413 | 0471 | 0528 | 0585 | 0642 | 0699 | 0756 | 57 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 760 | 880814 | 0871 | 0928 | 0985 | 1042 | 1099 | 1156 | 1213 | 1271 | 1328 | 57 |
| 761 | 1385 | 1442 | 1499 | 1556 | 1613 | 1670 | 1727 | 1784 | 1841 | 1898 | 57 |
| 762 | 1955 | 2012 | 2069 | 2126 | 2183 | 2240 | 2297 | 2354 | 2411 | 2468 | 57 |
| 763 | 2525 | 2581 | 2638 | 2695 | 2752 | 2809 | 2866 | 2923 | 2980 | 3037 | 57 |
| 764 | 3093 | 3150 | 3207 | 3264 | 3321 | 3377 | 3434 | 3491 | 3548 | 3605 | 57 |
| 765 | 3661 | 3718 | 3775 | 3832 | 3888 | 3945 | 4002 | 4059 | 4115 | 4172 | 57 |
| 766 | 4229 | 4285 | 4342 | 4399 | 4455 | 4512 | 4569 | 4625 | 4682 | 4739 | 57 |
| 767 | 4795 | 4852 | 4909 | 4965 | 5022 | 5078 | 5135 | 5192 | 5248 | 5305 | 57 |
| 768 | 5361 | 5418 | 5474 | 5531 | 5587 | 5644 | 5700 | 5757 | 5813 | 5870 | 57 |
| 769 | 5926 | 5983 | 6039 | 6096 | 6152 | 6209 | 6265 | 6321 | 6378 | 6434 | 56 |
| 770 | 886491 | 6547 | 6604 | 6660 | 6716 | 6773 | 6829 | 6885 | 6942 | 6998 | 56 |
| 771 | 7054 | 7111 | 7167 | 7223 | 7280 | 7336 | 7392 | 7449 | 7505 | 7561 | 56 |
| 772 | 7617 | 7674 | 7730 | 7786 | 7842 | 7898 | 7955 | 8011 | 8067 | 8123 | 56 |
| 773 | 8179 | 8236 | 8292 | 8348 | 8404 | 8460 | 8516 | 8573 | 8629 | 8685 | 56 |
| 774 | 8741 | 8797 | 8853 | 8909 | 8965 | 9021 | 9077 | 9134 | 9190 | 9246 | 56 |
| 775 | 9302 | 9358 | 9414 | 9470 | 9526 | 9582 | 9638 | 9694 | 9750 | 9806 | 56 |
| 776 | 9862 | 9918 | 9974 | ●●30 | ●●86 | ●141 | ●197 | ●253 | ●309 | ●365 | 56 |
| 777 | 890421 | 0477 | 0533 | 0589 | 0645 | 0700 | 0756 | 0812 | 0868 | 0924 | 56 |
| 778 | 0980 | 1035 | 1091 | 1147 | 1203 | 1259 | 1314 | 1370 | 1426 | 1482 | 56 |
| 779 | 1537 | 1593 | 1649 | 1705 | 1760 | 1816 | 1872 | 1928 | 1983 | 2039 | 56 |
| 780 | 892095 | 2150 | 2206 | 2262 | 2317 | 2373 | 2429 | 2484 | 2540 | 2595 | 56 |
| 781 | 2651 | 2707 | 2762 | 2818 | 2873 | 2929 | 2985 | 3040 | 3096 | 3151 | 56 |
| 782 | 3207 | 3262 | 3318 | 3373 | 3429 | 3484 | 3540 | 3595 | 3651 | 3706 | 56 |
| 783 | 3762 | 3817 | 3873 | 3928 | 3984 | 4039 | 4094 | 4150 | 4205 | 4261 | 55 |
| 784 | 4316 | 4371 | 4427 | 4482 | 4538 | 4593 | 4648 | 4704 | 4759 | 4814 | 55 |
| 785 | 4870 | 4925 | 4980 | 5036 | 5091 | 5146 | 5201 | 5257 | 5312 | 5367 | 55 |
| 786 | 5423 | 5478 | 5533 | 5588 | 5644 | 5699 | 5754 | 5809 | 5864 | 5920 | 55 |
| 787 | 5975 | 6030 | 6085 | 6140 | 6195 | 6251 | 6306 | 6361 | 6416 | 6471 | 55 |
| 788 | 6526 | 6581 | 6636 | 6692 | 6747 | 6802 | 6857 | 6912 | 6967 | 7022 | 55 |
| 789 | 7077 | 7132 | 7187 | 7242 | 7297 | 7352 | 7407 | 7462 | 7517 | 7572 | 55 |
| 790 | 897627 | 7682 | 7737 | 7792 | 7847 | 7902 | 7957 | 8012 | 8067 | 8122 | 55 |
| 791 | 8176 | 8231 | 8286 | 8341 | 8396 | 8451 | 8506 | 8561 | 8615 | 8670 | 55 |
| 792 | 8725 | 8780 | 8835 | 8890 | 8944 | 8999 | 9054 | 9109 | 9164 | 9218 | 55 |
| 793 | 9273 | 9328 | 9383 | 9437 | 9492 | 9547 | 9602 | 9656 | 9711 | 9766 | 55 |
| 794 | 9821 | 9875 | 9930 | 9985 | ●●39 | ●●94 | ●149 | ●203 | ●258 | ●312 | 55 |
| 795 | 900367 | 0422 | 0476 | 0531 | 0586 | 0640 | 0695 | 0749 | 0804 | 0859 | 55 |
| 796 | 0913 | 0968 | 1022 | 1077 | 1131 | 1186 | 1240 | 1295 | 1349 | 1404 | 55 |
| 797 | 1458 | 1513 | 1567 | 1622 | 1676 | 1731 | 1785 | 1840 | 1894 | 1948 | 54 |
| 798 | 2003 | 2057 | 2112 | 2166 | 2221 | 2275 | 2320 | 2384 | 2438 | 2492 | 54 |
| 799 | 2547 | 2601 | 2655 | 2710 | 2764 | 2818 | 2873 | 2927 | 2981 | 3036 | 54 |
| 800 | 903090 | 3144 | 3199 | 3253 | 3307 | 3361 | 3416 | 3470 | 3524 | 3578 | 54 |
| 801 | 3633 | 3687 | 3741 | 3795 | 3849 | 3904 | 3958 | 4012 | 4066 | 4120 | 54 |
| 802 | 4174 | 4229 | 4283 | 4337 | 4391 | 4445 | 4499 | 4553 | 4607 | 4661 | 54 |
| 803 | 4716 | 4770 | 4824 | 4878 | 4932 | 4986 | 5040 | 5094 | 5148 | 5202 | 54 |
| 804 | 5256 | 5310 | 5364 | 5418 | 5472 | 5526 | 5580 | 5634 | 5688 | 5742 | 54 |
| 805 | 5796 | 5850 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 | 54 |
| 806 | 6335 | 6389 | 6443 | 6497 | 6551 | 6604 | 6658 | 6712 | 6766 | 6820 | 54 |
| 807 | 6874 | 6927 | 6981 | 7035 | 7089 | 7143 | 7196 | 7250 | 7304 | 7358 | 54 |
| 808 | 7411 | 7465 | 7519 | 7573 | 7626 | 7680 | 7734 | 7787 | 7841 | 7895 | 54 |
| 809 | 7949 | 8002 | 8056 | 8110 | 8163 | 8217 | 8270 | 8324 | 8378 | 8431 | 54 |
| 810 | 908485 | 8539 | 8592 | 8646 | 8699 | 8753 | 8807 | 8860 | 8914 | 8967 | 54 |
| 811 | 9021 | 9074 | 9128 | 9181 | 9235 | 9289 | 9342 | 9396 | 9449 | 9503 | 54 |
| 812 | 9556 | 9610 | 9663 | 9716 | 9770 | 9823 | 9877 | 9930 | 9984 | ●●37 | 53 |
| 813 | 910091 | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 | 53 |
| 814 | 0624 | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 | 53 |
| 815 | 1158 | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 1530 | 1584 | 1637 | 53 |
| 816 | 1690 | 1743 | 1797 | 1850 | 1903 | 1956 | 2009 | 2063 | 2116 | 2169 | 53 |
| 817 | 2222 | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2647 | 2700 | 53 |
| 818 | 2753 | 2806 | 2859 | 2913 | 2966 | 3019 | 3072 | 3125 | 3178 | 3231 | 53 |
| 819 | 3284 | 3337 | 3390 | 3443 | 3496 | 3549 | 3602 | 3655 | 3708 | 3761 | 53 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 820 | 913814 | 3867 | 3920 | 3973 | 4026 | 4079 | 4132 | 4184 | 4237 | 4290 | 53 |
| 821 | 4343 | 4396 | 4449 | 4502 | 4555 | 4608 | 4660 | 4713 | 4766 | 4819 | 53 |
| 822 | 4872 | 4925 | 4977 | 5030 | 5083 | 5136 | 5189 | 5241 | 5294 | 5347 | 53 |
| 823 | 5400 | 5453 | 5505 | 5558 | 5611 | 5664 | 5716 | 5769 | 5822 | 5875 | 53 |
| 824 | 5927 | 5980 | 6033 | 6085 | 6138 | 6191 | 6243 | 6296 | 6349 | 6401 | 53 |
| 825 | 6454 | 6507 | 6559 | 6612 | 6664 | 6717 | 6770 | 6822 | 6875 | 6927 | 53 |
| 826 | 6980 | 7033 | 7085 | 7138 | 7190 | 7243 | 7295 | 7348 | 7400 | 7453 | 53 |
| 827 | 7506 | 7558 | 7611 | 7663 | 7716 | 7768 | 7820 | 7873 | 7925 | 7978 | 52 |
| 828 | 8030 | 8083 | 8135 | 8188 | 8240 | 8293 | 8345 | 8397 | 8450 | 8502 | 52 |
| 829 | 8555 | 8607 | 8659 | 8712 | 8764 | 8816 | 8869 | 8921 | 8973 | 9026 | 52 |
| 830 | 919078 | 9130 | 9183 | 9235 | 9287 | 9340 | 9392 | 9444 | 9496 | 9549 | 52 |
| 831 | 9601 | 9653 | 9706 | 9758 | 9810 | 9862 | 9914 | 9967 | ••19 | ••71 | 52 |
| 832 | 920123 | 0176 | 0228 | 0280 | 0332 | 0384 | 0436 | 0489 | 0541 | 0593 | 52 |
| 833 | 0645 | 0697 | 0749 | 0801 | 0853 | 0906 | 0958 | 1010 | 1062 | 1114 | 52 |
| 834 | 1166 | 1218 | 1270 | 1322 | 1374 | 1426 | 1478 | 1530 | 1582 | 1634 | 52 |
| 835 | 1686 | 1738 | 1790 | 1842 | 1894 | 1946 | 1998 | 2050 | 2102 | 2154 | 52 |
| 836 | 2206 | 2258 | 2310 | 2362 | 2414 | 2466 | 2518 | 2570 | 2622 | 2674 | 52 |
| 837 | 2725 | 2777 | 2829 | 2881 | 2933 | 2985 | 3037 | 3089 | 3140 | 3192 | 52 |
| 838 | 3244 | 3296 | 3348 | 3399 | 3451 | 3503 | 3555 | 3607 | 3658 | 3710 | 52 |
| 839 | 3762 | 3814 | 3865 | 3917 | 3969 | 4021 | 4072 | 4124 | 4176 | 4228 | 52 |
| 840 | 924279 | 4331 | 4383 | 4434 | 4486 | 4538 | 4589 | 4641 | 4693 | 4744 | 52 |
| 841 | 4796 | 4848 | 4899 | 4951 | 5003 | 5054 | 5106 | 5157 | 5209 | 5261 | 52 |
| 842 | 5312 | 5364 | 5415 | 5467 | 5518 | 5570 | 5621 | 5673 | 5725 | 5776 | 52 |
| 843 | 5828 | 5879 | 5931 | 5982 | 6034 | 6085 | 6137 | 6188 | 6240 | 6291 | 51 |
| 844 | 6342 | 6394 | 6445 | 6497 | 6548 | 6600 | 6651 | 6702 | 6754 | 6805 | 51 |
| 845 | 6857 | 6908 | 6959 | 7011 | 7062 | 7114 | 7165 | 7216 | 7268 | 7319 | 51 |
| 846 | 7370 | 7422 | 7473 | 7524 | 7576 | 7627 | 7678 | 7730 | 7781 | 7832 | 51 |
| 847 | 7883 | 7935 | 7986 | 8037 | 8088 | 8140 | 9191 | 8242 | 8293 | 8345 | 51 |
| 848 | 8396 | 8447 | 8498 | 8549 | 8601 | 8652 | 8703 | 8754 | 8805 | 8857 | 51 |
| 849 | 8908 | 8959 | 9010 | 9061 | 9112 | 9163 | 9215 | 9266 | 9317 | 9368 | 51 |
| 850 | 929419 | 9470 | 9521 | 9572 | 9623 | 9674 | 9725 | 9776 | 9827 | 9879 | 51 |
| 851 | 9930 | 9981 | ••32 | ••83 | •134 | •185 | •236 | •287 | •338 | •389 | 51 |
| 852 | 930440 | 0491 | 0542 | 0592 | 0643 | 0694 | 0745 | 0796 | 0847 | 0898 | 51 |
| 853 | 0949 | 1000 | 1051 | 1102 | 1153 | 1204 | 1254 | 1305 | 1356 | 1407 | 51 |
| 854 | 1458 | 1509 | 1560 | 1610 | 1661 | 1712 | 1763 | 1814 | 1865 | 1915 | 51 |
| 855 | 1966 | 2017 | 2068 | 2118 | 2169 | 2220 | 2271 | 2322 | 2372 | 2423 | 51 |
| 856 | 2474 | 2524 | 2575 | 2626 | 2677 | 2727 | 2778 | 2829 | 2879 | 2930 | 51 |
| 857 | 2981 | 3031 | 3082 | 3133 | 3183 | 3234 | 3285 | 3335 | 3386 | 3437 | 51 |
| 858 | 3487 | 3538 | 3589 | 3639 | 3690 | 3740 | 3791 | 3841 | 3892 | 3943 | 51 |
| 859 | 3993 | 4044 | 4094 | 4145 | 4195 | 4246 | 4296 | 4347 | 4397 | 4448 | 51 |
| 860 | 934498 | 4549 | 4599 | 4650 | 4700 | 4751 | 4801 | 4852 | 4902 | 4953 | 50 |
| 861 | 5003 | 5054 | 5104 | 5154 | 5205 | 5255 | 5306 | 5356 | 5406 | 5457 | 50 |
| 862 | 5507 | 5558 | 5608 | 5658 | 5709 | 5759 | 5809 | 5860 | 5910 | 5960 | 50 |
| 863 | 6011 | 6061 | 6111 | 6162 | 6212 | 6262 | 6313 | 6363 | 6413 | 6463 | 50 |
| 864 | 6514 | 6564 | 6614 | 6665 | 6715 | 6765 | 6815 | 6865 | 6916 | 6966 | 50 |
| 865 | 7016 | 7066 | 7117 | 7167 | 7217 | 7267 | 7317 | 7367 | 7418 | 7468 | 50 |
| 866 | 7518 | 7568 | 7618 | 7668 | 7718 | 7769 | 7819 | 7869 | 7919 | 7969 | 50 |
| 867 | 8019 | 8069 | 8119 | 8169 | 8219 | 8269 | 8320 | 8370 | 8420 | 8470 | 50 |
| 868 | 8520 | 8570 | 8620 | 8670 | 8720 | 8770 | 8820 | 8870 | 8920 | 8970 | 50 |
| 869 | 9020 | 9070 | 9120 | 9170 | 9220 | 9270 | 9320 | 9369 | 9419 | 9469 | 50 |
| 870 | 939519 | 9569 | 9619 | 9669 | 9719 | 9769 | 9819 | 9869 | 9918 | 9968 | 50 |
| 871 | 940018 | 0068 | 0118 | 0168 | 0218 | 0267 | 0317 | 0367 | 0417 | 0467 | 50 |
| 872 | 0516 | 0566 | 0616 | 0666 | 0716 | 0765 | 0815 | 0865 | 0915 | 0964 | 50 |
| 873 | 1014 | 1064 | 1114 | 1163 | 1213 | 1263 | 1313 | 1362 | 1412 | 1462 | 50 |
| 874 | 1511 | 1561 | 1611 | 1660 | 1710 | 1760 | 1809 | 1859 | 1909 | 1958 | 50 |
| 875 | 2008 | 2058 | 2107 | 2157 | 2207 | 2256 | 2306 | 2355 | 2405 | 2455 | 50 |
| 876 | 2504 | 2554 | 2603 | 2653 | 2702 | 2752 | 2801 | 2851 | 2901 | 2950 | 50 |
| 877 | 3000 | 3049 | 3099 | 3148 | 3198 | 3247 | 3297 | 3346 | 3396 | 3445 | 59 |
| 878 | 3495 | 3544 | 3593 | 3643 | 3692 | 3742 | 3791 | 3841 | 3890 | 3939 | 59 |
| 879 | 3989 | 4038 | 4088 | 4137 | 4186 | 4236 | 4285 | 4335 | 4384 | 4433 | 59 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 880 | 944483 | 4532 | 4581 | 4631 | 4680 | 4729 | 4779 | 4828 | 4877 | 4927 | 49 |
| 881 | 4976 | 5025 | 5074 | 5124 | 5173 | 5222 | 5272 | 5321 | 5370 | 5419 | 49 |
| 882 | 5469 | 5518 | 5567 | 5616 | 5665 | 5715 | 5764 | 5813 | 5862 | 5912 | 49 |
| 883 | 5961 | 6010 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | 49 |
| 884 | 6452 | 6501 | 6551 | 6600 | 6649 | 6698 | 6747 | 6796 | 6845 | 6894 | 49 |
| 885 | 6943 | 6992 | 7041 | 7090 | 7140 | 7189 | 7238 | 7287 | 7336 | 7385 | 49 |
| 886 | 7434 | 7483 | 7532 | 7581 | 7630 | 7679 | 7728 | 7777 | 7826 | 7875 | 49 |
| 887 | 7924 | 7973 | 8022 | 8070 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | 49 |
| 888 | 8413 | 8462 | 8511 | 8560 | 8609 | 8657 | 8706 | 8755 | 8804 | 8853 | 49 |
| 889 | 8902 | 8951 | 8999 | 9048 | 9097 | 9146 | 9195 | 9244 | 9292 | 9341 | 49 |
| 890 | 949390 | 9439 | 9488 | 9536 | 9585 | 9634 | 9683 | 9731 | 9780 | 9829 | 49 |
| 891 | 9878 | 9926 | 9975 | ●●24 | ●●73 | ●121 | ●170 | ●219 | ●267 | ●316 | 49 |
| 892 | 950365 | 0414 | 0462 | 0511 | 0560 | 0608 | 0657 | 0706 | 0754 | 0803 | 49 |
| 893 | 0851 | 0900 | 0949 | 0997 | 1046 | 1095 | 1143 | 1192 | 1240 | 1289 | 49 |
| 894 | 1338 | 1386 | 1435 | 1483 | 1532 | 1580 | 1629 | 1677 | 1726 | 1775 | 49 |
| 895 | 1823 | 1872 | 1920 | 1969 | 2017 | 2066 | 2114 | 2163 | 2211 | 2260 | 48 |
| 896 | 2308 | 2356 | 2405 | 2453 | 2502 | 2550 | 2599 | 2647 | 2696 | 2744 | 48 |
| 897 | 2792 | 2841 | 2889 | 2938 | 2986 | 3034 | 3083 | 3131 | 3180 | 3228 | 48 |
| 898 | 3276 | 3325 | 3373 | 3421 | 3470 | 3518 | 3566 | 3615 | 3663 | 3711 | 48 |
| 899 | 3760 | 3808 | 3856 | 3905 | 3953 | 4001 | 4049 | 4098 | 4146 | 4194 | 48 |
| 900 | 954243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 4580 | 4628 | 4677 | 48 |
| 901 | 4725 | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 5110 | 5158 | 48 |
| 902 | 5207 | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 5640 | 48 |
| 903 | 5688 | 5736 | 5784 | 5832 | 5880 | 5928 | 5976 | 6024 | 6072 | 6120 | 48 |
| 904 | 6168 | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 | 48 |
| 905 | 6649 | 6697 | 6745 | 6793 | 6840 | 6888 | 6936 | 6984 | 7032 | 7080 | 48 |
| 906 | 7128 | 7176 | 7224 | 7272 | 7320 | 7368 | 7416 | 7464 | 7512 | 7559 | 48 |
| 907 | 7607 | 7655 | 7703 | 7751 | 7799 | 7847 | 7894 | 7942 | 7990 | 8038 | 48 |
| 908 | 8086 | 8134 | 8181 | 8229 | 8277 | 8325 | 8373 | 8421 | 8468 | 8516 | 48 |
| 909 | 8564 | 8612 | 8659 | 8707 | 8755 | 8803 | 8850 | 8898 | 8946 | 8994 | 48 |
| 910 | 959041 | 9089 | 9137 | 9185 | 9232 | 9280 | 9328 | 9375 | 9423 | 9471 | 48 |
| 911 | 9518 | 9566 | 9614 | 9661 | 9709 | 9757 | 9804 | 9852 | 9900 | 9947 | 48 |
| 912 | 9995 | ●●42 | ●●90 | ●138 | ●185 | ●233 | ●280 | ●328 | ●376 | ●423 | 48 |
| 913 | 960471 | 0518 | 0566 | 0613 | 0661 | 0709 | 0756 | 0804 | 0851 | 0899 | 48 |
| 914 | 0946 | 0994 | 1041 | 1089 | 1136 | 1184 | 1231 | 1279 | 1326 | 1374 | 47 |
| 915 | 1421 | 1469 | 1516 | 1563 | 1611 | 1658 | 1706 | 1753 | 1801 | 1848 | 47 |
| 916 | 1895 | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 | 47 |
| 917 | 2369 | 2417 | 2464 | 2511 | 2559 | 2606 | 2653 | 2701 | 2748 | 2795 | 47 |
| 918 | 2843 | 2890 | 2937 | 2985 | 3032 | 3079 | 3126 | 3174 | 3221 | 3268 | 47 |
| 919 | 3316 | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 | 47 |
| 920 | 963788 | 3835 | 3882 | 3929 | 3977 | 4024 | 4071 | 4118 | 4165 | 4212 | 47 |
| 921 | 4260 | 4307 | 4354 | 4401 | 4448 | 4495 | 4542 | 4590 | 4637 | 4684 | 47 |
| 922 | 4731 | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 | 47 |
| 923 | 5202 | 5249 | 5296 | 5343 | 5390 | 5437 | 5484 | 5531 | 5578 | 5625 | 47 |
| 924 | 5672 | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 | 47 |
| 925 | 6142 | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 | 47 |
| 926 | 6611 | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 | 47 |
| 927 | 7080 | 7127 | 7173 | 7220 | 7267 | 7314 | 7361 | 7408 | 7454 | 7501 | 47 |
| 928 | 7548 | 7595 | 7642 | 7688 | 7735 | 7782 | 7829 | 7875 | 7922 | 7969 | 47 |
| 929 | 8016 | 8062 | 8109 | 8156 | 8203 | 8249 | 8296 | 8343 | 8390 | 8436 | 47 |
| 930 | 968483 | 8530 | 8576 | 8623 | 8670 | 8716 | 8763 | 8810 | 8856 | 8903 | 47 |
| 931 | 8950 | 8996 | 9043 | 9090 | 9136 | 9183 | 9229 | 9276 | 9323 | 9369 | 47 |
| 932 | 9416 | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 | 47 |
| 933 | 9882 | 9928 | 9975 | ●●21 | ●●68 | ●114 | ●161 | ●207 | ●254 | ●300 | 47 |
| 934 | 970347 | 0393 | 0440 | 0486 | 0533 | 0579 | 0626 | 0672 | 0719 | 0765 | 46 |
| 935 | 0812 | 0858 | 0904 | 0951 | 0997 | 1044 | 1090 | 1137 | 1183 | 1229 | 46 |
| 936 | 1276 | 1322 | 1369 | 1415 | 1461 | 1508 | 1554 | 1601 | 1647 | 1693 | 46 |
| 937 | 1740 | 1786 | 1832 | 1879 | 1925 | 1971 | 2018 | 2064 | 2110 | 2157 | 46 |
| 938 | 2203 | 2249 | 2295 | 2342 | 2388 | 2434 | 2481 | 2527 | 2573 | 2619 | 46 |
| 939 | 2666 | 2712 | 2758 | 2804 | 2851 | 2897 | 2943 | 2989 | 3035 | 3082 | 46 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 940 | 973128 | 3174 | 3220 | 3266 | 3313 | 3359 | 3405 | 3451 | 3497 | 3543 | 46 |
| 941 | 3590 | 3636 | 3682 | 3728 | 3774 | 3820 | 3866 | 3913 | 3959 | 4005 | 46 |
| 942 | 4051 | 4097 | 4143 | 4189 | 4235 | 4281 | 4327 | 4374 | 4420 | 4466 | 46 |
| 943 | 4512 | 4558 | 4604 | 4650 | 4696 | 4742 | 4788 | 4834 | 4880 | 4926 | 46 |
| 944 | 4972 | 5018 | 5064 | 5110 | 5156 | 5202 | 5248 | 5294 | 5340 | 5386 | 46 |
| 945 | 5432 | 5478 | 5524 | 5570 | 5616 | 5662 | 5707 | 5753 | 5799 | 5845 | 46 |
| 946 | 5891 | 5937 | 5983 | 6029 | 6075 | 6121 | 6167 | 6212 | 6258 | 6304 | 46 |
| 947 | 6350 | 6396 | 6442 | 6488 | 6533 | 6579 | 6625 | 6671 | 6717 | 6763 | 46 |
| 948 | 6808 | 6854 | 6900 | 6946 | 6992 | 7037 | 7083 | 7129 | 7175 | 7220 | 46 |
| 949 | 7266 | 7312 | 7358 | 7403 | 7449 | 7495 | 7541 | 7586 | 7632 | 7678 | 46 |
| 950 | 977724 | 7769 | 7815 | 7861 | 7906 | 7952 | 7998 | 8043 | 8089 | 8135 | 46 |
| 951 | 8181 | 8226 | 8272 | 8317 | 8363 | 8409 | 8454 | 8500 | 8546 | 8591 | 46 |
| 952 | 8637 | 8683 | 8728 | 8774 | 8819 | 8865 | 8911 | 8956 | 9002 | 9047 | 46 |
| 953 | 9093 | 9138 | 9184 | 9230 | 9275 | 9321 | 9366 | 9412 | 9457 | 9503 | 46 |
| 954 | 9548 | 9594 | 9639 | 9685 | 9730 | 9776 | 9821 | 9867 | 9912 | 9958 | 46 |
| 955 | 980003 | 0049 | 0094 | 0140 | 0185 | 0231 | 0276 | 0322 | 0367 | 0412 | 45 |
| 956 | 0458 | 0503 | 0549 | 0594 | 0640 | 0685 | 0730 | 0776 | 0821 | 0867 | 45 |
| 957 | 0912 | 0957 | 1003 | 1048 | 1093 | 1139 | 1184 | 1229 | 1275 | 1320 | 45 |
| 958 | 1366 | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 | 45 |
| 959 | 1819 | 1864 | 1909 | 1954 | 2000 | 2045 | 2090 | 2135 | 2181 | 2226 | 45 |
| 960 | 982271 | 2316 | 2362 | 2407 | 2452 | 2497 | 2543 | 2588 | 2633 | 2678 | 45 |
| 961 | 2723 | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 3040 | 3085 | 3130 | 45 |
| 962 | 3175 | 3220 | 3265 | 3310 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 | 45 |
| 963 | 3626 | 3671 | 3716 | 3762 | 3807 | 3852 | 3897 | 3942 | 3987 | 4032 | 45 |
| 964 | 4077 | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 | 45 |
| 965 | 4527 | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 | 45 |
| 966 | 4977 | 5022 | 5067 | 5112 | 5157 | 5202 | 5247 | 5292 | 5337 | 5382 | 45 |
| 967 | 5426 | 5471 | 5516 | 5561 | 5606 | 5651 | 5696 | 5741 | 5786 | 5830 | 45 |
| 968 | 5875 | 5920 | 5965 | 6010 | 6055 | 6100 | 6144 | 6189 | 6234 | 6279 | 45 |
| 969 | 6324 | 6369 | 6413 | 6458 | 6503 | 6548 | 6593 | 6637 | 6682 | 6727 | 45 |
| 970 | 986772 | 6817 | 6861 | 6906 | 6951 | 6996 | 7040 | 7085 | 7130 | 7175 | 45 |
| 971 | 7219 | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 | 45 |
| 972 | 7666 | 7711 | 7756 | 7800 | 7845 | 7890 | 7934 | 7979 | 8024 | 8068 | 45 |
| 973 | 8113 | 8157 | 8202 | 8247 | 8291 | 8336 | 8381 | 8425 | 8470 | 8514 | 45 |
| 974 | 8559 | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 8960 | 45 |
| 975 | 9005 | 9049 | 9094 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 | 45 |
| 976 | 9450 | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 9850 | 44 |
| 977 | 9895 | 9939 | 9983 | ●●28 | ●●72 | ●117 | ●161 | ●206 | ●250 | ●294 | 44 |
| 978 | 990339 | 0383 | 0428 | 0472 | 0516 | 0561 | 0605 | 0650 | 0694 | 0738 | 44 |
| 979 | 0783 | 0827 | 0871 | 0916 | 0960 | 1004 | 1049 | 1093 | 1137 | 1182 | 44 |
| 980 | 991226 | 1270 | 1315 | 1359 | 1403 | 1448 | 1492 | 1536 | 1580 | 1625 | 44 |
| 981 | 1669 | 1713 | 1758 | 1802 | 1846 | 1890 | 1935 | 1979 | 2023 | 2067 | 44 |
| 982 | 2111 | 2156 | 2200 | 2244 | 2288 | 2333 | 2377 | 2421 | 2465 | 2509 | 44 |
| 983 | 2554 | 2598 | 2642 | 2686 | 2730 | 2774 | 2819 | 2863 | 2907 | 2951 | 44 |
| 984 | 2995 | 3039 | 3083 | 3127 | 3172 | 3216 | 3260 | 3304 | 3348 | 3392 | 44 |
| 985 | 3436 | 3480 | 3524 | 3568 | 3613 | 3657 | 3701 | 3745 | 3789 | 3833 | 44 |
| 986 | 3877 | 3921 | 3965 | 4009 | 4053 | 4097 | 4141 | 4185 | 4229 | 4273 | 44 |
| 987 | 4317 | 4361 | 4405 | 4449 | 4493 | 4537 | 4581 | 4625 | 4669 | 4713 | 44 |
| 988 | 4757 | 4801 | 4845 | 4889 | 4933 | 4977 | 5021 | 5065 | 5108 | 5152 | 44 |
| 989 | 5196 | 5240 | 5284 | 5328 | 5372 | 5416 | 5460 | 5504 | 5547 | 5591 | 44 |
| 990 | 995635 | 5679 | 5723 | 5767 | 5811 | 5854 | 5898 | 5942 | 5986 | 6030 | 44 |
| 991 | 6074 | 6117 | 6161 | 6205 | 6249 | 6293 | 6337 | 6380 | 6424 | 6468 | 44 |
| 992 | 6512 | 6555 | 6599 | 6643 | 6687 | 6731 | 6774 | 6818 | 6862 | 6906 | 44 |
| 993 | 6949 | 6993 | 7037 | 7080 | 7124 | 7168 | 7212 | 7255 | 7299 | 7343 | 44 |
| 994 | 7386 | 7430 | 7474 | 7517 | 7561 | 7605 | 7648 | 7692 | 7736 | 7779 | 44 |
| 995 | 7823 | 7867 | 7910 | 7954 | 7998 | 8041 | 8085 | 8129 | 8172 | 8216 | 44 |
| 996 | 8259 | 8303 | 8347 | 8390 | 8434 | 8477 | 8521 | 8564 | 8608 | 8652 | 44 |
| 997 | 8695 | 8739 | 8782 | 8826 | 8869 | 8913 | 8956 | 9000 | 9043 | 9087 | 44 |
| 998 | 9131 | 9174 | 9218 | 9261 | 9305 | 9348 | 9392 | 9435 | 9479 | 9522 | 44 |
| 999 | 9565 | 9609 | 9652 | 9696 | 9739 | 9783 | 9826 | 9870 | 9913 | 9957 | 43 |
| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |

# A TABLE

OF

# LOGARITHMIC

# SINES AND TANGENTS

FOR EVERY

# DEGREE AND MINUTE

OF THE QUADRANT.

---

REMARK. The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0·000000 | | 10·000000 | | 0·000000 | | Infinite. | 60 |
| 1 | 6·463726 | 5017·17 | 000000 | ·00 | 6·463726 | 5017·17 | 13·536274 | 59 |
| 2 | 764756 | 2934·85 | 000000 | ·00 | 764756 | 2934·83 | 235244 | 58 |
| 3 | 940847 | 2082·31 | 000000 | ·00 | 940847 | 2082·31 | 059153 | 57 |
| 4 | 7·065786 | 1615·17 | 000000 | ·00 | 7·065786 | 1615·17 | 12·934214 | 56 |
| 5 | 162696 | 1319·68 | 000000 | ·00 | 162696 | 1319·69 | 837304 | 55 |
| 6 | 241877 | 1115·75 | 9·999999 | ·01 | 241878 | 1115·78 | 758122 | 54 |
| 7 | 308824 | 966·53 | 999999 | ·01 | 308825 | 996·53 | 691175 | 53 |
| 8 | 366816 | 852·54 | 999999 | ·01 | 366817 | 852·54 | 633183 | 52 |
| 9 | 417968 | 762·63 | 999999 | ·01 | 417970 | 762·63 | 582030 | 51 |
| 10 | 463725 | 689·88 | 999998 | ·01 | 463727 | 689·88 | 536273 | 50 |
| 11 | 7·505118 | 629·81 | 9·999998 | ·01 | 7·505120 | 629·81 | 12·494880 | 49 |
| 12 | 542906 | 579·36 | 999997 | ·01 | 542909 | 579·33 | 457091 | 48 |
| 13 | 577668 | 536·41 | 999997 | ·01 | 577672 | 536·42 | 422328 | 47 |
| 14 | 609853 | 499·38 | 999996 | ·01 | 609857 | 499·39 | 390143 | 46 |
| 15 | 639816 | 467·14 | 999996 | ·01 | 639820 | 467·15 | 360180 | 45 |
| 16 | 667845 | 438·81 | 999995 | ·01 | 667849 | 438·82 | 332151 | 44 |
| 17 | 694173 | 413·72 | 999995 | ·01 | 694179 | 413·73 | 305821 | 43 |
| 18 | 718997 | 391·35 | 999994 | ·01 | 719004 | 391·36 | 280997 | 42 |
| 19 | 742477 | 371·27 | 999993 | ·01 | 742484 | 371·28 | 257516 | 41 |
| 20 | 764754 | 353·15 | 999993 | ·01 | 764761 | 351·36 | 235239 | 40 |
| 21 | 7·785943 | 336·72 | 9·999992 | ·01 | 7·785951 | 336·73 | 12·214049 | 39 |
| 22 | 806146 | 321·75 | 999991 | ·01 | 806155 | 321·76 | 193845 | 38 |
| 23 | 825451 | 308·05 | 999990 | ·01 | 825460 | 308·06 | 174540 | 37 |
| 24 | 843934 | 295·47 | 999989 | ·02 | 843944 | 295·49 | 156056 | 36 |
| 25 | 861662 | 283·88 | 999988 | ·02 | 861674 | 283·90 | 138326 | 35 |
| 26 | 878695 | 273·17 | 999988 | ·02 | 878708 | 273·18 | 121292 | 34 |
| 27 | 895085 | 263·23 | 999987 | ·02 | 895099 | 263·25 | 104901 | 33 |
| 28 | 910879 | 253·99 | 999986 | ·02 | 910894 | 254·01 | 089106 | 32 |
| 29 | 926119 | 245·38 | 999985 | ·02 | 926134 | 245·40 | 073866 | 31 |
| 30 | 940842 | 237·33 | 999983 | ·02 | 940858 | 237·35 | 059142 | 30 |
| 31 | 7·955082 | 229·80 | 9·999982 | ·02 | 7·955100 | 229·81 | 12·044900 | 29 |
| 32 | 968870 | 222·73 | 999981 | ·02 | 968889 | 222·75 | 031111 | 28 |
| 33 | 982233 | 216·08 | 999980 | ·02 | 982253 | 216·10 | 017747 | 27 |
| 34 | 995198 | 209·81 | 999979 | ·02 | 995219 | 209·83 | 004781 | 26 |
| 35 | 8·007787 | 203·90 | 999977 | ·02 | 8·007809 | 203·92 | 11·992191 | 25 |
| 36 | 020021 | 198·31 | 999976 | ·02 | 020045 | 198·33 | 979955 | 24 |
| 37 | 031919 | 193·02 | 999975 | ·02 | 031945 | 193·05 | 968055 | 23 |
| 38 | 043501 | 188·01 | 999973 | ·02 | 043527 | 188·03 | 956473 | 22 |
| 39 | 054781 | 183·25 | 999972 | ·02 | 054809 | 183·27 | 945191 | 21 |
| 40 | 065776 | 178·72 | 999971 | ·02 | 065806 | 178·74 | 934194 | 20 |
| 41 | 8·076500 | 174·41 | 9·999969 | ·02 | 8·076531 | 174·44 | 11·923469 | 19 |
| 42 | 086965 | 170·31 | 999968 | ·02 | 086997 | 170·34 | 913003 | 18 |
| 43 | 097183 | 166·39 | 999966 | ·02 | 097217 | 166·42 | 902783 | 17 |
| 44 | 107167 | 162·65 | 999964 | ·03 | 107202 | 162·68 | 892797 | 16 |
| 45 | 116926 | 159·08 | 999963 | ·03 | 116963 | 159·10 | 883037 | 15 |
| 46 | 126471 | 155·66 | 999961 | ·03 | 126510 | 155·68 | 873490 | 14 |
| 47 | 135810 | 152·38 | 999959 | ·03 | 135851 | 152·41 | 864149 | 13 |
| 48 | 144953 | 149·24 | 999958 | ·03 | 144996 | 149·27 | 855004 | 12 |
| 49 | 153907 | 146·22 | 999956 | ·03 | 153952 | 146·27 | 846048 | 11 |
| 50 | 162681 | 143·33 | 999954 | ·03 | 162727 | 143 36 | 837273 | 10 |
| 51 | 8·171280 | 140·54 | 9·999952 | ·03 | 8·171328 | 140 57 | 11·828672 | 9 |
| 52 | 179713 | 137·86 | 999950 | ·03 | 179763 | 137·90 | 820237 | 8 |
| 53 | 187985 | 135·29 | 999948 | ·03 | 188036 | 135·32 | 811964 | 7 |
| 54 | 196102 | 132·80 | 999946 | ·03 | 196156 | 132·84 | 803844 | 6 |
| 55 | 204070 | 130·41 | 999944 | ·03 | 204126 | 130·44 | 795874 | 5 |
| 56 | 211895 | 128·10 | 999942 | ·04 | 211953 | 128·14 | 788047 | 4 |
| 57 | 219581 | 125·87 | 999940 | ·04 | 219641 | 125·90 | 780359 | 3 |
| 58 | 227134 | 123·72 | 999938 | ·04 | 227195 | 123·76 | 772805 | 2 |
| 59 | 234557 | 121·64 | 999936 | ·04 | 234621 | 121·68 | 765379 | 1 |
| 60 | 241855 | 119·63 | 999934 | ·04 | 241921 | 119·67 | 758079 | 0 |
| | Cosine | D. | Sine | 89° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 8·241855 | 119·63 | 9·999934 | ·04 | 8·241921 | 119·67 | 11·758079 | 60 |
| 1 | 249033 | 117·68 | 999932 | ·04 | 249102 | 117·72 | 750898 | 59 |
| 2 | 256094 | 115·80 | 999929 | ·04 | 256165 | 115·84 | 743835 | 58 |
| 3 | 263042 | 113·98 | 999927 | ·04 | 263115 | 114·02 | 736885 | 57 |
| 4 | 269881 | 112·21 | 999925 | ·04 | 269956 | 112·25 | 730044 | 56 |
| 5 | 276614 | 110·50 | 999922 | ·04 | 276691 | 110·54 | 723309 | 55 |
| 6 | 283243 | 108·83 | 999920 | ·04 | 283323 | 108·87 | 716677 | 54 |
| 7 | 289773 | 107·21 | 999918 | ·04 | 289856 | 107·26 | 710144 | 53 |
| 8 | 296207 | 105·65 | 999915 | ·04 | 296292 | 105·70 | 703708 | 52 |
| 9 | 302546 | 104·13 | 999913 | ·04 | 302634 | 104·18 | 697366 | 51 |
| 10 | 308794 | 102·66 | 999910 | ·04 | 308884 | 102·70 | 691116 | 50 |
| 11 | 8·314904 | 101·22 | 9·999907 | ·04 | 8·315046 | 101·26 | 11·684954 | 49 |
| 12 | 321027 | 99·82 | 999905 | ·04 | 321122 | 99·87 | 678878 | 48 |
| 13 | 327016 | 98·47 | 999902 | ·04 | 327114 | 98·51 | 672886 | 47 |
| 14 | 332924 | 97·14 | 999899 | ·05 | 333025 | 97·19 | 666975 | 46 |
| 15 | 338753 | 95·86 | 999897 | ·05 | 338856 | 95·90 | 661144 | 45 |
| 16 | 344504 | 94·60 | 999894 | ·05 | 344610 | 94·65 | 655390 | 44 |
| 17 | 350181 | 93·38 | 999891 | ·05 | 350289 | 93·43 | 649711 | 43 |
| 18 | 355783 | 92·19 | 999888 | ·05 | 355895 | 92·24 | 644105 | 42 |
| 19 | 361315 | 91·03 | 999885 | ·05 | 361430 | 91·08 | 638570 | 41 |
| 20 | 366777 | 89·90 | 999882 | ·05 | 366895 | 89·95 | 633105 | 40 |
| 21 | 8·372171 | 88·80 | 9·999879 | ·05 | 8·372292 | 88·85 | 11·627708 | 39 |
| 22 | 377499 | 87·72 | 999876 | ·05 | 377622 | 87·77 | 622378 | 38 |
| 23 | 382762 | 86·67 | 999873 | ·05 | 382889 | 86·72 | 617111 | 37 |
| 24 | 387962 | 85·64 | 999870 | ·05 | 388092 | 85·70 | 611908 | 36 |
| 25 | 393101 | 84·64 | 999867 | ·05 | 393234 | 84·70 | 606766 | 35 |
| 26 | 398179 | 83·66 | 999864 | ·05 | 398315 | 83·71 | 601685 | 34 |
| 27 | 403199 | 82·71 | 999861 | ·05 | 403338 | 82·76 | 596662 | 33 |
| 28 | 408161 | 81·77 | 999858 | ·05 | 408304 | 81·82 | 591696 | 32 |
| 29 | 413068 | 80·86 | 999854 | ·05 | 413213 | 80·91 | 586787 | 31 |
| 30 | 417919 | 79·96 | 999851 | ·06 | 418068 | 80·02 | 581932 | 30 |
| 31 | 8·422717 | 79·09 | 9·999848 | ·06 | 8·422869 | 79·14 | 11·577131 | 29 |
| 32 | 427462 | 78·23 | 999844 | ·06 | 427618 | 78·30 | 572382 | 28 |
| 33 | 432156 | 77·40 | 999841 | ·06 | 432315 | 77·45 | 567685 | 27 |
| 34 | 436800 | 76·57 | 999838 | ·06 | 436962 | 76·63 | 563038 | 26 |
| 35 | 441394 | 75·77 | 999834 | ·06 | 441560 | 75·83 | 558440 | 25 |
| 36 | 445941 | 74·99 | 999831 | ·06 | 446110 | 75·05 | 553890 | 24 |
| 37 | 450440 | 74·22 | 999827 | ·06 | 450613 | 74·28 | 549387 | 23 |
| 38 | 454893 | 73·46 | 999823 | ·06 | 455070 | 73·52 | 544930 | 22 |
| 39 | 459301 | 72·73 | 999820 | ·06 | 459481 | 72·79 | 540519 | 21 |
| 40 | 463665 | 72·00 | 999816 | ·06 | 463849 | 72·06 | 536151 | 20 |
| 41 | 8·467985 | 71·29 | 9·999812 | ·06 | 8·468172 | 71·35 | 11·531828 | 19 |
| 42 | 472263 | 70·60 | 999809 | ·06 | 472454 | 70·66 | 527546 | 18 |
| 43 | 476498 | 69·91 | 999805 | ·06 | 476693 | 69·98 | 523307 | 17 |
| 44 | 480693 | 69·24 | 999801 | ·06 | 480892 | 69·31 | 519108 | 16 |
| 45 | 484848 | 68·59 | 999797 | ·07 | 485050 | 68·65 | 514950 | 15 |
| 46 | 488963 | 67·94 | 999793 | ·07 | 489170 | 68·01 | 510830 | 14 |
| 47 | 493040 | 67·31 | 999790 | ·07 | 493250 | 67·38 | 506750 | 13 |
| 48 | 497078 | 66·69 | 999786 | ·07 | 497293 | 66·76 | 502707 | 12 |
| 49 | 501080 | 66·08 | 999782 | ·07 | 501298 | 66·15 | 498702 | 11 |
| 50 | 505045 | 65·48 | 999778 | ·07 | 505267 | 65·55 | 494733 | 10 |
| 51 | 8·508974 | 64·89 | 9·999774 | ·07 | 8·509200 | 64·96 | 11·490800 | 9 |
| 52 | 512867 | 64·31 | 999769 | ·07 | 513098 | 64·39 | 486902 | 8 |
| 53 | 516726 | 63·75 | 999765 | ·07 | 516961 | 63·82 | 483039 | 7 |
| 54 | 520551 | 63·19 | 999761 | ·07 | 520790 | 63·26 | 479210 | 6 |
| 55 | 524343 | 62·64 | 999757 | ·07 | 524586 | 62·72 | 475414 | 5 |
| 56 | 528102 | 62·11 | 999753 | ·07 | 528349 | 62·18 | 471651 | 4 |
| 57 | 531828 | 61·58 | 999748 | ·07 | 532080 | 61·65 | 467920 | 3 |
| 58 | 535523 | 61·06 | 999744 | ·07 | 535779 | 61·13 | 464221 | 2 |
| 59 | 539186 | 60·55 | 999740 | ·07 | 539447 | 60·62 | 460553 | 1 |
| 60 | 542819 | 60·04 | 999735 | ·07 | 543084 | 60·12 | 456916 | 0 |
| | Cosine | D. | Sine | 88° | Cotang. | D. | Tang. | |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 8·542819 | 60·04 | 9·999735 | ·07 | 8·543084 | 60·12 | 11·456916 | 60 |
| 1 | 546422 | 59·55 | 999731 | ·07 | 546691 | 59·62 | 453309 | 59 |
| 2 | 549995 | 59·06 | 999726 | ·07 | 550268 | 59·14 | 449732 | 58 |
| 3 | 553539 | 58·58 | 999722 | ·08 | 553817 | 58·66 | 446183 | 57 |
| 4 | 557054 | 58·11 | 999717 | ·08 | 557336 | 58·19 | 442664 | 56 |
| 5 | 560540 | 57·65 | 999713 | ·08 | 560828 | 57·73 | 439172 | 55 |
| 6 | 563999 | 57·19 | 999708 | ·08 | 564291 | 57·27 | 435709 | 54 |
| 7 | 567431 | 56·74 | 999704 | ·08 | 567727 | 56·82 | 432273 | 53 |
| 8 | 570836 | 56·30 | 999699 | ·08 | 571137 | 56·38 | 428863 | 52 |
| 9 | 574214 | 55·87 | 999694 | ·08 | 574520 | 55·95 | 425480 | 51 |
| 10 | 577566 | 55·44 | 999689 | ·08 | 577877 | 55·52 | 422123 | 50 |
| 11 | 8·580892 | 55·02 | 9·999685 | ·08 | 8·581208 | 55·10 | 11·418792 | 49 |
| 12 | 584193 | 54·60 | 999680 | ·08 | 584514 | 54·68 | 415486 | 48 |
| 13 | 587469 | 54·19 | 999675 | ·08 | 587795 | 54·27 | 412205 | 47 |
| 14 | 590721 | 53·79 | 999670 | ·08 | 591051 | 53·87 | 408949 | 46 |
| 15 | 593948 | 53·39 | 999665 | ·08 | 594283 | 53·47 | 405717 | 45 |
| 16 | 597152 | 53·00 | 999660 | ·08 | 597492 | 53·08 | 402508 | 44 |
| 17 | 600332 | 52·61 | 999655 | ·08 | 600677 | 52·70 | 399323 | 43 |
| 18 | 603489 | 52·23 | 999650 | ·08 | 603839 | 52·32 | 396161 | 42 |
| 19 | 606623 | 51·86 | 999645 | 09 | 606978 | 51·94 | 393022 | 41 |
| 20 | 609734 | 51·49 | 999640 | 09 | 610094 | 51·58 | 389906 | 40 |
| 21 | 8·612823 | 51·12 | 9·999635 | ·09 | 8·613189 | 51·21 | 11·386811 | 39 |
| 22 | 615891 | 50·76 | 999629 | ·09 | 616262 | 50·85 | 383738 | 38 |
| 23 | 618937 | 50·41 | 999624 | ·09 | 619313 | 50·50 | 380687 | 37 |
| 24 | 621962 | 50·06 | 999619 | ·09 | 622343 | 50·15 | 377657 | 36 |
| 25 | 624965 | 49·72 | 999614 | ·09 | 625352 | 49·81 | 374648 | 35 |
| 26 | 627948 | 49·38 | 999608 | ·09 | 628340 | 49·47 | 371660 | 34 |
| 27 | 630911 | 49·04 | 999603 | ·09 | 631308 | 49·13 | 368692 | 33 |
| 28 | 633854 | 48·71 | 999597 | ·09 | 634256 | 48·80 | 365744 | 32 |
| 29 | 636776 | 48·39 | 999592 | ·09 | 637184 | 48·48 | 362816 | 31 |
| 30 | 639680 | 48·06 | 999586 | ·09 | 640093 | 48·16 | 359907 | 30 |
| 31 | 8·642563 | 47·75 | 9·999581 | ·09 | 8·642982 | 47·84 | 11·357018 | 29 |
| 32 | 645428 | 47·43 | 999575 | ·09 | 645853 | 47·53 | 354147 | 28 |
| 33 | 648274 | 47·12 | 999570 | ·09 | 648704 | 47·22 | 351296 | 27 |
| 34 | 651102 | 46·82 | 999564 | ·09 | 651537 | 46·91 | 348463 | 26 |
| 35 | 653911 | 46·52 | 999558 | ·10 | 654352 | 46·61 | 345648 | 25 |
| 36 | 656702 | 46·22 | 999553 | ·10 | 657149 | 46·31 | 342851 | 24 |
| 37 | 659475 | 45·92 | 999547 | ·10 | 659928 | 46·02 | 340072 | 23 |
| 38 | 662230 | 45·63 | 999541 | ·10 | 662689 | 45·73 | 337311 | 22 |
| 39 | 664968 | 45·35 | 999535 | ·10 | 665433 | 45·44 | 334567 | 21 |
| 40 | 667689 | 45·06 | 999529 | ·10 | 668160 | 45·26 | 331840 | 20 |
| 41 | 8·670393 | 44·79 | 9·999524 | ·10 | 8·670870 | 44·88 | 11·329130 | 19 |
| 42 | 673080 | 44·51 | 999518 | ·10 | 673563 | 44·61 | 326437 | 18 |
| 43 | 675751 | 44·24 | 999512 | ·10 | 676239 | 44·34 | 323761 | 17 |
| 44 | 678405 | 43·97 | 999506 | ·10 | 678900 | 44·17 | 321100 | 16 |
| 45 | 681043 | 43·70 | 999500 | ·10 | 681544 | 43·80 | 318456 | 15 |
| 46 | 683665 | 43·44 | 999493 | ·10 | 684172 | 43·54 | 315828 | 14 |
| 47 | 686272 | 43·18 | 999487 | ·10 | 686784 | 43·28 | 313216 | 13 |
| 48 | 688863 | 42·92 | 999481 | ·10 | 689381 | 43·03 | 310619 | 12 |
| 49 | 691438 | 42·67 | 999475 | ·10 | 691963 | 42·77 | 308037 | 11 |
| 50 | 693998 | 42·42 | 999469 | ·10 | 694529 | 42·52 | 305471 | 10 |
| 51 | 8·696543 | 42·17 | 9·999463 | ·11 | 8·697081 | 42·28 | 11·302919 | 9 |
| 52 | 699073 | 41·92 | 999456 | ·11 | 699617 | 42·03 | 300383 | 8 |
| 53 | 701589 | 41·68 | 999450 | ·11 | 702139 | 41·79 | 297861 | 7 |
| 54 | 704090 | 41·44 | 999443 | ·11 | 704646 | 41·55 | 295354 | 6 |
| 55 | 706577 | 41·21 | 999437 | ·11 | 707140 | 41·32 | 292860 | 5 |
| 56 | 709049 | 40·97 | 999431 | ·11 | 709618 | 41·08 | 290382 | 4 |
| 57 | 711507 | 40·74 | 999424 | ·11 | 712083 | 40·85 | 287917 | 3 |
| 58 | 713952 | 40·51 | 999418 | ·11 | 714534 | 40·62 | 285465 | 2 |
| 59 | 716383 | 40·29 | 999411 | ·11 | 716972 | 40·40 | 283028 | 1 |
| 60 | 718800 | 40 06 | 999404 | ·11 | 719396 | 40·17 | 280604 | 0 |
| | Cosine | D. | Sine | 87° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 8·718800 | 40·06 | 9·999404 | ·11 | 8·719396 | 40·17 | 11·280604 | 60 |
| 1 | 721204 | 39·84 | 999398 | ·11 | 721806 | 39·95 | 278194 | 59 |
| 2 | 723595 | 39·62 | 999391 | ·11 | 724204 | 39·74 | 275796 | 58 |
| 3 | 725972 | 39·41 | 999384 | ·11 | 726588 | 39·52 | 273412 | 57 |
| 4 | 728337 | 39·19 | 999378 | ·11 | 728959 | 39·30 | 271041 | 56 |
| 5 | 730688 | 38·98 | 999371 | ·11 | 731317 | 39·09 | 268683 | 55 |
| 6 | 733027 | 38·77 | 999364 | ·12 | 733663 | 38·89 | 266337 | 54 |
| 7 | 735354 | 38·57 | 999357 | ·12 | 735996 | 38·68 | 264004 | 53 |
| 8 | 737667 | 38·36 | 999350 | ·12 | 738317 | 38·48 | 261683 | 52 |
| 9 | 739969 | 38·16 | 999343 | ·12 | 740626 | 38·27 | 259374 | 51 |
| 10 | 742259 | 37·96 | 999336 | ·12 | 742922 | 38·07 | 257078 | 50 |
| 11 | 8·744536 | 37·76 | 9·999329 | ·12 | 8·745207 | 37·87 | 11·254793 | 49 |
| 12 | 746802 | 37·56 | 999322 | ·12 | 747479 | 37·68 | 252521 | 48 |
| 13 | 749055 | 37·37 | 999315 | ·12 | 749740 | 37·49 | 250260 | 47 |
| 14 | 751297 | 37·17 | 999308 | ·12 | 751989 | 37·29 | 248011 | 46 |
| 15 | 753528 | 36·98 | 999301 | ·12 | 754227 | 37·10 | 245773 | 45 |
| 16 | 755747 | 36·79 | 999294 | ·12 | 756453 | 36·92 | 243547 | 44 |
| 17 | 757955 | 36·61 | 999286 | ·12 | 758668 | 36·73 | 241332 | 43 |
| 18 | 760151 | 36·42 | 999279 | ·12 | 760872 | 36·55 | 239128 | 42 |
| 19 | 762337 | 36·24 | 999272 | ·12 | 763065 | 36·35 | 236935 | 41 |
| 20 | 764511 | 36·06 | 999265 | ·12 | 765246 | 36·18 | 234754 | 40 |
| 21 | 8·766675 | 35·88 | 9·999257 | ·12 | 8·767417 | 36·00 | 11·232583 | 39 |
| 22 | 768828 | 35·70 | 999250 | ·13 | 769578 | 35·83 | 230422 | 38 |
| 23 | 770970 | 35·53 | 999242 | ·13 | 771727 | 35·65 | 228273 | 37 |
| 24 | 773101 | 35·35 | 999235 | ·13 | 773866 | 35·48 | 226134 | 36 |
| 25 | 775223 | 35·18 | 999227 | ·13 | 775995 | 35·31 | 224005 | 35 |
| 26 | 777333 | 35·01 | 999220 | ·13 | 778114 | 35·14 | 221886 | 34 |
| 27 | 779434 | 34·84 | 999212 | ·13 | 780222 | 34·97 | 219778 | 33 |
| 28 | 781524 | 34·67 | 999205 | ·13 | 782320 | 34·80 | 217680 | 32 |
| 29 | 783605 | 34·51 | 999197 | ·13 | 784408 | 34·64 | 215592 | 31 |
| 30 | 785675 | 34·31 | 999189 | ·13 | 786486 | 34·47 | 213514 | 30 |
| 31 | 8·787736 | 34·18 | 9·999181 | ·13 | 8·788554 | 34·31 | 11·211446 | 29 |
| 32 | 789787 | 34·02 | 999174 | ·13 | 790613 | 34·15 | 209387 | 28 |
| 33 | 791828 | 33·86 | 999166 | ·13 | 792662 | 33·99 | 207338 | 27 |
| 34 | 793859 | 33·70 | 999158 | ·13 | 794701 | 33·83 | 205299 | 26 |
| 35 | 795881 | 33·54 | 999150 | ·13 | 796731 | 33·68 | 203269 | 25 |
| 36 | 797894 | 33·39 | 999142 | ·13 | 798752 | 33·52 | 201248 | 24 |
| 37 | 799897 | 33·23 | 999134 | ·13 | 800763 | 33·37 | 199237 | 23 |
| 38 | 801892 | 33·08 | 999126 | ·13 | 802765 | 33·22 | 197235 | 22 |
| 39 | 803876 | 32·93 | 999118 | ·13 | 804758 | 33·07 | 195242 | 21 |
| 40 | 805852 | 32·78 | 999110 | ·13 | 806742 | 32·92 | 193258 | 20 |
| 41 | 8·807819 | 32·63 | 9·999102 | ·13 | 8·808717 | 32·78 | 11·191283 | 19 |
| 42 | 809777 | 32·49 | 999094 | ·14 | 810683 | 32·62 | 189317 | 18 |
| 43 | 811726 | 32·34 | 999086 | ·14 | 812641 | 32·48 | 187359 | 17 |
| 44 | 813667 | 32·19 | 999077 | ·14 | 814589 | 32·33 | 185411 | 16 |
| 45 | 815599 | 32·05 | 999069 | ·14 | 816529 | 32·19 | 183471 | 15 |
| 46 | 817522 | 31·91 | 999061 | ·14 | 818461 | 32·05 | 181539 | 14 |
| 47 | 819436 | 31·77 | 999053 | ·14 | 820384 | 31·91 | 179616 | 13 |
| 48 | 821343 | 31·63 | 999044 | ·14 | 822298 | 31·77 | 177702 | 12 |
| 49 | 823240 | 31·49 | 999036 | ·14 | 824205 | 31·63 | 175795 | 11 |
| 50 | 825130 | 31·35 | 999027 | ·14 | 826103 | 31·50 | 173897 | 10 |
| 51 | 8·827011 | 31·22 | 9·999019 | ·14 | 8·827992 | 31·36 | 11·172008 | 9 |
| 52 | 828884 | 31·08 | 999010 | ·14 | 829874 | 31·23 | 170126 | 8 |
| 53 | 830749 | 30·95 | 999002 | ·14 | 831748 | 31·10 | 168252 | 7 |
| 54 | 832607 | 30·82 | 998993 | ·14 | 833613 | 30·96 | 166387 | 6 |
| 55 | 834456 | 30·69 | 998984 | ·14 | 835471 | 30·83 | 164529 | 5 |
| 56 | 836297 | 30·56 | 998976 | ·14 | 837321 | 30·70 | 162679 | 4 |
| 57 | 838130 | 30·43 | 998967 | ·15 | 839163 | 30·57 | 160837 | 3 |
| 58 | 839956 | 30·30 | 998958 | ·15 | 840998 | 30·45 | 159002 | 2 |
| 59 | 841774 | 30·17 | 998950 | ·15 | 842825 | 30·32 | 157175 | 1 |
| 60 | 843585 | 30·00 | 998941 | ·15 | 844644 | 30·19 | 155356 | 0 |
| | Cosine | D. | Sine | 86° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 8·843585 | 30·05 | 9·998941 | ·15 | 8·844644 | 30·19 | 11·155356 | 60 |
| 1 | 845387 | 29·92 | 998932 | ·15 | 846455 | 30·07 | 153545 | 59 |
| 2 | 847183 | 29·80 | 998923 | ·15 | 848260 | 29·95 | 151740 | 58 |
| 3 | 848971 | 29·67 | 998914 | ·15 | 850057 | 29·82 | 149943 | 57 |
| 4 | 850751 | 29·55 | 998905 | ·15 | 851846 | 29·70 | 148154 | 56 |
| 5 | 852525 | 29·43 | 998896 | ·15 | 853628 | 29·58 | 146372 | 55 |
| 6 | 854291 | 29·31 | 998887 | ·15 | 855403 | 29·46 | 144597 | 54 |
| 7 | 856049 | 29·19 | 998878 | ·15 | 857171 | 29·35 | 142829 | 53 |
| 8 | 857801 | 29·07 | 998869 | ·15 | 858932 | 29·23 | 141068 | 52 |
| 9 | 859546 | 28·96 | 998860 | ·15 | 860686 | 29·11 | 139314 | 51 |
| 10 | 861283 | 28·84 | 998851 | ·15 | 862433 | 29·00 | 137567 | 50 |
| 11 | 8·863014 | 28·73 | 9·998841 | ·15 | 8·864173 | 28·88 | 11·135827 | 49 |
| 12 | 864738 | 28·61 | 998832 | ·15 | 865906 | 28·77 | 134094 | 48 |
| 13 | 866455 | 28·50 | 998823 | ·16 | 867632 | 28·66 | 132368 | 47 |
| 14 | 868165 | 28·39 | 998813 | ·16 | 869351 | 28·54 | 130649 | 46 |
| 15 | 869868 | 28·28 | 998804 | ·16 | 871064 | 28·43 | 128936 | 45 |
| 16 | 871565 | 28·17 | 998795 | ·16 | 872770 | 28·32 | 127230 | 44 |
| 17 | 873255 | 28·06 | 998785 | ·16 | 874469 | 28·21 | 125531 | 43 |
| 18 | 874938 | 27·95 | 998776 | ·16 | 876162 | 28·11 | 123838 | 42 |
| 19 | 876615 | 27·86 | 998766 | ·16 | 877849 | 28·00 | 122151 | 41 |
| 20 | 878285 | 27·73 | 998757 | ·16 | 879529 | 27·89 | 120471 | 40 |
| 21 | 8·879949 | 27·63 | 9·998747 | ·16 | 8·881202 | 27·79 | 11 118798 | 39 |
| 22 | 881607 | 27·52 | 998738 | ·16 | 882869 | 27·68 | 117131 | 38 |
| 23 | 883258 | 27·42 | 998728 | ·16 | 884530 | 27·58 | 115470 | 37 |
| 24 | 884903 | 27·31 | 998718 | ·16 | 886185 | 27·47 | 113815 | 36 |
| 25 | 886542 | 27·21 | 998708 | ·16 | 887833 | 27·37 | 112167 | 35 |
| 26 | 888174 | 27·11 | 998699 | ·16 | 889476 | 27·27 | 110524 | 34 |
| 27 | 889801 | 27·00 | 998689 | ·16 | 891112 | 27·17 | 108888 | 33 |
| 28 | 891421 | 26·90 | 998679 | ·16 | 892742 | 27·07 | 107258 | 32 |
| 29 | 893035 | 26·80 | 998669 | ·17 | 894366 | 26·97 | 105634 | 31 |
| 30 | 894643 | 26·70 | 998659 | ·17 | 895984 | 26·87 | 104016 | 30 |
| 31 | 8·896246 | 26·60 | 9·998649 | ·17 | 8·897596 | 26·77 | 11·102404 | 29 |
| 32 | 897842 | 26·51 | 998639 | ·17 | 899203 | 26·67 | 100797 | 28 |
| 33 | 899432 | 26·41 | 998629 | ·17 | 900803 | 26·58 | 099197 | 27 |
| 34 | 901017 | 26·31 | 998619 | ·17 | 902398 | 26·48 | 097602 | 26 |
| 35 | 902596 | 26·22 | 998609 | ·17 | 903987 | 26·38 | 096013 | 25 |
| 36 | 904169 | 26·12 | 998599 | ·17 | 905570 | 26·29 | 094430 | 24 |
| 37 | 905736 | 26·03 | 998589 | ·17 | 907147 | 26·20 | 092853 | 23 |
| 38 | 907297 | 25·93 | 998578 | ·17 | 908719 | 26·10 | 091281 | 22 |
| 39 | 908853 | 25·84 | 998568 | ·17 | 910285 | 26·01 | 089715 | 21 |
| 40 | 910404 | 25·75 | 998558 | ·17 | 911846 | 25·92 | 088154 | 20 |
| 41 | 8·911949 | 25·65 | 9·998548 | ·17 | 8·913401 | 25·83 | 11·086599 | 19 |
| 42 | 913488 | 25·56 | 998537 | ·17 | 914951 | 25·74 | 085049 | 18 |
| 43 | 915022 | 25·47 | 998527 | ·17 | 916495 | 25·65 | 083505 | 17 |
| 44 | 916550 | 25·38 | 998516 | ·18 | 918034 | 25·56 | 081966 | 16 |
| 45 | 918073 | 25·29 | 998506 | ·18 | 919568 | 25·47 | 080432 | 15 |
| 46 | 919591 | 25·20 | 998495 | ·18 | 921096 | 25·38 | 078904 | 14 |
| 47 | 921103 | 25·12 | 998485 | ·18 | 922619 | 25·30 | 077381 | 13 |
| 48 | 922610 | 25·03 | 998474 | ·18 | 924136 | 25·21 | 075864 | 12 |
| 49 | 924112 | 24·94 | 998464 | ·18 | 925649 | 25·12 | 074351 | 11 |
| 50 | 925609 | 24·86 | 998453 | ·18 | 927156 | 25·03 | 072844 | 10 |
| 51 | 8·927100 | 24·77 | 9·998442 | ·18 | 8·928658 | 24·95 | 11·071342 | 9 |
| 52 | 928587 | 24·69 | 998431 | ·18 | 930155 | 24·86 | 069845 | 8 |
| 53 | 930068 | 24·60 | 998421 | ·18 | 931647 | 24·78 | 068353 | 7 |
| 54 | 931544 | 24·52 | 998410 | ·18 | 933134 | 24·70 | 066866 | 6 |
| 55 | 933015 | 24·43 | 998399 | ·18 | 934616 | 24·61 | 065384 | 5 |
| 56 | 934481 | 24·35 | 998388 | ·18 | 936093 | 24·53 | 063907 | 4 |
| 57 | 935942 | 24·27 | 998377 | ·18 | 937565 | 24·45 | 062435 | 3 |
| 58 | 937398 | 24·19 | 998366 | ·18 | 939032 | 24·37 | 060968 | 2 |
| 59 | 938850 | 24·11 | 998355 | ·18 | 940494 | 24·30 | 059506 | 1 |
| 60 | 940296 | 24·03 | 998344 | ·18 | 941952 | 24·21 | 058048 | 0 |
| | Cosine | D. | Sine | 85° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 8·940296 | 24·03 | 9·998344 | ·19 | 8·941952 | 24·21 | 11·058048 | 60 |
| 1 | 941738 | 23·94 | 998333 | ·19 | 943404 | 24·13 | 056596 | 59 |
| 2 | 943174 | 23·87 | 998322 | ·19 | 944852 | 24·05 | 055148 | 58 |
| 3 | 944606 | 23·79 | 998311 | ·19 | 946295 | 23·97 | 053705 | 57 |
| 4 | 946034 | 23·71 | 998300 | ·19 | 947734 | 23·90 | 052266 | 56 |
| 5 | 947456 | 23·63 | 998289 | ·19 | 949168 | 23·82 | 050832 | 55 |
| 6 | 948874 | 23·55 | 998277 | ·19 | 950597 | 23·74 | 049403 | 54 |
| 7 | 950287 | 23·48 | 998266 | ·19 | 952021 | 23·66 | 047979 | 53 |
| 8 | 951696 | 23·40 | 998255 | ·19 | 953441 | 23·60 | 046559 | 52 |
| 9 | 953100 | 23·32 | 998243 | ·19 | 954856 | 23·51 | 045144 | 51 |
| 10 | 954499 | 23·25 | 998232 | ·19 | 956267 | 23·44 | 043733 | 50 |
| 11 | 8·955894 | 23·17 | 9·998220 | ·19 | 8·957674 | 23·37 | 11·042326 | 49 |
| 12 | 957284 | 23·10 | 998209 | ·19 | 959075 | 23·29 | 040925 | 48 |
| 13 | 958670 | 23·02 | 998197 | ·19 | 960473 | 23·23 | 039527 | 47 |
| 14 | 960052 | 22·95 | 998186 | ·19 | 961866 | 23·14 | 038134 | 46 |
| 15 | 961429 | 22·88 | 998174 | ·19 | 963255 | 23·07 | 036745 | 45 |
| 16 | 962801 | 22·80 | 998163 | ·19 | 964639 | 23·00 | 035361 | 44 |
| 17 | 964170 | 22·73 | 998151 | ·19 | 966019 | 22·93 | 033981 | 43 |
| 18 | 965534 | 22·66 | 998139 | ·20 | 967394 | 22·86 | 032606 | 42 |
| 19 | 966893 | 22·59 | 998128 | ·20 | 968766 | 22·79 | 031234 | 41 |
| 20 | 968249 | 22·52 | 998116 | ·20 | 970133 | 22·71 | 029867 | 40 |
| 21 | 8·969600 | 22·44 | 9·998104 | ·20 | 8·071496 | 22·65 | 11·028504 | 39 |
| 22 | 970947 | 22·38 | 998092 | ·20 | 972855 | 22·57 | 027145 | 38 |
| 23 | 972289 | 22·31 | 998080 | ·20 | 974209 | 22·51 | 025791 | 37 |
| 24 | 973628 | 22·24 | 998068 | ·20 | 975560 | 22·44 | 024440 | 36 |
| 25 | 974962 | 22·17 | 998056 | ·20 | 976906 | 22·37 | 023094 | 35 |
| 26 | 976293 | 22·10 | 998044 | ·20 | 978248 | 22·30 | 021752 | 34 |
| 27 | 977619 | 22·03 | 998032 | ·20 | 979586 | 22·23 | 020414 | 33 |
| 28 | 978941 | 21·97 | 998020 | ·20 | 980921 | 22·17 | 019079 | 32 |
| 29 | 980259 | 21·90 | 998008 | ·20 | 982251 | 22·10 | 017749 | 31 |
| 30 | 981573 | 21·83 | 997996 | ·20 | 983577 | 22·04 | 016423 | 30 |
| 31 | 8·982883 | 21·77 | 9·997985 | ·20 | 8·984899 | 21·97 | 11·015101 | 29 |
| 32 | 984189 | 21·70 | 997972 | ·20 | 986217 | 21·91 | 013783 | 28 |
| 33 | 985491 | 21·63 | 997959 | ·20 | 987532 | 21·84 | 012468 | 27 |
| 34 | 986789 | 21·57 | 997947 | ·20 | 988842 | 21·78 | 011158 | 26 |
| 35 | 988083 | 21·50 | 997935 | ·21 | 990149 | 21·71 | 009851 | 25 |
| 36 | 989374 | 21·44 | 997922 | ·21 | 991451 | 21·65 | 008549 | 24 |
| 37 | 990660 | 21·38 | 997910 | ·21 | 992750 | 21·58 | 007250 | 23 |
| 38 | 991943 | 21·31 | 997897 | ·21 | 994045 | 21·52 | 005955 | 22 |
| 39 | 993222 | 21·25 | 997885 | ·21 | 995337 | 21·46 | 004663 | 21 |
| 40 | 994497 | 21·19 | 997872 | ·21 | 996624 | 21·40 | 003376 | 20 |
| 41 | 8·995768 | 21·12 | 9·997860 | ·21 | 8·997908 | 21·34 | 11·002092 | 19 |
| 42 | 997036 | 21·06 | 997847 | ·21 | 999188 | 21·27 | 000812 | 18 |
| 43 | 998299 | 21·00 | 997835 | ·21 | 9·000465 | 21·21 | 10·999535 | 17 |
| 44 | 999560 | 20·94 | 997822 | ·21 | 001738 | 21·15 | 998262 | 16 |
| 45 | 9·000816 | 20·87 | 997809 | ·21 | 003007 | 21·09 | 996993 | 15 |
| 46 | 002069 | 20·82 | 997797 | ·21 | 004272 | 21·03 | 995728 | 14 |
| 47 | 003318 | 20·76 | 997784 | ·21 | 005534 | 20·97 | 994466 | 13 |
| 48 | 004563 | 20·70 | 997771 | ·21 | 006792 | 20·91 | 993208 | 12 |
| 49 | 005805 | 20·64 | 997758 | ·21 | 008047 | 20·85 | 991953 | 11 |
| 50 | 007044 | 20·58 | 997745 | ·21 | 009298 | 20·80 | 990702 | 10 |
| 51 | 9·008278 | 20·52 | 9·997732 | ·21 | 9·010546 | 20·74 | 10·989454 | 9 |
| 52 | 009510 | 20·46 | 997719 | ·21 | 011790 | 20·68 | 988210 | 8 |
| 53 | 010737 | 20·40 | 997706 | ·21 | 013031 | 20·62 | 986969 | 7 |
| 54 | 011962 | 20·34 | 997693 | ·22 | 014268 | 20·56 | 985732 | 6 |
| 55 | 013182 | 20·29 | 997680 | ·22 | 015502 | 20·51 | 984498 | 5 |
| 56 | 014400 | 20·23 | 997667 | ·22 | 016732 | 20·45 | 983268 | 4 |
| 57 | 015613 | 20·17 | 997654 | ·22 | 017959 | 20·40 | 982041 | 3 |
| 58 | 016824 | 20·12 | 997641 | ·22 | 019183 | 20·33 | 980817 | 2 |
| 59 | 018031 | 20·06 | 997628 | ·22 | 020403 | 20·28 | 979597 | 1 |
| 60 | 019235 | 20·00 | 997614 | ·22 | 021620 | 20·23 | 978380 | 0 |
| | Cosine | D. | Sine | 84° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·019235 | 20·00 | 9·997614 | ·22 | 9·021620 | 20·23 | 10·978380 | 60 |
| 1 | 020435 | 19·95 | 997601 | ·22 | 022834 | 20·17 | 977166 | 59 |
| 2 | 021632 | 19·89 | 997588 | ·22 | 024044 | 20·11 | 975956 | 58 |
| 3 | 022825 | 19·84 | 997574 | ·22 | 025251 | 20·06 | 974749 | 57 |
| 4 | 024016 | 19·78 | 997561 | ·22 | 026455 | 20·00 | 973545 | 56 |
| 5 | 025203 | 19·73 | 997547 | ·22 | 027655 | 19·95 | 972345 | 55 |
| 6 | 026386 | 19·67 | 997534 | ·23 | 028852 | 19·90 | 971148 | 54 |
| 7 | 027567 | 19·62 | 997520 | ·23 | 030046 | 19·85 | 969954 | 53 |
| 8 | 028744 | 19·57 | 997507 | ·23 | 031237 | 19·79 | 968763 | 52 |
| 9 | 029918 | 19·51 | 997493 | ·23 | 032425 | 19·74 | 967575 | 51 |
| 10 | 031089 | 19·47 | 997480 | ·23 | 033609 | 19·69 | 966391 | 50 |
| 11 | 9·032257 | 19·41 | 9·997466 | ·23 | 9·034791 | 19·64 | 10·965209 | 49 |
| 12 | 033421 | 19·36 | 997452 | ·23 | 035969 | 19·58 | 964031 | 48 |
| 13 | 034582 | 19·30 | 997439 | ·23 | 037144 | 19·53 | 962856 | 47 |
| 14 | 035741 | 19·25 | 997425 | ·23 | 038316 | 19·48 | 961684 | 46 |
| 15 | 036896 | 19·20 | 997411 | ·23 | 039485 | 19·43 | 960515 | 45 |
| 16 | 038048 | 19·15 | 997397 | ·23 | 040651 | 19·38 | 959349 | 44 |
| 17 | 039197 | 19·10 | 997383 | ·23 | 041813 | 19·33 | 958187 | 43 |
| 18 | 040342 | 19·05 | 997369 | ·23 | 042973 | 19·28 | 957027 | 42 |
| 19 | 041485 | 18·99 | 997355 | ·23 | 044130 | 19·23 | 955870 | 41 |
| 20 | 042625 | 18·94 | 997341 | ·23 | 045284 | 19·18 | 954716 | 40 |
| 21 | 9·043762 | 18·89 | 9·997327 | ·24 | 9·046434 | 19·13 | 10·953566 | 39 |
| 22 | 044895 | 18·84 | 997313 | ·24 | 047582 | 19·08 | 952418 | 38 |
| 23 | 046026 | 18·79 | 997299 | ·24 | 048727 | 19·03 | 951273 | 37 |
| 24 | 047154 | 18·75 | 997285 | ·24 | 049869 | 18·98 | 950131 | 36 |
| 25 | 048279 | 18·70 | 997271 | ·24 | 051008 | 18·93 | 948992 | 35 |
| 26 | 049400 | 18·65 | 997257 | ·24 | 052144 | 18·89 | 947856 | 34 |
| 27 | 050519 | 18·60 | 997242 | ·24 | 053277 | 18·84 | 946723 | 33 |
| 28 | 051635 | 18·55 | 997228 | ·24 | 054407 | 18·79 | 945593 | 32 |
| 29 | 052749 | 18·50 | 997214 | ·24 | 055535 | 18·74 | 944465 | 31 |
| 30 | 053859 | 18·45 | 997199 | ·24 | 056659 | 18·70 | 943341 | 30 |
| 31 | 9·054966 | 18·41 | 9·997185 | ·24 | 9·057781 | 18·65 | 10·942219 | 29 |
| 32 | 056071 | 18·36 | 997170 | ·24 | 058900 | 18·60 | 941100 | 28 |
| 33 | 057172 | 18·31 | 997156 | ·24 | 060016 | 18·55 | 939984 | 27 |
| 34 | 058271 | 18·27 | 997141 | ·24 | 061130 | 18·51 | 938870 | 26 |
| 35 | 059367 | 18·22 | 997127 | ·24 | 062240 | 18·46 | 937760 | 25 |
| 36 | 060460 | 18·17 | 997112 | ·24 | 063348 | 18·42 | 936652 | 24 |
| 37 | 061551 | 18·13 | 997098 | ·24 | 064453 | 18·37 | 935547 | 23 |
| 38 | 062639 | 18·08 | 997083 | ·25 | 065556 | 18·33 | 934444 | 22 |
| 39 | 063724 | 18·04 | 997068 | ·25 | 066655 | 18·28 | 933345 | 21 |
| 40 | 064806 | 17·99 | 997053 | ·25 | 067752 | 18·24 | 932248 | 20 |
| 41 | 9·065885 | 17·94 | 9·997039 | ·25 | 9·068846 | 18·19 | 10·931154 | 19 |
| 42 | 066962 | 17·90 | 997024 | ·25 | 069938 | 18·15 | 930062 | 18 |
| 43 | 068036 | 17·86 | 997009 | ·25 | 071027 | 18·10 | 928973 | 17 |
| 44 | 069107 | 17·81 | 996994 | ·25 | 072113 | 18·06 | 927887 | 16 |
| 45 | 070176 | 17·77 | 996979 | ·25 | 073197 | 18·02 | 926803 | 15 |
| 46 | 071242 | 17·72 | 996964 | ·25 | 074278 | 17·97 | 925722 | 14 |
| 47 | 072306 | 17·68 | 996949 | ·25 | 075356 | 17·93 | 924644 | 13 |
| 48 | 073366 | 17·63 | 996934 | ·25 | 076432 | 17·89 | 923568 | 12 |
| 49 | 074424 | 17·59 | 996919 | ·25 | 077505 | 17·84 | 922495 | 11 |
| 50 | 075480 | 17·55 | 996904 | ·25 | 078576 | 17·80 | 921424 | 10 |
| 51 | 9·076533 | 17·50 | 9·996889 | ·25 | 9·079644 | 17·76 | 10·920356 | 9 |
| 52 | 077583 | 17·46 | 996874 | ·25 | 080710 | 17·72 | 919290 | 8 |
| 53 | 078631 | 17·42 | 996858 | ·25 | 081773 | 17·67 | 918227 | 7 |
| 54 | 079676 | 17·38 | 996843 | ·25 | 082833 | 17·63 | 917167 | 6 |
| 55 | 080719 | 17·33 | 996828 | ·25 | 083891 | 17·59 | 916109 | 5 |
| 56 | 081759 | 17·29 | 996812 | ·26 | 084947 | 17·55 | 915053 | 4 |
| 57 | 082797 | 17·25 | 996797 | ·26 | 086000 | 17·51 | 914000 | 3 |
| 58 | 083832 | 17·21 | 996782 | ·26 | 087050 | 17·47 | 912950 | 2 |
| 59 | 084864 | 17·17 | 996766 | ·26 | 088098 | 17·43 | 911902 | 1 |
| 60 | 085894 | 17·13 | 996751 | ·26 | 089144 | 17·38 | 910856 | 0 |
| | Cosine | D. | Sine | 83°. | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·085894 | 17·13 | 9·996751 | ·26 | 9·089144 | 17·38 | 10·910856 | 60 |
| 1 | 086922 | 17·09 | 996735 | ·26 | 090187 | 17·34 | 909813 | 59 |
| 2 | 087947 | 17·04 | 996720 | ·26 | 091228 | 17·30 | 908772 | 58 |
| 3 | 088970 | 17·00 | 996704 | ·26 | 092266 | 17·27 | 907734 | 57 |
| 4 | 089990 | 16·96 | 996688 | ·26 | 093302 | 17·22 | 906698 | 56 |
| 5 | 091008 | 16·92 | 996673 | ·26 | 094336 | 17·19 | 905664 | 55 |
| 6 | 092024 | 16·88 | 996657 | ·26 | 095367 | 17·15 | 904633 | 54 |
| 7 | 093037 | 16·84 | 996641 | ·26 | 096395 | 17·11 | 903605 | 53 |
| 8 | 094047 | 16·80 | 996625 | ·26 | 097422 | 17·07 | 902578 | 52 |
| 9 | 095056 | 16·76 | 996610 | ·26 | 098446 | 17·03 | 901554 | 51 |
| 10 | 096062 | 16·73 | 996594 | ·26 | 099468 | 16·99 | 900532 | 50 |
| 11 | 9·097065 | 16·68 | 9·996578 | ·27 | 9·100487 | 16·95 | 10·899513 | 49 |
| 12 | 098066 | 16·65 | 996562 | ·27 | 101504 | 16·91 | 898496 | 48 |
| 13 | 099065 | 16·61 | 996546 | ·27 | 102519 | 16·87 | 897481 | 47 |
| 14 | 100062 | 16·57 | 996530 | ·27 | 103532 | 16·84 | 896468 | 46 |
| 15 | 101056 | 16·53 | 996514 | ·27 | 104542 | 16·80 | 895458 | 45 |
| 16 | 102048 | 16·49 | 996498 | ·27 | 105550 | 16·76 | 894450 | 44 |
| 17 | 103037 | 16·45 | 996482 | ·27 | 106556 | 16·72 | 893444 | 43 |
| 18 | 104025 | 16·41 | 996465 | ·27 | 107559 | 16·69 | 892441 | 42 |
| 19 | 105010 | 16·38 | 996449 | ·27 | 108560 | 16·65 | 891440 | 41 |
| 20 | 105992 | 16·34 | 996433 | ·27 | 109559 | 16·61 | 890441 | 40 |
| 21 | 9·106973 | 16·30 | 9·996417 | ·27 | 9·110556 | 16·58 | 10·889444 | 39 |
| 22 | 107951 | 16·27 | 996400 | ·27 | 111551 | 16·54 | 888449 | 38 |
| 23 | 108927 | 16·23 | 996384 | ·27 | 112543 | 16·50 | 887457 | 37 |
| 24 | 109901 | 16·19 | 996368 | ·27 | 113533 | 16·46 | 886467 | 36 |
| 25 | 110873 | 16·16 | 996351 | ·27 | 114521 | 16·43 | 885479 | 35 |
| 26 | 111842 | 16·12 | 996335 | ·27 | 115507 | 16·39 | 884493 | 34 |
| 27 | 112809 | 16·08 | 996318 | ·27 | 116491 | 16·36 | 883509 | 33 |
| 28 | 113774 | 16·05 | 996302 | ·28 | 117472 | 16·32 | 882528 | 32 |
| 29 | 114737 | 16·01 | 996285 | ·28 | 118452 | 16·29 | 881548 | 31 |
| 30 | 115698 | 15·97 | 996269 | ·28 | 119429 | 16·25 | 880571 | 30 |
| 31 | 9·116656 | 15·94 | 9·996252 | ·28 | 9·120404 | 16·22 | 10·879596 | 29 |
| 32 | 117613 | 15·90 | 996235 | ·28 | 121377 | 16·18 | 878623 | 28 |
| 33 | 118567 | 15·87 | 996219 | ·28 | 122348 | 16·15 | 877652 | 27 |
| 34 | 119519 | 15·83 | 996202 | ·28 | 123317 | 16·11 | 876683 | 26 |
| 35 | 120469 | 15·80 | 996185 | ·28 | 124284 | 16·07 | 875716 | 25 |
| 36 | 121417 | 15·76 | 996168 | ·28 | 125249 | 16·04 | 874751 | 24 |
| 37 | 122362 | 15·73 | 996151 | ·28 | 126211 | 16·01 | 873789 | 23 |
| 38 | 123306 | 15·69 | 996134 | ·28 | 127172 | 15·97 | 872828 | 22 |
| 39 | 124248 | 15·66 | 996117 | ·28 | 128130 | 15·94 | 871870 | 21 |
| 40 | 125187 | 15·62 | 996100 | ·28 | 129087 | 15·91 | 870913 | 20 |
| 41 | 9·126125 | 15·59 | 9·996083 | ·29 | 9·130041 | 15·87 | 10·869959 | 19 |
| 42 | 127060 | 15·56 | 996066 | ·29 | 130994 | 15·84 | 869006 | 18 |
| 43 | 127993 | 15·52 | 996049 | ·29 | 131944 | 15·81 | 868056 | 17 |
| 44 | 128925 | 15·49 | 996032 | ·29 | 132893 | 15·77 | 867107 | 16 |
| 45 | 129854 | 15·45 | 996015 | ·29 | 133839 | 15·74 | 866161 | 15 |
| 46 | 130781 | 15·42 | 995998 | ·29 | 134784 | 15·71 | 865216 | 14 |
| 47 | 131706 | 15·39 | 995980 | ·29 | 135726 | 15·67 | 864274 | 13 |
| 48 | 132630 | 15·35 | 995963 | ·29 | 136667 | 15·64 | 863333 | 12 |
| 49 | 133551 | 15·32 | 995946 | ·29 | 137605 | 15·61 | 862395 | 11 |
| 50 | 134470 | 15·29 | 995928 | ·29 | 138542 | 15·58 | 861458 | 10 |
| 51 | 9·135387 | 15·25 | 9·995911 | ·29 | 9·139476 | 15·55 | 10·860524 | 9 |
| 52 | 136303 | 15·22 | 995894 | ·29 | 140409 | 15·51 | 859591 | 8 |
| 53 | 137216 | 15·19 | 995876 | ·29 | 141340 | 15·48 | 858660 | 7 |
| 54 | 138128 | 15·16 | 995859 | ·29 | 142269 | 15·45 | 857731 | 6 |
| 55 | 139037 | 15·12 | 995841 | ·29 | 143196 | 15·42 | 856804 | 5 |
| 56 | 139944 | 15·09 | 995823 | ·29 | 144121 | 15·39 | 855879 | 4 |
| 57 | 140850 | 15·06 | 995806 | ·29 | 145044 | 15·35 | 854956 | 3 |
| 58 | 141754 | 15·03 | 995788 | ·29 | 145966 | 15·32 | 854034 | 2 |
| 59 | 142655 | 15·00 | 995771 | ·29 | 146885 | 15·29 | 853115 | 1 |
| 60 | 143555 | 14·96 | 995753 | ·29 | 147803 | 15·26 | 852197 | 0 |
| | Cosine | D. | Sine | 82° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·143555 | 14·96 | 9·995753 | ·30 | 9·147803 | 15·26 | 10·852197 | 60 |
| 1 | 144453 | 14·93 | 995735 | ·30 | 148718 | 15·23 | 851282 | 59 |
| 2 | 145349 | 14·90 | 995717 | ·30 | 149632 | 15·20 | 850368 | 58 |
| 3 | 146243 | 14·87 | 995699 | ·30 | 150544 | 15·17 | 849456 | 57 |
| 4 | 147136 | 14·84 | 995681 | ·30 | 151454 | 15·14 | 848546 | 56 |
| 5 | 148026 | 14·81 | 995664 | ·30 | 152363 | 15·11 | 847637 | 55 |
| 6 | 148915 | 14·78 | 995646 | ·30 | 153269 | 15·08 | 846731 | 54 |
| 7 | 149802 | 14·75 | 995628 | ·30 | 154174 | 15·05 | 845826 | 53 |
| 8 | 150686 | 14·72 | 995610 | ·30 | 155077 | 15·02 | 844923 | 52 |
| 9 | 151569 | 14·69 | 995591 | ·30 | 155978 | 14·99 | 844022 | 51 |
| 10 | 152451 | 14·66 | 995573 | ·30 | 156877 | 14·96 | 843123 | 50 |
| 11 | 9·153330 | 14·63 | 9·995555 | ·30 | 9·157775 | 14·93 | 10·842225 | 49 |
| 12 | 154208 | 14·60 | 995537 | ·30 | 158671 | 14·90 | 841329 | 48 |
| 13 | 155083 | 14·57 | 995519 | ·30 | 159565 | 14·87 | 840435 | 47 |
| 14 | 155957 | 14·54 | 995501 | ·31 | 160457 | 14·84 | 839543 | 46 |
| 15 | 156830 | 14·51 | 995482 | ·31 | 161347 | 14·81 | 838653 | 45 |
| 16 | 157700 | 14·48 | 995464 | ·31 | 162236 | 14·79 | 837764 | 44 |
| 17 | 158569 | 14·45 | 995446 | ·31 | 163123 | 14·76 | 836877 | 43 |
| 18 | 159435 | 14·42 | 995427 | ·31 | 164008 | 14·73 | 835992 | 42 |
| 19 | 160301 | 14·39 | 995409 | ·31 | 164892 | 14·70 | 835108 | 41 |
| 20 | 161164 | 14·36 | 995390 | ·31 | 165774 | 14·67 | 834226 | 40 |
| 21 | 9·162025 | 14·33 | 9·995372 | ·31 | 9·166654 | 14·64 | 10·833346 | 39 |
| 22 | 162885 | 14·30 | 995353 | ·31 | 167532 | 14·61 | 832468 | 38 |
| 23 | 163743 | 14·27 | 995334 | ·31 | 168409 | 14·58 | 831591 | 37 |
| 24 | 164600 | 14·24 | 995316 | ·31 | 169284 | 14·55 | 830716 | 36 |
| 25 | 165454 | 14·22 | 995297 | ·31 | 170157 | 14·53 | 829843 | 35 |
| 26 | 166307 | 14·19 | 995278 | ·31 | 171029 | 14·50 | 828971 | 34 |
| 27 | 167159 | 14·16 | 995260 | ·31 | 171899 | 14·47 | 828101 | 33 |
| 28 | 168008 | 14·13 | 995241 | ·32 | 172767 | 14·44 | 827233 | 32 |
| 29 | 168856 | 14·10 | 995222 | ·32 | 173634 | 14·42 | 826366 | 31 |
| 30 | 169702 | 14·07 | 995203 | ·32 | 174499 | 14·39 | 825501 | 30 |
| 31 | 9·170547 | 14·05 | 9·995184 | ·32 | 9·175362 | 14·36 | 10·824638 | 29 |
| 32 | 171389 | 14·02 | 995165 | ·32 | 176224 | 14·33 | 823776 | 28 |
| 33 | 172230 | 13·99 | 995146 | ·32 | 177084 | 14·31 | 822916 | 27 |
| 34 | 173070 | 13·96 | 995127 | ·32 | 177942 | 14·28 | 822058 | 26 |
| 35 | 173908 | 13·94 | 995108 | ·32 | 178799 | 14·25 | 821201 | 25 |
| 36 | 174744 | 13·91 | 995089 | ·32 | 179655 | 14·23 | 820345 | 24 |
| 37 | 175578 | 13·88 | 995070 | ·32 | 180508 | 14·20 | 819492 | 23 |
| 38 | 176411 | 13·86 | 995051 | ·32 | 181360 | 14·17 | 818640 | 22 |
| 39 | 177242 | 13·83 | 995032 | ·32 | 182211 | 14·15 | 817789 | 21 |
| 40 | 178072 | 13·80 | 995013 | ·32 | 183059 | 14·12 | 816941 | 20 |
| 41 | 9·178900 | 13·77 | 9·994993 | ·32 | 9·183907 | 14·09 | 10·816093 | 19 |
| 42 | 179726 | 13·74 | 994974 | ·32 | 184752 | 14·07 | 815248 | 18 |
| 43 | 180551 | 13·72 | 994955 | ·32 | 185597 | 14·04 | 814403 | 17 |
| 44 | 181374 | 13·69 | 994935 | ·32 | 186439 | 14·02 | 813561 | 16 |
| 45 | 182196 | 13·66 | 994916 | ·33 | 187280 | 13·99 | 812720 | 15 |
| 46 | 183016 | 13·64 | 994896 | ·33 | 188120 | 13·96 | 811880 | 14 |
| 47 | 183834 | 13·61 | 994877 | ·33 | 188958 | 13·93 | 811042 | 13 |
| 48 | 184651 | 13·59 | 994857 | ·33 | 189794 | 13·91 | 810206 | 12 |
| 49 | 185466 | 13·56 | 994838 | ·33 | 190629 | 13·89 | 809371 | 11 |
| 50 | 186280 | 13·53 | 994818 | ·33 | 191462 | 13·86 | 808538 | 10 |
| 51 | 9·187092 | 13·51 | 9·994798 | ·33 | 9·192294 | 13·84 | 10·807706 | 9 |
| 52 | 187903 | 13·48 | 994779 | ·33 | 193124 | 13·81 | 806876 | 8 |
| 53 | 188712 | 13·46 | 994759 | ·33 | 193953 | 13·79 | 806047 | 7 |
| 54 | 189519 | 13·43 | 994739 | ·33 | 194780 | 13·76 | 805220 | 6 |
| 55 | 190325 | 13·41 | 994719 | ·33 | 195606 | 13·74 | 804394 | 5 |
| 56 | 191130 | 13·38 | 994700 | ·33 | 196430 | 13·71 | 803570 | 4 |
| 57 | 191933 | 13·36 | 994680 | ·33 | 197253 | 13·69 | 802747 | 3 |
| 58 | 192734 | 13·33 | 994660 | ·33 | 198074 | 13·66 | 801926 | 2 |
| 59 | 193534 | 13·30 | 994640 | ·33 | 198894 | 13·64 | 801106 | 1 |
| 60 | 194332 | 13·28 | 994620 | ·33 | 199713 | 13·61 | 800287 | 0 |
| | Cosine | D. | Sine | 81° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·194332 | 13·28 | 9·994620 | ·33 | 9·199713 | 13·61 | 10·800287 | 60 |
| 1 | 195129 | 13·26 | 994600 | ·33 | 200529 | 13·59 | 799471 | 59 |
| 2 | 195925 | 13·23 | 994580 | ·33 | 201345 | 13·56 | 798655 | 58 |
| 3 | 196719 | 13·21 | 994560 | ·34 | 202159 | 13·54 | 797841 | 57 |
| 4 | 197511 | 13·18 | 994540 | ·34 | 202971 | 13·52 | 797029 | 56 |
| 5 | 198302 | 13·16 | 994519 | ·34 | 203782 | 13·49 | 796218 | 55 |
| 6 | 199091 | 13·13 | 994499 | ·34 | 204592 | 13·47 | 795408 | 54 |
| 7 | 199879 | 13·11 | 994479 | ·34 | 205400 | 13·45 | 794600 | 53 |
| 8 | 200666 | 13·08 | 994459 | ·34 | 206207 | 13·42 | 793793 | 52 |
| 9 | 201451 | 13·06 | 994438 | ·34 | 207013 | 13·40 | 792987 | 51 |
| 10 | 202234 | 13·04 | 994418 | ·34 | 207817 | 13·38 | 792183 | 50 |
| 11 | 9·203017 | 13·01 | 9·994397 | ·34 | 9·208619 | 13·35 | 10·791381 | 49 |
| 12 | 203797 | 12·99 | 994377 | ·34 | 209420 | 13·33 | 790580 | 48 |
| 13 | 204577 | 12·96 | 994357 | ·34 | 210220 | 13·31 | 789780 | 47 |
| 14 | 205354 | 12·94 | 994336 | ·34 | 211018 | 13·28 | 788982 | 46 |
| 15 | 206131 | 12·92 | 994316 | ·34 | 211815 | 13·26 | 788185 | 45 |
| 16 | 206906 | 12·89 | 994295 | ·34 | 212611 | 13·24 | 787389 | 44 |
| 17 | 207679 | 12·87 | 994274 | ·35 | 213405 | 13·21 | 786595 | 43 |
| 18 | 208452 | 12·85 | 994254 | ·35 | 214198 | 13·19 | 785802 | 42 |
| 19 | 209222 | 12·82 | 994233 | ·35 | 214989 | 13·17 | 785011 | 41 |
| 20 | 209992 | 12·80 | 994212 | ·35 | 215780 | 13·15 | 784220 | 40 |
| 21 | 9·210760 | 12·78 | 9·994191 | ·35 | 9·216568 | 13·12 | 10·783432 | 39 |
| 22 | 211526 | 12·75 | 994171 | ·35 | 217356 | 13·10 | 782644 | 38 |
| 23 | 212291 | 12·73 | 994150 | ·35 | 218142 | 13·08 | 781858 | 37 |
| 24 | 213055 | 12·71 | 994129 | ·35 | 218926 | 13·05 | 781074 | 36 |
| 25 | 213818 | 12·68 | 994108 | ·35 | 219710 | 13·03 | 780290 | 35 |
| 26 | 214579 | 12·66 | 994087 | ·35 | 220492 | 13·01 | 779508 | 34 |
| 27 | 215338 | 12·64 | 994066 | ·35 | 221272 | 12·99 | 778728 | 33 |
| 28 | 216097 | 12·61 | 994045 | ·35 | 222052 | 12·97 | 777948 | 32 |
| 29 | 216854 | 12·59 | 994024 | ·35 | 222830 | 12·94 | 777170 | 31 |
| 30 | 217609 | 12·57 | 994003 | ·35 | 223606 | 12·92 | 776394 | 30 |
| 31 | 9·218363 | 12·55 | 9·993981 | ·35 | 9·224382 | 12·90 | 10·775618 | 29 |
| 32 | 219116 | 12·53 | 993960 | ·35 | 225156 | 12·88 | 774844 | 28 |
| 33 | 219868 | 12·50 | 993939 | ·35 | 225929 | 12·86 | 774071 | 27 |
| 34 | 220618 | 12·48 | 993918 | ·35 | 226700 | 12·84 | 773300 | 26 |
| 35 | 221367 | 12·46 | 993896 | ·36 | 227471 | 12·81 | 772529 | 25 |
| 36 | 222115 | 12·44 | 993875 | ·36 | 228239 | 12·79 | 771761 | 24 |
| 37 | 222861 | 12·42 | 993854 | ·36 | 229007 | 12·77 | 770993 | 23 |
| 38 | 223606 | 12·39 | 993832 | ·36 | 229773 | 12·75 | 770227 | 22 |
| 39 | 224349 | 12·37 | 993811 | ·36 | 230539 | 12·73 | 769461 | 21 |
| 40 | 225092 | 12·35 | 993789 | ·36 | 231302 | 12·71 | 768698 | 20 |
| 41 | 9·225833 | 12·33 | 9·993768 | ·36 | 9·232065 | 12·69 | 10·767935 | 19 |
| 42 | 226573 | 12·31 | 993746 | ·36 | 232826 | 12·67 | 767174 | 18 |
| 43 | 227311 | 12·28 | 993725 | ·36 | 233586 | 12·65 | 766414 | 17 |
| 44 | 228048 | 12·26 | 993703 | ·36 | 234345 | 12·62 | 765655 | 16 |
| 45 | 228784 | 12·24 | 993681 | ·36 | 235103 | 12·60 | 764897 | 15 |
| 46 | 229518 | 12·22 | 993660 | ·36 | 235859 | 12·58 | 764141 | 14 |
| 47 | 230252 | 12·20 | 993638 | ·36 | 236614 | 12·56 | 763386 | 13 |
| 48 | 230984 | 12·18 | 993616 | ·36 | 237368 | 12·54 | 762632 | 12 |
| 49 | 231714 | 12·16 | 993594 | ·37 | 238120 | 12·52 | 761880 | 11 |
| 50 | 232444 | 12·14 | 993572 | ·37 | 238872 | 12·50 | 761128 | 10 |
| 51 | 9·233172 | 12·12 | 9·993550 | ·37 | 9·239622 | 12·48 | 10·760378 | 9 |
| 52 | 233899 | 12·09 | 993528 | ·37 | 240371 | 12·46 | 759629 | 8 |
| 53 | 234625 | 12·07 | 993506 | ·37 | 241118 | 12·44 | 758882 | 7 |
| 54 | 235349 | 12·05 | 993484 | ·37 | 241865 | 12·42 | 758135 | 6 |
| 55 | 236073 | 12·03 | 993462 | ·37 | 242610 | 12·40 | 757390 | 5 |
| 56 | 236795 | 12·01 | 993440 | ·37 | 243354 | 12·38 | 756646 | 4 |
| 57 | 237515 | 11·99 | 993418 | ·37 | 244097 | 12·36 | 755903 | 3 |
| 58 | 238235 | 11·97 | 993396 | ·37 | 244839 | 12·34 | 755161 | 2 |
| 59 | 238953 | 11·95 | 993374 | ·37 | 245579 | 12·32 | 754421 | 1 |
| 60 | 239670 | 11·93 | 993351 | ·37 | 246319 | 12·30 | 753681 | 0 |
| | Cosine | D. | Sine | 80° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. |  |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·239670 | 11·93 | 9·993351 | ·37 | 9·246319 | 12·30 | 10·753681 | 60 |
| 1 | 240386 | 11·91 | 993329 | ·37 | 247057 | 12·28 | 752943 | 59 |
| 2 | 241101 | 11·89 | 993307 | ·37 | 247794 | 12·26 | 752206 | 58 |
| 3 | 241814 | 11·87 | 993285 | ·37 | 248530 | 12·24 | 751470 | 57 |
| 4 | 242526 | 11·85 | 993262 | ·37 | 249264 | 12·22 | 750736 | 56 |
| 5 | 243237 | 11·83 | 993240 | ·37 | 249998 | 12·20 | 750002 | 55 |
| 6 | 243947 | 11·81 | 993217 | ·38 | 250730 | 12·18 | 749270 | 54 |
| 7 | 244656 | 11·79 | 993195 | ·38 | 251461 | 12·17 | 748539 | 53 |
| 8 | 245363 | 11·77 | 993172 | ·38 | 252191 | 12·15 | 747809 | 52 |
| 9 | 246069 | 11·75 | 993149 | ·38 | 252920 | 12·13 | 747080 | 51 |
| 10 | 246775 | 11·73 | 993127 | ·38 | 253648 | 12·11 | 746352 | 50 |
| 11 | 9·247478 | 11·71 | 9·993104 | ·38 | 9·254374 | 12·09 | 10·745626 | 49 |
| 12 | 248181 | 11·69 | 993081 | ·38 | 255100 | 12·07 | 744900 | 48 |
| 13 | 248883 | 11·67 | 993059 | ·38 | 255824 | 12·05 | 744176 | 47 |
| 14 | 249583 | 11·65 | 993036 | ·38 | 256547 | 12·03 | 743453 | 46 |
| 15 | 250282 | 11·63 | 993013 | ·38 | 257269 | 12·01 | 742731 | 45 |
| 16 | 250980 | 11·61 | 992990 | ·38 | 257990 | 12·00 | 742010 | 44 |
| 17 | 251677 | 11·59 | 992967 | ·38 | 258710 | 11·98 | 741290 | 43 |
| 18 | 252373 | 11·58 | 992944 | ·38 | 259429 | 11·96 | 740571 | 42 |
| 19 | 253067 | 11·56 | 992921 | ·38 | 260146 | 11·94 | 739854 | 41 |
| 20 | 253761 | 11·54 | 992898 | ·38 | 260863 | 11·92 | 739137 | 40 |
| 21 | 9·254453 | 11·52 | 9·992875 | ·38 | 9·261578 | 11·90 | 10·738422 | 39 |
| 22 | 255144 | 11·50 | 992852 | ·38 | 262292 | 11·89 | 737708 | 38 |
| 23 | 255834 | 11·48 | 992829 | ·39 | 263005 | 11·87 | 736995 | 37 |
| 24 | 256523 | 11·46 | 992806 | ·39 | 263717 | 11·85 | 736283 | 36 |
| 25 | 257211 | 11·44 | 992783 | ·39 | 264428 | 11·83 | 735572 | 35 |
| 26 | 257898 | 11·42 | 992759 | ·39 | 265138 | 11·81 | 734862 | 34 |
| 27 | 258583 | 11·41 | 992736 | ·39 | 265847 | 11·79 | 734153 | 33 |
| 28 | 259268 | 11·39 | 992713 | ·39 | 266555 | 11·78 | 733445 | 32 |
| 29 | 259951 | 11·37 | 992690 | ·39 | 267261 | 11·76 | 732739 | 31 |
| 30 | 260633 | 11·35 | 992666 | ·39 | 267967 | 11·74 | 732033 | 30 |
| 31 | 9·261314 | 11·33 | 9·992643 | ·39 | 9·268671 | 11·72 | 10·731329 | 29 |
| 32 | 261994 | 11·31 | 992619 | ·39 | 269375 | 11·70 | 730625 | 28 |
| 33 | 262673 | 11·30 | 992596 | ·39 | 270077 | 11·69 | 729923 | 27 |
| 34 | 263351 | 11·28 | 992572 | ·39 | 270779 | 11·67 | 729221 | 26 |
| 35 | 264027 | 11·26 | 992549 | ·39 | 271479 | 11·66 | 728521 | 25 |
| 36 | 264703 | 11·24 | 992525 | ·39 | 272178 | 11·64 | 727822 | 24 |
| 37 | 265377 | 11·22 | 992501 | ·39 | 272876 | 11·62 | 727124 | 23 |
| 38 | 266051 | 11·20 | 992478 | ·40 | 273573 | 11·60 | 726427 | 22 |
| 39 | 266723 | 11·19 | 992454 | ·40 | 274269 | 11·58 | 725731 | 21 |
| 40 | 267395 | 11·17 | 992430 | ·40 | 274964 | 11·57 | 725036 | 20 |
| 41 | 9·268065 | 11·15 | 9·992406 | ·40 | 9·275658 | 11·55 | 10·724342 | 19 |
| 42 | 268734 | 11·13 | 992382 | ·40 | 276351 | 11·53 | 723649 | 18 |
| 43 | 269402 | 11·11 | 992359 | ·40 | 277043 | 11·51 | 722957 | 17 |
| 44 | 270069 | 11·10 | 992335 | ·40 | 277734 | 11·50 | 722266 | 16 |
| 45 | 270735 | 11·08 | 992311 | ·40 | 278424 | 11·48 | 721576 | 15 |
| 46 | 271400 | 11·06 | 992287 | ·40 | 279113 | 11·47 | 720887 | 14 |
| 47 | 272064 | 11·05 | 992263 | ·40 | 279801 | 11·45 | 720199 | 13 |
| 48 | 272726 | 11·03 | 992239 | ·40 | 280488 | 11·43 | 719512 | 12 |
| 49 | 273388 | 11·01 | 992214 | ·40 | 281174 | 11·41 | 718826 | 11 |
| 50 | 274049 | 10·99 | 992190 | ·40 | 281858 | 11·40 | 718142 | 10 |
| 51 | 9 274708 | 10·98 | 9·992166 | ·40 | 9·282542 | 11·38 | 10·717458 | 9 |
| 52 | 275367 | 10·96 | 992142 | ·40 | 283225 | 11·36 | 716775 | 8 |
| 53 | 276024 | 10·94 | 992117 | ·41 | 283907 | 11·35 | 716093 | 7 |
| 54 | 276681 | 10·92 | 992093 | ·41 | 284588 | 11·33 | 715412 | 6 |
| 55 | 277337 | 10·91 | 992069 | ·41 | 285268 | 11·31 | 714732 | 5 |
| 56 | 277991 | 10·89 | 992044 | ·41 | 285947 | 11·30 | 714053 | 4 |
| 57 | 278644 | 10·87 | 992020 | ·41 | 286624 | 11·28 | 713376 | 3 |
| 58 | 279297 | 10·86 | 991996 | ·41 | 287301 | 11·26 | 712699 | 2 |
| 59 | 279948 | 10·84 | 991971 | ·41 | 287977 | 11·25 | 712023 | 1 |
| 60 | 280599 | 10·82 | 991947 | ·41 | 288652 | 11·23 | 711348 | 0 |
|  | Cosine | D. | Sine | 79° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·280599 | 10·82 | 9·991947 | ·41 | 9·288652 | 11·23 | 10·711348 | 60 |
| 1 | 281248 | 10·81 | 991922 | ·41 | 289326 | 11·22 | 710674 | 59 |
| 2 | 281897 | 10·79 | 991897 | ·41 | 289999 | 11·20 | 710001 | 58 |
| 3 | 282544 | 10·77 | 991873 | ·41 | 290671 | 11·18 | 709329 | 57 |
| 4 | 283190 | 10·76 | 991848 | ·41 | 291342 | 11·17 | 708658 | 56 |
| 5 | 283836 | 10·74 | 991823 | ·41 | 292013 | 11·15 | 707987 | 55 |
| 6 | 284480 | 10·72 | 991799 | ·41 | 292682 | 11·14 | 707318 | 54 |
| 7 | 285124 | 10·71 | 991774 | ·42 | 293350 | 11·12 | 706650 | 53 |
| 8 | 285766 | 10·69 | 991749 | ·42 | 294017 | 11·11 | 705983 | 52 |
| 9 | 286408 | 10·67 | 991724 | ·42 | 294684 | 11·09 | 705316 | 51 |
| 10 | 287048 | 10·66 | 991699 | ·42 | 295349 | 11·07 | 704651 | 50 |
| 11 | 9·287687 | 10·64 | 9·991674 | ·42 | 9·296013 | 11·06 | 10·703987 | 49 |
| 12 | 288326 | 10·63 | 991649 | ·42 | 296677 | 11·04 | 703323 | 48 |
| 13 | 288964 | 10·61 | 991624 | ·42 | 297339 | 11·03 | 702661 | 47 |
| 14 | 289600 | 10·59 | 991599 | ·42 | 298001 | 11·01 | 701999 | 46 |
| 15 | 290236 | 10·58 | 991574 | ·42 | 298662 | 11·00 | 701338 | 45 |
| 16 | 290870 | 10·56 | 991549 | ·42 | 299322 | 10·98 | 700678 | 44 |
| 17 | 291504 | 10·54 | 991524 | ·42 | 299980 | 10·96 | 700020 | 43 |
| 18 | 292137 | 10·53 | 991498 | ·42 | 300638 | 10·95 | 699362 | 42 |
| 19 | 292768 | 10·51 | 991473 | ·42 | 301295 | 10·93 | 698705 | 41 |
| 20 | 293399 | 10·50 | 991448 | ·42 | 301951 | 10·92 | 698049 | 40 |
| 21 | 9·294029 | 10·48 | 9·991422 | ·42 | 9·302607 | 10·90 | 10·697393 | 39 |
| 22 | 294658 | 10·46 | 991397 | ·42 | 303261 | 10·89 | 696739 | 38 |
| 23 | 295286 | 10·45 | 991372 | ·43 | 303914 | 10·87 | 696086 | 37 |
| 24 | 295913 | 10·43 | 991346 | ·43 | 304567 | 10·86 | 695433 | 36 |
| 25 | 296539 | 10·42 | 991321 | ·43 | 305218 | 10·84 | 694782 | 35 |
| 26 | 297164 | 10·40 | 991295 | ·43 | 305869 | 10·83 | 694131 | 34 |
| 27 | 297788 | 10·39 | 991270 | ·43 | 306519 | 10·81 | 693481 | 33 |
| 28 | 298412 | 10·37 | 991244 | ·43 | 307168 | 10·80 | 692832 | 32 |
| 29 | 299034 | 10·36 | 991218 | ·43 | 307815 | 10·78 | 692185 | 31 |
| 30 | 299655 | 10·34 | 991193 | ·43 | 308463 | 10·77 | 691537 | 30 |
| 31 | 9·300276 | 10·32 | 9·991167 | ·43 | 9·309109 | 10·75 | 10·690891 | 29 |
| 32 | 300895 | 10·31 | 991141 | ·43 | 309754 | 10·74 | 690246 | 28 |
| 33 | 301514 | 10·29 | 991115 | ·43 | 310398 | 10·73 | 689602 | 27 |
| 34 | 302132 | 10·28 | 991090 | ·43 | 311042 | 10·71 | 688958 | 26 |
| 35 | 302748 | 10·26 | 991064 | ·43 | 311685 | 10·70 | 688315 | 25 |
| 36 | 303364 | 10·25 | 991038 | ·43 | 312327 | 10·68 | 687673 | 24 |
| 37 | 303979 | 10·23 | 991012 | ·43 | 312967 | 10·67 | 687033 | 23 |
| 38 | 304593 | 10·22 | 990986 | ·43 | 313608 | 10·65 | 686392 | 22 |
| 39 | 305207 | 10·20 | 990960 | ·43 | 314247 | 10·64 | 685753 | 21 |
| 40 | 305819 | 10·19 | 990934 | ·44 | 314885 | 10·62 | 685115 | 20 |
| 41 | 9·306430 | 10·17 | 9·990908 | ·44 | 9·315523 | 10·61 | 10·684477 | 19 |
| 42 | 307041 | 10·16 | 990882 | ·44 | 316159 | 10·60 | 683841 | 18 |
| 43 | 307650 | 10·14 | 990855 | ·44 | 316795 | 10·58 | 683205 | 17 |
| 44 | 308259 | 10·13 | 990829 | ·44 | 317430 | 10·57 | 682570 | 16 |
| 45 | 308867 | 10·11 | 990803 | ·44 | 318064 | 10·55 | 681936 | 15 |
| 46 | 309474 | 10·10 | 990777 | ·44 | 318697 | 10·54 | 681303 | 14 |
| 47 | 310080 | 10·08 | 990750 | ·44 | 319329 | 10·53 | 680671 | 13 |
| 48 | 310685 | 10·07 | 990724 | ·44 | 319961 | 10·51 | 680039 | 12 |
| 49 | 311289 | 10·05 | 990697 | ·44 | 320592 | 10·50 | 679408 | 11 |
| 50 | 311893 | 10·04 | 990671 | ·44 | 321222 | 10·48 | 678778 | 10 |
| 51 | 9·312495 | 10·03 | 9·990644 | ·44 | 9·321851 | 10·47 | 10·678149 | 9 |
| 52 | 313097 | 10·01 | 990618 | ·44 | 322479 | 10·45 | 677521 | 8 |
| 53 | 313698 | 10·00 | 990591 | ·44 | 323106 | 10·44 | 676894 | 7 |
| 54 | 314297 | 9·98 | 990565 | ·44 | 323733 | 10·43 | 676267 | 6 |
| 55 | 314897 | 9·97 | 990538 | ·44 | 324358 | 10·41 | 675642 | 5 |
| 56 | 315495 | 9·96 | 990511 | ·45 | 324983 | 10·40 | 675017 | 4 |
| 57 | 316092 | 9·94 | 990485 | ·45 | 325607 | 10·39 | 674393 | 3 |
| 58 | 316689 | 9·93 | 990458 | ·45 | 326231 | 10·37 | 673769 | 2 |
| 59 | 317284 | 9·91 | 990431 | ·45 | 326853 | 10·36 | 673147 | 1 |
| 60 | 317879 | 9·90 | 990404 | ·45 | 327475 | 10·35 | 672525 | 0 |
| | Cosine | D. | Sine | 78° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·317879 | 9·90 | 9·990404 | ·45 | 9·327474 | 10·35 | 10·672526 | 60 |
| 1 | 318473 | 9·88 | 990378 | ·45 | 328095 | 10·33 | 671905 | 59 |
| 2 | 319066 | 9·87 | 990351 | ·45 | 328715 | 10·32 | 671285 | 58 |
| 3 | 319658 | 9·86 | 990324 | ·45 | 329334 | 10·30 | 670666 | 57 |
| 4 | 320249 | 9·84 | 990207 | ·45 | 329953 | 10·29 | 670047 | 56 |
| 5 | 320840 | 9·83 | 990270 | ·45 | 330570 | 10·28 | 669430 | 55 |
| 6 | 321430 | 9·82 | 990243 | ·45 | 331187 | 10·26 | 668813 | 54 |
| 7 | 322019 | 9·80 | 990215 | ·45 | 331803 | 10·25 | 668197 | 53 |
| 8 | 322607 | 9·79 | 990188 | ·45 | 332418 | 10·24 | 667582 | 52 |
| 9 | 323194 | 9·77 | 990161 | ·45 | 333033 | 10·23 | 666967 | 51 |
| 10 | 323780 | 9·76 | 990134 | ·45 | 333646 | 10·21 | 666354 | 50 |
| 11 | 9·324366 | 9·75 | 9·990107 | ·46 | 9·334259 | 10·20 | 10·665741 | 49 |
| 12 | 324950 | 9·73 | 990079 | ·46 | 334871 | 10·19 | 665129 | 48 |
| 13 | 325534 | 9·72 | 990052 | ·46 | 335482 | 10·17 | 664518 | 47 |
| 14 | 326117 | 9·70 | 990025 | ·46 | 336093 | 10·16 | 663907 | 46 |
| 15 | 326700 | 9·69 | 989997 | ·46 | 336702 | 10·15 | 663298 | 45 |
| 16 | 327281 | 9·68 | 989970 | ·46 | 337311 | 10·13 | 662689 | 44 |
| 17 | 327862 | 9·66 | 989942 | ·46 | 337919 | 10·12 | 662081 | 43 |
| 18 | 328442 | 9·65 | 989915 | ·46 | 338527 | 10·11 | 661473 | 42 |
| 19 | 329021 | 9·64 | 989887 | ·46 | 339133 | 10·10 | 560867 | 41 |
| 20 | 329599 | 9·62 | 989860 | ·46 | 339739 | 10·08 | 660261 | 40 |
| 21 | 9·330176 | 9·61 | 9·989832 | ·46 | 9·340344 | 10·07 | 10·659656 | 39 |
| 22 | 330753 | 9·60 | 989804 | ·46 | 340948 | 10·06 | 659052 | 38 |
| 23 | 331329 | 9·58 | 989777 | ·46 | 341552 | 10·04 | 658448 | 37 |
| 24 | 331903 | 9·57 | 989749 | ·47 | 342155 | 10·03 | 657845 | 36 |
| 25 | 332478 | 9·56 | 989721 | ·47 | 342757 | 10·02 | 657243 | 35 |
| 26 | 333051 | 9·54 | 989693 | ·47 | 343358 | 10·00 | 656642 | 34 |
| 27 | 333624 | 9·53 | 989665 | ·47 | 343958 | 9·99 | 656042 | 33 |
| 28 | 334195 | 9·52 | 989637 | ·47 | 344558 | 9·98 | 655442 | 32 |
| 29 | 334766 | 9·50 | 989609 | ·47 | 345157 | 9·97 | 654843 | 31 |
| 30 | 335337 | 9·49 | 989582 | ·47 | 345755 | 9·96 | 654245 | 30 |
| 31 | 9·335906 | 9·48 | 9·989553 | ·47 | 9·346353 | 9·94 | 10·653647 | 29 |
| 32 | 336475 | 9·46 | 989525 | ·47 | 346949 | 9·93 | 653051 | 28 |
| 33 | 337043 | 9·45 | 989497 | ·47 | 347545 | 9·92 | 652455 | 27 |
| 34 | 337610 | 9·44 | 989469 | ·47 | 348141 | 9·91 | 651859 | 26 |
| 35 | 338176 | 9·43 | 989441 | ·47 | 348735 | 9·90 | 651265 | 25 |
| 36 | 338742 | 9·41 | 989413 | ·47 | 349329 | 9·88 | 650671 | 24 |
| 37 | 339306 | 9·40 | 989384 | ·47 | 349922 | 9·87 | 650078 | 23 |
| 38 | 339871 | 9·39 | 989356 | ·47 | 350514 | 9·86 | 649486 | 22 |
| 39 | 340434 | 9·37 | 989328 | ·47 | 351106 | 9·85 | 648894 | 21 |
| 40 | 340996 | 9·36 | 989300 | ·47 | 351697 | 9·83 | 648303 | 20 |
| 41 | 9·341558 | 9·35 | 9·989271 | ·47 | 9·352287 | 9·82 | 10·647713 | 19 |
| 42 | 342119 | 9·34 | 989243 | ·47 | 352876 | 9·81 | 647124 | 18 |
| 43 | 342679 | 9·32 | 989214 | ·47 | 353465 | 9·80 | 646535 | 17 |
| 44 | 343239 | 9·31 | 989186 | ·47 | 354053 | 9·79 | 645947 | 16 |
| 45 | 343797 | 9·30 | 989157 | ·47 | 354640 | 9·77 | 645360 | 15 |
| 46 | 344355 | 9·29 | 989128 | ·48 | 355227 | 9·76 | 644773 | 14 |
| 47 | 344912 | 9·27 | 989100 | ·48 | 355813 | 9·75 | 644187 | 13 |
| 48 | 345469 | 9·26 | 989071 | ·48 | 356398 | 9·74 | 643602 | 12 |
| 49 | 346024 | 9·25 | 989042 | ·48 | 356982 | 9·73 | 643018 | 11 |
| 50 | 346579 | 9·24 | 989014 | ·48 | 357566 | 9·71 | 642434 | 10 |
| 51 | 9·347134 | 9·22 | 9·988985 | ·48 | 9·358149 | 9·70 | 10·641851 | 9 |
| 52 | 347687 | 9·21 | 988956 | ·48 | 358731 | 9·69 | 641269 | 8 |
| 53 | 348240 | 9·20 | 988927 | ·48 | 359313 | 9·68 | 640687 | 7 |
| 54 | 348792 | 9·19 | 988898 | ·48 | 359893 | 9·67 | 640107 | 6 |
| 55 | 349343 | 9·17 | 988869 | ·48 | 360474 | 9·66 | 639526 | 5 |
| 56 | 349893 | 9·16 | 988840 | ·48 | 361053 | 9·65 | 638947 | 4 |
| 57 | 350443 | 9·15 | 988811 | ·49 | 361632 | 9·63 | 638368 | 3 |
| 58 | 350992 | 9·14 | 988782 | ·49 | 362210 | 9·62 | 637790 | 2 |
| 59 | 351540 | 9·13 | 988753 | ·49 | 362787 | 9·61 | 637213 | 1 |
| 60 | 352088 | 9·11 | 988724 | ·49 | 363364 | 9·60 | 636636 | 0 |
| | Cosine | D. | Sine | 77° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·352088 | 9·11 | 9·988724 | ·49 | 9·363364 | 9·60 | 10·636636 | 60 |
| 1 | 352635 | 9·10 | 988695 | ·49 | 363940 | 9·59 | 636060 | 59 |
| 2 | 353181 | 9·09 | 988666 | ·49 | 364515 | 9·58 | 635485 | 58 |
| 3 | 353726 | 9·08 | 988636 | ·49 | 365090 | 9·57 | 634910 | 57 |
| 4 | 354271 | 9·07 | 988607 | ·49 | 365664 | 9·55 | 634336 | 56 |
| 5 | 354815 | 9·05 | 988578 | ·49 | 366237 | 9·54 | 633763 | 55 |
| 6 | 355358 | 9·04 | 988548 | ·49 | 366810 | 9·53 | 633190 | 54 |
| 7 | 355901 | 9·03 | 988519 | ·49 | 367382 | 9·52 | 632618 | 53 |
| 8 | 356443 | 9·02 | 988489 | ·49 | 367953 | 9·51 | 632047 | 52 |
| 9 | 356984 | 9·01 | 988460 | ·49 | 368524 | 9·50 | 631476 | 51 |
| 10 | 357524 | 8·99 | 988430 | ·49 | 369094 | 9·49 | 630906 | 50 |
| 11 | 9·358064 | 8·98 | 9·988401 | ·49 | 9·369663 | 9·48 | 10·630337 | 49 |
| 12 | 358603 | 8·97 | 988371 | ·49 | 370232 | 9·46 | 629768 | 48 |
| 13 | 359141 | 8·96 | 988342 | ·49 | 370799 | 9·45 | 629201 | 47 |
| 14 | 359678 | 8·95 | 988312 | ·50 | 371367 | 9·44 | 628633 | 46 |
| 15 | 360215 | 8·93 | 988282 | ·50 | 371933 | 9·43 | 628067 | 45 |
| 16 | 360752 | 8·92 | 988252 | ·50 | 372499 | 9·42 | 627501 | 44 |
| 17 | 361287 | 8·91 | 988223 | ·50 | 373064 | 9·41 | 626936 | 43 |
| 18 | 361822 | 8·90 | 988193 | ·50 | 373629 | 9·40 | 626371 | 42 |
| 19 | 362356 | 8·89 | 988163 | ·50 | 374193 | 9·39 | 625807 | 41 |
| 20 | 362889 | 8·88 | 988133 | ·50 | 374756 | 9·38 | 625244 | 40 |
| 21 | 9·363422 | 8·87 | 9·988103 | ·50 | 9·375319 | 9·37 | 10·624681 | 39 |
| 22 | 363954 | 8·85 | 988073 | ·50 | 375881 | 9·35 | 624119 | 38 |
| 23 | 364485 | 8·84 | 988043 | ·50 | 376442 | 9·34 | 623558 | 37 |
| 24 | 365016 | 8·83 | 988013 | ·50 | 377003 | 9·33 | 622997 | 36 |
| 25 | 365546 | 8·82 | 987983 | ·50 | 377563 | 9·32 | 622437 | 35 |
| 26 | 366075 | 8·81 | 987953 | ·50 | 378122 | 9·31 | 621878 | 34 |
| 27 | 366604 | 8·80 | 987922 | ·50 | 378681 | 9·30 | 621319 | 33 |
| 28 | 367131 | 8·79 | 987892 | ·50 | 379239 | 9·29 | 620761 | 32 |
| 29 | 367659 | 8·77 | 987862 | ·50 | 379797 | 9·28 | 620203 | 31 |
| 30 | 368185 | 8·76 | 987832 | ·51 | 380354 | 9·27 | 619646 | 30 |
| 31 | 9·368711 | 8·75 | 9·987801 | ·51 | 9·380910 | 9·26 | 10·619090 | 29 |
| 32 | 369236 | 8·74 | 987771 | ·51 | 381466 | 9·25 | 618534 | 28 |
| 33 | 369761 | 8·73 | 987740 | ·51 | 382020 | 9·24 | 617980 | 27 |
| 34 | 370285 | 8·72 | 987710 | ·51 | 382575 | 9·23 | 617425 | 26 |
| 35 | 370808 | 8·71 | 987679 | ·51 | 383129 | 9·22 | 616871 | 25 |
| 36 | 371330 | 8·70 | 987649 | ·51 | 383682 | 9·21 | 616318 | 24 |
| 37 | 371852 | 8·69 | 987618 | ·51 | 384234 | 9·20 | 615766 | 23 |
| 38 | 372373 | 8·67 | 987588 | ·51 | 384786 | 9·19 | 615214 | 22 |
| 39 | 372894 | 8·66 | 987557 | ·51 | 385337 | 9·18 | 614663 | 21 |
| 40 | 373414 | 8·65 | 987526 | ·51 | 385888 | 9·17 | 614112 | 20 |
| 41 | 9·373933 | 8·64 | 9·987496 | ·51 | 9·386438 | 9·15 | 10·613562 | 19 |
| 42 | 374452 | 8·63 | 987465 | ·51 | 386987 | 9·14 | 613013 | 18 |
| 43 | 374970 | 8·62 | 987434 | ·51 | 387536 | 9·13 | 612464 | 17 |
| 44 | 375487 | 8·61 | 987403 | ·52 | 388084 | 9·12 | 611916 | 16 |
| 45 | 376003 | 8·60 | 987372 | ·52 | 388631 | 9·11 | 611369 | 15 |
| 46 | 376519 | 8·59 | 987341 | ·52 | 389178 | 9·10 | 610822 | 14 |
| 47 | 377035 | 8·58 | 987310 | ·52 | 389724 | 9·09 | 610276 | 13 |
| 48 | 377549 | 8·57 | 987279 | ·52 | 390270 | 9·08 | 609730 | 12 |
| 49 | 378063 | 8·56 | 987248 | ·52 | 390815 | 9·07 | 609185 | 11 |
| 50 | 378577 | 8·54 | 987217 | ·52 | 391360 | 9·06 | 608640 | 10 |
| 51 | 9·379089 | 8·53 | 9·987186 | ·52 | 9·391903 | 9·05 | 10·608097 | 9 |
| 52 | 379601 | 8·52 | 987155 | ·52 | 392447 | 9·04 | 607553 | 8 |
| 53 | 380113 | 8·51 | 987124 | ·52 | 392989 | 9·03 | 607011 | 7 |
| 54 | 380624 | 8·50 | 987092 | ·52 | 393531 | 9·02 | 606469 | 6 |
| 55 | 381134 | 8·49 | 987061 | ·52 | 394073 | 9·01 | 605927 | 5 |
| 56 | 381643 | 8·48 | 987030 | ·52 | 394614 | 9·00 | 605386 | 4 |
| 57 | 382152 | 8·47 | 986998 | ·52 | 395154 | 8·99 | 604846 | 3 |
| 58 | 382661 | 8·46 | 986967 | ·52 | 395694 | 8·98 | 604306 | 2 |
| 59 | 383168 | 8·45 | 986936 | ·52 | 396233 | 8·97 | 603767 | 1 |
| 60 | 383675 | 8·44 | 986904 | ·52 | 396771 | 8·96 | 603229 | 0 |
| | Cosine | D. | Sine | 76° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·383675 | 8·44 | 9·986904 | ·52 | 9·396771 | 8·96 | 10·603229 | 60 |
| 1 | 384182 | 8·43 | 986873 | ·53 | 397309 | 8·96 | 602691 | 59 |
| 2 | 384687 | 8·42 | 986841 | ·53 | 397846 | 8·95 | 602154 | 58 |
| 3 | 385192 | 8·41 | 986809 | ·53 | 398383 | 8·94 | 601617 | 57 |
| 4 | 385697 | 8·40 | 986778 | ·53 | 398919 | 8·93 | 601081 | 56 |
| 5 | 386201 | 8·39 | 986746 | ·53 | 399455 | 8·92 | 600545 | 55 |
| 6 | 386704 | 8·38 | 986714 | ·53 | 399990 | 8·91 | 600010 | 54 |
| 7 | 387207 | 8·37 | 986683 | ·53 | 400524 | 8·90 | 599476 | 53 |
| 8 | 387709 | 8·36 | 986651 | ·53 | 401058 | 8·89 | 598942 | 52 |
| 9 | 388210 | 8·35 | 986619 | ·53 | 401591 | 8·88 | 598409 | 51 |
| 10 | 388711 | 8·34 | 986587 | ·53 | 402124 | 8·87 | 597876 | 50 |
| 11 | 9·389211 | 8·33 | 9·986555 | ·53 | 9·402656 | 8·86 | 10·597344 | 49 |
| 12 | 389711 | 8·32 | 986523 | ·53 | 403187 | 8·85 | 596813 | 48 |
| 13 | 390210 | 8·31 | 986491 | ·53 | 403718 | 8·84 | 596282 | 47 |
| 14 | 390708 | 8·30 | 986459 | ·53 | 404249 | 8·83 | 595751 | 46 |
| 15 | 391206 | 8·28 | 986427 | ·53 | 404778 | 8·82 | 595222 | 45 |
| 16 | 391703 | 8·27 | 986395 | ·53 | 405308 | 8·81 | 594692 | 44 |
| 17 | 392199 | 8·26 | 986363 | ·54 | 405836 | 8·80 | 594164 | 43 |
| 18 | 392695 | 8·25 | 986331 | ·54 | 406364 | 8·79 | 593636 | 42 |
| 19 | 393191 | 8·24 | 986299 | ·54 | 406892 | 8·78 | 593108 | 41 |
| 20 | 393685 | 8·23 | 986266 | ·54 | 407419 | 8·77 | 592581 | 40 |
| 21 | 9·394179 | 8·22 | 9·986234 | ·54 | 9·407945 | 8·76 | 10·592055 | 39 |
| 22 | 394673 | 8·21 | 986202 | ·54 | 408471 | 8·75 | 591529 | 38 |
| 23 | 395166 | 8·20 | 986169 | ·54 | 408997 | 8·74 | 591003 | 37 |
| 24 | 395658 | 8·19 | 986137 | ·54 | 409521 | 8·74 | 590479 | 36 |
| 25 | 396150 | 8·18 | 986104 | ·54 | 410045 | 8·73 | 589955 | 35 |
| 26 | 396641 | 8·17 | 986072 | ·54 | 410569 | 8·72 | 589431 | 34 |
| 27 | 397132 | 8·17 | 986039 | ·54 | 411092 | 8·71 | 588908 | 33 |
| 28 | 397621 | 8·16 | 986007 | ·54 | 411615 | 8·70 | 588385 | 32 |
| 29 | 398111 | 8·15 | 985974 | ·54 | 412137 | 8·69 | 587863 | 31 |
| 30 | 398600 | 8·14 | 985942 | ·54 | 412658 | 8·68 | 587342 | 30 |
| 31 | 9·399088 | 8·13 | 9·985909 | ·55 | 9·413179 | 8·67 | 10·586821 | 29 |
| 32 | 399575 | 8·12 | 985876 | ·55 | 413699 | 8·66 | 586301 | 28 |
| 33 | 400062 | 8·11 | 985843 | ·55 | 414219 | 8·65 | 585781 | 27 |
| 34 | 400549 | 8·10 | 985811 | ·55 | 414738 | 8·64 | 585262 | 26 |
| 35 | 401035 | 8·09 | 985778 | ·55 | 415257 | 8·64 | 584743 | 25 |
| 36 | 401520 | 8·08 | 985745 | ·55 | 415775 | 8·63 | 584225 | 24 |
| 37 | 402005 | 8·07 | 985712 | ·55 | 416293 | 8·62 | 583707 | 23 |
| 38 | 402489 | 8·06 | 985679 | ·55 | 416810 | 8·61 | 583190 | 22 |
| 39 | 402972 | 8·05 | 985646 | ·55 | 417326 | 8·60 | 582674 | 21 |
| 40 | 403455 | 8·04 | 985613 | ·55 | 417842 | 8·59 | 582158 | 20 |
| 41 | 9·403938 | 8·03 | 9·985580 | ·55 | 9·418358 | 8·58 | 10·581642 | 19 |
| 42 | 404420 | 8·02 | 985547 | ·55 | 418873 | 8·57 | 581127 | 18 |
| 43 | 404901 | 8·01 | 985514 | ·55 | 419387 | 8·56 | 580613 | 17 |
| 44 | 405382 | 8·00 | 985480 | ·55 | 419901 | 8·55 | 580099 | 16 |
| 45 | 405862 | 7·99 | 985447 | ·55 | 420415 | 8·55 | 579585 | 15 |
| 46 | 406341 | 7·98 | 985414 | ·56 | 420927 | 8·54 | 579073 | 14 |
| 47 | 406820 | 7·97 | 985380 | ·56 | 421440 | 8·53 | 578560 | 13 |
| 48 | 407299 | 7·96 | 985347 | ·56 | 421952 | 8·52 | 578048 | 12 |
| 49 | 407777 | 7·95 | 985314 | ·56 | 422463 | 8·51 | 577537 | 11 |
| 50 | 408254 | 7·94 | 985280 | ·56 | 422974 | 8·50 | 577026 | 10 |
| 51 | 9·408731 | 7·94 | 9·985247 | ·56 | 9·423484 | 8·49 | 10·576516 | 9 |
| 52 | 409207 | 7·93 | 985213 | ·56 | 423993 | 8·48 | 576007 | 8 |
| 53 | 409682 | 7·92 | 985180 | ·56 | 424503 | 8·48 | 575497 | 7 |
| 54 | 410157 | 7·91 | 985146 | ·56 | 425011 | 8·47 | 574989 | 6 |
| 55 | 410632 | 7·90 | 985113 | ·56 | 425519 | 8·46 | 574481 | 5 |
| 56 | 411106 | 7·89 | 985079 | ·56 | 426027 | 8·45 | 573973 | 4 |
| 57 | 411579 | 7·88 | 985045 | ·56 | 426534 | 8·44 | 573466 | 3 |
| 58 | 412052 | 7·87 | 985011 | ·56 | 427041 | 8·43 | 572959 | 2 |
| 59 | 412524 | 7·86 | 984978 | ·56 | 427547 | 8·43 | 572453 | 1 |
| 60 | 412996 | 7·85 | 984944 | 56 | 428052 | 8·42 | 571948 | 0 |
| | Cosine | D. | Sine | 75° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·412996 | 7·85 | 9·984944 | ·57 | 9·428052 | 8·42 | 10·571948 | 60 |
| 1 | 413467 | 7·84 | 984910 | ·57 | 428557 | 8·41 | 571443 | 59 |
| 2 | 413938 | 7·83 | 984876 | ·57 | 429062 | 8·40 | 570938 | 58 |
| 3 | 414408 | 7·83 | 984842 | ·57 | 429566 | 8·39 | 570434 | 57 |
| 4 | 414878 | 7·82 | 984808 | ·57 | 430070 | 8·38 | 569930 | 56 |
| 5 | 415347 | 7·81 | 984774 | ·57 | 430573 | 8·38 | 569427 | 55 |
| 6 | 415815 | 7·80 | 984740 | ·57 | 431075 | 8·37 | 568925 | 54 |
| 7 | 416283 | 7·79 | 984706 | ·57 | 431577 | 8·36 | 568423 | 53 |
| 8 | 416751 | 7·78 | 984672 | ·57 | 432079 | 8·35 | 567921 | 52 |
| 9 | 417217 | 7·77 | 984637 | ·57 | 432580 | 8·34 | 567420 | 51 |
| 10 | 417684 | 7·76 | 984603 | ·57 | 433080 | 8·33 | 566920 | 50 |
| 11 | 9·418150 | 7·75 | 9·984569 | ·57 | 9·433580 | 8·32 | 10·566420 | 49 |
| 12 | 418615 | 7·74 | 984535 | ·57 | 434080 | 8·32 | 565920 | 48 |
| 13 | 419079 | 7·73 | 984500 | ·57 | 434579 | 8·31 | 565421 | 47 |
| 14 | 419544 | 7·73 | 984466 | ·57 | 435078 | 8·30 | 564922 | 46 |
| 15 | 420007 | 7·72 | 984432 | ·58 | 435576 | 8·29 | 564424 | 45 |
| 16 | 420470 | 7·71 | 984397 | ·58 | 436073 | 8·28 | 563927 | 44 |
| 17 | 420933 | 7·70 | 984363 | ·58 | 436570 | 8·28 | 563430 | 43 |
| 18 | 421395 | 7·69 | 984328 | ·58 | 437067 | 8·27 | 562933 | 42 |
| 19 | 421857 | 7·68 | 984294 | ·58 | 437563 | 8·26 | 562437 | 41 |
| 20 | 422318 | 7·67 | 984259 | ·58 | 438059 | 8·25 | 561941 | 40 |
| 21 | 9·422778 | 7·67 | 9·984224 | ·58 | 9·438554 | 8·24 | 10·561446 | 39 |
| 22 | 423238 | 7·66 | 984190 | ·58 | 439048 | 8·23 | 560952 | 38 |
| 23 | 423697 | 7·65 | 984155 | ·58 | 439543 | 8·23 | 560457 | 37 |
| 24 | 424156 | 7·64 | 984120 | ·58 | 440036 | 8·22 | 559964 | 36 |
| 25 | 424615 | 7·63 | 984085 | ·58 | 440529 | 8·21 | 559471 | 35 |
| 26 | 425073 | 7·62 | 984050 | ·58 | 441022 | 8·20 | 558978 | 34 |
| 27 | 425530 | 7·61 | 984015 | ·58 | 441514 | 8·19 | 558486 | 33 |
| 28 | 425987 | 7·60 | 983981 | ·58 | 442006 | 8·19 | 557994 | 32 |
| 29 | 426443 | 7·60 | 983946 | ·58 | 442497 | 8·18 | 557503 | 31 |
| 30 | 426899 | 7·59 | 983911 | ·58 | 442988 | 8·17 | 557012 | 30 |
| 31 | 9·427354 | 7·58 | 9·983875 | ·58 | 9·443479 | 8·16 | 10·556521 | 29 |
| 32 | 427809 | 7·57 | 983840 | ·59 | 443968 | 8·16 | 556032 | 28 |
| 33 | 428263 | 7·56 | 983805 | ·59 | 444458 | 8·15 | 555542 | 27 |
| 34 | 428717 | 7·55 | 983770 | ·59 | 444947 | 8·14 | 555053 | 26 |
| 35 | 429170 | 7·54 | 983735 | ·59 | 445435 | 8·13 | 554565 | 25 |
| 36 | 429623 | 7·53 | 983700 | ·59 | 445923 | 8·12 | 554077 | 24 |
| 37 | 430075 | 7·52 | 983664 | ·59 | 446411 | 8·12 | 553589 | 23 |
| 38 | 430527 | 7·52 | 983629 | ·59 | 446898 | 8·11 | 553102 | 22 |
| 39 | 430978 | 7·51 | 983594 | ·59 | 447384 | 8·10 | 552616 | 21 |
| 40 | 431429 | 7·50 | 983558 | ·59 | 447870 | 8·09 | 552130 | 20 |
| 41 | 9·431879 | 7·49 | 9·983523 | ·59 | 9·448356 | 8·09 | 10·551644 | 19 |
| 42 | 432329 | 7·49 | 983487 | ·59 | 448841 | 8·08 | 551159 | 18 |
| 43 | 432778 | 7·48 | 983452 | ·59 | 449326 | 8·07 | 550674 | 17 |
| 44 | 433226 | 7·47 | 983416 | ·59 | 449810 | 8·06 | 550190 | 16 |
| 45 | 433675 | 7·46 | 983381 | ·59 | 450294 | 8·06 | 549706 | 15 |
| 46 | 434122 | 7·45 | 983345 | ·59 | 450777 | 8·05 | 549223 | 14 |
| 47 | 434569 | 7·44 | 983309 | ·59 | 451260 | 8·04 | 548740 | 13 |
| 48 | 435016 | 7·44 | 983273 | ·60 | 451743 | 8·03 | 548257 | 12 |
| 49 | 435462 | 7·43 | 983238 | ·60 | 452225 | 8·02 | 547775 | 11 |
| 50 | 435908 | 7·42 | 983202 | ·60 | 452706 | 8·02 | 547294 | 10 |
| 51 | 9·436353 | 7·41 | 9·983166 | ·60 | 9·453187 | 8·01 | 10·546813 | 9 |
| 52 | 436798 | 7·40 | 983130 | ·60 | 453668 | 8·00 | 546332 | 8 |
| 53 | 437242 | 7·40 | 983094 | ·60 | 454148 | 7·99 | 545852 | 7 |
| 54 | 437686 | 7·39 | 983058 | ·60 | 454628 | 7·99 | 545372 | 6 |
| 55 | 438129 | 7·38 | 983022 | ·60 | 455107 | 7·98 | 544893 | 5 |
| 56 | 438572 | 7·37 | 982986 | ·60 | 455586 | 7·97 | 544414 | 4 |
| 57 | 439014 | 7·36 | 982950 | ·60 | 456064 | 7·96 | 543936 | 3 |
| 58 | 439456 | 7·36 | 982914 | ·60 | 456542 | 7·96 | 543458 | 2 |
| 59 | 439897 | 7·35 | 982878 | ·60 | 457019 | 7·95 | 542981 | 1 |
| 60 | 440338 | 7·34 | 982842 | ·60 | 457496 | 7·94 | 542504 | 0 |
| | Cosine | D. | Sine | 74° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·440338 | 7·34 | 9·982842 | ·60 | 9·457496 | 7·94 | 10·542504 | 60 |
| 1 | 440778 | 7·33 | 982805 | ·60 | 457973 | 7·93 | 542027 | 59 |
| 2 | 441218 | 7·32 | 982769 | ·61 | 458449 | 7·93 | 541551 | 58 |
| 3 | 441658 | 7·31 | 982733 | ·61 | 458925 | 7·92 | 541075 | 57 |
| 4 | 442096 | 7·31 | 982696 | ·61 | 459400 | 7·91 | 540600 | 56 |
| 5 | 442535 | 7·30 | 982660 | ·61 | 459875 | 7·90 | 540125 | 55 |
| 6 | 442973 | 7·29 | 982624 | ·61 | 460349 | 7·90 | 539651 | 54 |
| 7 | 443410 | 7·28 | 982587 | ·61 | 460823 | 7·89 | 539177 | 53 |
| 8 | 443847 | 7·27 | 982551 | ·61 | 461297 | 7·88 | 538703 | 52 |
| 9 | 444284 | 7·27 | 982514 | ·61 | 461770 | 7·88 | 538230 | 51 |
| 10 | 444720 | 7·26 | 982477 | ·61 | 462242 | 7·87 | 537758 | 50 |
| 11 | 9·445155 | 7·25 | 9·982441 | ·61 | 9·462714 | 7·86 | 10·537286 | 49 |
| 12 | 445590 | 7·24 | 982404 | ·61 | 463186 | 7·85 | 536814 | 48 |
| 13 | 446025 | 7·23 | 982367 | ·61 | 463658 | 7·85 | 536342 | 47 |
| 14 | 446459 | 7·23 | 982331 | ·61 | 464129 | 7·84 | 535871 | 46 |
| 15 | 446893 | 7·22 | 982294 | ·61 | 464599 | 7·83 | 535401 | 45 |
| 16 | 447326 | 7·21 | 982257 | ·61 | 465069 | 7·83 | 534931 | 44 |
| 17 | 447759 | 7·20 | 982220 | ·62 | 465539 | 7·82 | 534461 | 43 |
| 18 | 448191 | 7·20 | 982183 | ·62 | 466008 | 7·81 | 533992 | 42 |
| 19 | 448623 | 7·19 | 982146 | ·62 | 466476 | 7·80 | 533524 | 41 |
| 20 | 449054 | 7·18 | 982109 | ·62 | 466945 | 7·80 | 533055 | 40 |
| 21 | 9·449485 | 7·17 | 9·982072 | ·62 | 9·467413 | 7·79 | 10·532587 | 39 |
| 22 | 449915 | 7·16 | 982035 | ·62 | 467880 | 7·78 | 532120 | 38 |
| 23 | 450345 | 7·16 | 981998 | ·62 | 468347 | 7·78 | 531653 | 37 |
| 24 | 450775 | 7·15 | 981961 | ·62 | 468814 | 7·77 | 531186 | 36 |
| 25 | 451204 | 7·14 | 981924 | ·62 | 469280 | 7·76 | 530720 | 35 |
| 26 | 451632 | 7·13 | 981886 | ·62 | 469746 | 7·75 | 530254 | 34 |
| 27 | 452060 | 7·13 | 981849 | ·62 | 470211 | 7·75 | 529789 | 33 |
| 28 | 452488 | 7·12 | 981812 | ·62 | 470676 | 7·74 | 529324 | 32 |
| 29 | 452915 | 7·11 | 981774 | ·62 | 471141 | 7·73 | 528859 | 31 |
| 30 | 453342 | 7·10 | 981737 | ·62 | 471605 | 7·73 | 528395 | 30 |
| 31 | 9·453768 | 7·10 | 9·981699 | ·63 | 9·472068 | 7·72 | 10·527932 | 29 |
| 32 | 454194 | 7·09 | 981662 | ·63 | 472532 | 7·71 | 527468 | 28 |
| 33 | 454619 | 7·08 | 981625 | ·63 | 472995 | 7·71 | 527005 | 27 |
| 34 | 455044 | 7·07 | 981587 | ·63 | 473457 | 7·70 | 526543 | 26 |
| 35 | 455469 | 7·07 | 981549 | ·63 | 473919 | 7·69 | 526081 | 25 |
| 36 | 455893 | 7·06 | 981512 | ·63 | 474381 | 7·69 | 525619 | 24 |
| 37 | 456316 | 7·05 | 981474 | ·63 | 474842 | 7·68 | 525158 | 23 |
| 38 | 456739 | 7·04 | 981436 | ·63 | 475303 | 7·67 | 524697 | 22 |
| 39 | 457162 | 7·04 | 981399 | ·63 | 475763 | 7·67 | 524237 | 21 |
| 40 | 457584 | 7·03 | 981361 | ·63 | 476223 | 7·66 | 523777 | 20 |
| 41 | 9·458006 | 7·02 | 9·981323 | ·63 | 9·476683 | 7·65 | 10·523317 | 19 |
| 42 | 458427 | 7·01 | 981285 | ·63 | 477142 | 7·65 | 522858 | 18 |
| 43 | 458848 | 7·01 | 981247 | ·63 | 477601 | 7·64 | 522399 | 17 |
| 44 | 459268 | 7·00 | 981209 | ·63 | 478059 | 7·63 | 521941 | 16 |
| 45 | 459688 | 6·99 | 981171 | ·63 | 478517 | 7·63 | 521483 | 15 |
| 46 | 460108 | 6·98 | 981133 | ·64 | 478975 | 7·62 | 521025 | 14 |
| 47 | 460527 | 6·98 | 981095 | ·64 | 479432 | 7·61 | 520568 | 13 |
| 48 | 460946 | 6·97 | 981057 | ·64 | 479889 | 7·61 | 520111 | 12 |
| 49 | 461364 | 6·96 | 981019 | ·64 | 480345 | 7·60 | 519655 | 11 |
| 50 | 461782 | 6·95 | 980981 | ·64 | 480801 | 7·59 | 519199 | 10 |
| 51 | 9·462199 | 6·95 | 9·980942 | ·64 | 9·481257 | 7·59 | 10·518743 | 9 |
| 52 | 462616 | 6·94 | 980904 | ·64 | 481712 | 7·58 | 518288 | 8 |
| 53 | 463032 | 6·93 | 980866 | ·64 | 482167 | 7·57 | 517833 | 7 |
| 54 | 463448 | 6·93 | 980827 | ·64 | 482621 | 7·57 | 517379 | 6 |
| 55 | 463864 | 6·92 | 980789 | ·64 | 483075 | 7·56 | 516925 | 5 |
| 56 | 464279 | 6·91 | 980750 | ·64 | 483529 | 7·55 | 516471 | 4 |
| 57 | 464694 | 6·90 | 980712 | ·64 | 483982 | 7·55 | 516018 | 3 |
| 58 | 465108 | 6·90 | 980673 | ·64 | 484435 | 7·54 | 515565 | 2 |
| 59 | 465522 | 6·89 | 980635 | ·64 | 484887 | 7·53 | 515113 | 1 |
| 60 | 465935 | 6·88 | 980596 | ·64 | 485339 | 7·53 | 514661 | 0 |
| | Cosine | D. | Sine | 73° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·465935 | 6·88 | 9·980596 | ·64 | 9·485339 | 7·55 | 10·514661 | 60 |
| 1 | 466348 | 6·88 | 980558 | ·64 | 485791 | 7·52 | 514209 | 59 |
| 2 | 466761 | 6·87 | 980519 | ·65 | 486242 | 7·51 | 513758 | 58 |
| 3 | 467173 | 6·86 | 980480 | ·65 | 486693 | 7·51 | 513307 | 57 |
| 4 | 467585 | 6·85 | 980442 | ·65 | 487143 | 7·50 | 512857 | 56 |
| 5 | 467996 | 6·85 | 980403 | ·65 | 487593 | 7·49 | 512407 | 55 |
| 6 | 468407 | 6·84 | 980364 | ·65 | 488043 | 7·49 | 511957 | 54 |
| 7 | 468817 | 6·83 | 980325 | ·65 | 488492 | 7·48 | 511508 | 53 |
| 8 | 469227 | 6·83 | 980286 | ·65 | 488941 | 7·47 | 511059 | 52 |
| 9 | 469637 | 6·82 | 980247 | ·65 | 489390 | 7·47 | 510610 | 51 |
| 10 | 470046 | 6·81 | 980208 | ·65 | 489838 | 7·46 | 510162 | 50 |
| 11 | 9·470455 | 6·80 | 9·980169 | ·65 | 9·490286 | 7·46 | 10·509714 | 49 |
| 12 | 470863 | 6·80 | 980130 | ·65 | 490733 | 7·45 | 509267 | 48 |
| 13 | 471271 | 6·79 | 980091 | ·65 | 491180 | 7·44 | 508820 | 47 |
| 14 | 471679 | 6·78 | 980052 | ·65 | 491627 | 7·44 | 508373 | 46 |
| 15 | 472086 | 6·78 | 980012 | ·65 | 492073 | 7·43 | 507927 | 45 |
| 16 | 472492 | 6·77 | 979973 | ·65 | 492519 | 7·43 | 507481 | 44 |
| 17 | 472898 | 6·76 | 979934 | ·66 | 492965 | 7·42 | 507035 | 43 |
| 18 | 473304 | 6·76 | 979895 | ·66 | 493410 | 7·41 | 506590 | 42 |
| 19 | 473710 | 6·75 | 979855 | ·66 | 493854 | 7·40 | 506146 | 41 |
| 20 | 474115 | 6·74 | 979816 | ·66 | 494299 | 7·40 | 505701 | 40 |
| 21 | 9·474519 | 6·74 | 9·979776 | ·66 | 9·494743 | 7·40 | 10·505257 | 39 |
| 22 | 474923 | 6·73 | 979737 | ·66 | 495186 | 7·39 | 504814 | 38 |
| 23 | 475327 | 6·72 | 979697 | ·66 | 495630 | 7·38 | 504370 | 37 |
| 24 | 475730 | 6·72 | 979658 | ·66 | 496073 | 7·37 | 503927 | 36 |
| 25 | 476133 | 6·71 | 979618 | ·66 | 496515 | 7·37 | 503485 | 35 |
| 26 | 476536 | 6·70 | 979579 | ·66 | 496957 | 7·36 | 503043 | 34 |
| 27 | 476938 | 6·69 | 979539 | ·66 | 497399 | 7·36 | 502601 | 33 |
| 28 | 477340 | 6·69 | 979499 | ·66 | 497841 | 7·35 | 502159 | 32 |
| 29 | 477741 | 6·68 | 979459 | ·66 | 498282 | 7·34 | 501718 | 31 |
| 30 | 478142 | 6·67 | 979420 | ·66 | 498722 | 7·34 | 501278 | 30 |
| 31 | 9·478542 | 6·67 | 9·979380 | ·66 | 9·499163 | 7·33 | 10·500837 | 29 |
| 32 | 478942 | 6·66 | 979340 | ·66 | 499603 | 7·33 | 500397 | 28 |
| 33 | 479342 | 6·65 | 979300 | ·67 | 500042 | 7·32 | 499958 | 27 |
| 34 | 479741 | 6·65 | 979260 | ·67 | 500481 | 7·31 | 499519 | 26 |
| 35 | 480140 | 6·64 | 979220 | ·67 | 500920 | 7·31 | 499080 | 25 |
| 36 | 480539 | 6·63 | 979180 | ·67 | 501359 | 7·30 | 498641 | 24 |
| 37 | 480937 | 6·63 | 979140 | ·67 | 501797 | 7·30 | 498203 | 23 |
| 38 | 481334 | 6·62 | 979100 | ·67 | 502235 | 7·29 | 497765 | 22 |
| 39 | 481731 | 6·61 | 979059 | ·67 | 502672 | 7·28 | 497328 | 21 |
| 40 | 482128 | 6·61 | 979019 | ·67 | 503109 | 7·28 | 496891 | 20 |
| 41 | 9·482525 | 6·60 | 9·978979 | ·67 | 9·503546 | 7·27 | 10·496454 | 19 |
| 42 | 482921 | 6·59 | 978939 | ·67 | 503982 | 7·27 | 496018 | 18 |
| 43 | 483316 | 6·59 | 978898 | ·67 | 504418 | 7·26 | 495582 | 17 |
| 44 | 483712 | 6·58 | 978858 | ·67 | 504854 | 7·25 | 495146 | 16 |
| 45 | 484107 | 6·57 | 978817 | ·67 | 505289 | 7·25 | 494711 | 15 |
| 46 | 484501 | 6·57 | 978777 | ·67 | 505724 | 7·24 | 494276 | 14 |
| 47 | 484895 | 6·56 | 978736 | ·67 | 506159 | 7·24 | 493841 | 13 |
| 48 | 485289 | 6·55 | 978696 | ·68 | 506593 | 7·23 | 493407 | 12 |
| 49 | 485682 | 6·55 | 978655 | ·68 | 507027 | 7·22 | 492973 | 11 |
| 50 | 486075 | 6·54 | 978615 | ·68 | 507460 | 7·22 | 492540 | 10 |
| 51 | 9·486467 | 6·53 | 9·978574 | ·68 | 9·507893 | 7·21 | 10·492107 | 9 |
| 52 | 486860 | 6·53 | 978533 | ·68 | 508326 | 7·21 | 491674 | 8 |
| 53 | 487251 | 6·52 | 978493 | ·68 | 508759 | 7·20 | 491241 | 7 |
| 54 | 487643 | 6·51 | 978452 | ·68 | 509191 | 7·19 | 490809 | 6 |
| 55 | 488034 | 6·51 | 978411 | ·68 | 509622 | 7·19 | 490378 | 5 |
| 56 | 488424 | 6·50 | 978370 | ·68 | 510054 | 7·18 | 489946 | 4 |
| 57 | 488814 | 6·50 | 978329 | ·68 | 510485 | 7·18 | 489515 | 3 |
| 58 | 489204 | 6·49 | 978288 | ·68 | 510916 | 7·17 | 489084 | 2 |
| 59 | 489593 | 6·48 | 978247 | ·68 | 511346 | 7·16 | 488654 | 1 |
| 60 | 489982 | 6·48 | 978206 | ·68 | 511776 | 7·16 | 488224 | 0 |
| | Cosine | D. | Sine | 72° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·489982 | 6·48 | 9·978206 | ·68 | 9·511776 | 7·16 | 10·488224 | 60 |
| 1 | 490371 | 6·48 | 978165 | ·68 | 512206 | 7·16 | 487794 | 59 |
| 2 | 490759 | 6·47 | 978124 | ·68 | 512635 | 7·15 | 487365 | 58 |
| 3 | 491147 | 6·46 | 978083 | ·69 | 513064 | 7·14 | 486936 | 57 |
| 4 | 491535 | 6·46 | 978042 | ·69 | 513493 | 7·14 | 486507 | 56 |
| 5 | 491922 | 6·45 | 978001 | ·69 | 513921 | 7·13 | 486079 | 55 |
| 6 | 492308 | 6·44 | 977959 | ·69 | 514349 | 7·13 | 485651 | 54 |
| 7 | 492695 | 6·44 | 977918 | ·69 | 514777 | 7·12 | 485223 | 53 |
| 8 | 493081 | 6·43 | 977877 | ·69 | 515204 | 7·12 | 484796 | 52 |
| 9 | 493466 | 6·42 | 977835 | ·69 | 515631 | 7·11 | 484369 | 51 |
| 10 | 493851 | 6·42 | 977794 | ·69 | 516057 | 7·10 | 483943 | 50 |
| 11 | 9·494236 | 6·41 | 9·977752 | ·69 | 9·516484 | 7·10 | 10·483516 | 49 |
| 12 | 494621 | 6·41 | 977711 | ·69 | 516910 | 7·09 | 483090 | 48 |
| 13 | 495005 | 6·40 | 977669 | ·69 | 517335 | 7·09 | 482665 | 47 |
| 14 | 495388 | 6·39 | 977628 | ·69 | 517761 | 7·08 | 482239 | 46 |
| 15 | 495772 | 6·39 | 977586 | ·69 | 518185 | 7·08 | 481815 | 45 |
| 16 | 496154 | 6·38 | 977544 | ·70 | 518610 | 7·07 | 481390 | 44 |
| 17 | 496537 | 6·37 | 977503 | ·70 | 519034 | 7·06 | 480966 | 43 |
| 18 | 496919 | 6·37 | 977461 | ·70 | 519458 | 7·06 | 480542 | 42 |
| 19 | 497301 | 6·36 | 977419 | ·70 | 519882 | 7·05 | 480118 | 41 |
| 20 | 497682 | 6·36 | 977377 | ·70 | 520305 | 7·05 | 479695 | 40 |
| 21 | 9·498064 | 6·35 | 9·977335 | ·70 | 9·520728 | 7·04 | 10·479272 | 39 |
| 22 | 498444 | 6·34 | 977293 | ·70 | 521151 | 7·03 | 478849 | 38 |
| 23 | 498825 | 6·34 | 977251 | ·70 | 521573 | 7·03 | 478427 | 37 |
| 24 | 499204 | 6·33 | 977209 | ·70 | 521995 | 7·03 | 478005 | 36 |
| 25 | 499584 | 6·32 | 977167 | ·70 | 522417 | 7·02 | 477583 | 35 |
| 26 | 499963 | 6·32 | 977125 | ·70 | 522838 | 7·02 | 477162 | 34 |
| 27 | 500342 | 6·31 | 977083 | ·70 | 523259 | 7·01 | 476741 | 33 |
| 28 | 500721 | 6·31 | 977041 | ·70 | 523680 | 7·01 | 476320 | 32 |
| 29 | 501099 | 6·30 | 976999 | ·70 | 524100 | 7·00 | 475900 | 31 |
| 30 | 501476 | 6·29 | 976957 | ·70 | 524520 | 6·99 | 475480 | 30 |
| 31 | 9·501854 | 6·29 | 9·976914 | ·70 | 9·524939 | 6·99 | 10·475061 | 29 |
| 32 | 502231 | 6·28 | 976872 | ·71 | 525359 | 6·98 | 474641 | 28 |
| 33 | 502607 | 6·28 | 976830 | ·71 | 525778 | 6·98 | 474222 | 27 |
| 34 | 502984 | 6·27 | 976787 | ·71 | 526197 | 6·97 | 473803 | 26 |
| 35 | 503360 | 6·26 | 976745 | ·71 | 526615 | 6·97 | 473385 | 25 |
| 36 | 503735 | 6·26 | 976702 | ·71 | 527033 | 6·96 | 472967 | 24 |
| 37 | 504110 | 6·25 | 976660 | ·71 | 527451 | 6·96 | 472549 | 23 |
| 38 | 504485 | 6·25 | 976617 | ·71 | 527868 | 6·95 | 472132 | 22 |
| 39 | 504860 | 6·24 | 976574 | ·71 | 528285 | 6·95 | 471715 | 21 |
| 40 | 505234 | 6·23 | 976532 | ·71 | 528702 | 6·94 | 471298 | 20 |
| 41 | 9·505608 | 6·23 | 9·976489 | ·71 | 9·529119 | 6·93 | 10·470881 | 19 |
| 42 | 505981 | 6·22 | 976446 | ·71 | 529535 | 6·93 | 470465 | 18 |
| 43 | 506354 | 6·22 | 976404 | ·71 | 529950 | 6·93 | 470050 | 17 |
| 44 | 506727 | 6·21 | 976361 | ·71 | 530366 | 6·92 | 469634 | 16 |
| 45 | 507099 | 6·20 | 676318 | ·71 | 530781 | 6·91 | 469219 | 15 |
| 46 | 507471 | 6·20 | 976275 | ·71 | 531196 | 6·91 | 468804 | 14 |
| 47 | 507843 | 6·19 | 976232 | ·72 | 531611 | 6·90 | 468389 | 13 |
| 48 | 508214 | 6·19 | 976189 | ·72 | 532025 | 6·90 | 467975 | 12 |
| 49 | 508585 | 6·18 | 976146 | ·72 | 532439 | 6·89 | 467561 | 11 |
| 50 | 508956 | 6·18 | 976103 | ·72 | 532853 | 6·89 | 467147 | 10 |
| 51 | 9·509326 | 6·17 | 9·976060 | ·72 | 9·533266 | 6·88 | 10·466734 | 9 |
| 52 | 509696 | 6·16 | 976017 | ·72 | 533679 | 6·88 | 466321 | 8 |
| 53 | 510065 | 6·16 | 975974 | ·72 | 534092 | 6·87 | 465908 | 7 |
| 54 | 510434 | 6·15 | 975930 | ·72 | 534504 | 6·87 | 465496 | 6 |
| 55 | 510803 | 6·15 | 975887 | ·72 | 534916 | 6·86 | 465084 | 5 |
| 56 | 511172 | 6·14 | 975844 | ·72 | 535328 | 6·86 | 464672 | 4 |
| 57 | 511540 | 6·13 | 975800 | ·72 | 535739 | 6·85 | 464261 | 3 |
| 58 | 511907 | 6·13 | 975757 | ·72 | 536150 | 6·85 | 463850 | 2 |
| 59 | 512275 | 6·12 | 975714 | ·72 | 536561 | 6·84 | 463439 | 1 |
| 60 | 512642 | 6·12 | 975670 | ·72 | 536972 | 6·84 | 463028 | 0 |
| | Cosine | D. | Sine | 71° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·512642 | 6·12 | 9·975670 | ·73 | 9·536972 | 6·84 | 10·463028 | 60 |
| 1 | 513009 | 6·11 | 975627 | ·73 | 537382 | 6·83 | 462618 | 59 |
| 2 | 513375 | 6·11 | 975583 | ·73 | 537792 | 6·83 | 462208 | 58 |
| 3 | 513741 | 6·10 | 975539 | ·73 | 538202 | 6·82 | 461798 | 57 |
| 4 | 514107 | 6·09 | 975496 | ·73 | 538611 | 6·82 | 461389 | 56 |
| 5 | 514472 | 6·09 | 975452 | ·73 | 539020 | 6·81 | 460980 | 55 |
| 6 | 514837 | 6·08 | 975408 | ·73 | 539429 | 6·81 | 460571 | 54 |
| 7 | 515202 | 6·08 | 975365 | ·73 | 539837 | 6·80 | 460163 | 53 |
| 8 | 515566 | 6·07 | 975321 | ·73 | 540245 | 6·80 | 459755 | 52 |
| 9 | 515930 | 6·07 | 975277 | ·73 | 540653 | 6·79 | 459347 | 51 |
| 10 | 516294 | 6·06 | 975233 | ·73 | 541061 | 6·79 | 458939 | 50 |
| 11 | 9·516657 | 6·05 | 9·975189 | ·73 | 9·541468 | 6·78 | 10·458532 | 49 |
| 12 | 517020 | 6·05 | 975145 | ·73 | 541875 | 6·78 | 458125 | 48 |
| 13 | 517382 | 6·04 | 975101 | ·73 | 542281 | 6·77 | 457719 | 47 |
| 14 | 517745 | 6·04 | 975057 | ·73 | 542688 | 6·77 | 457312 | 46 |
| 15 | 518107 | 6·03 | 975013 | ·73 | 543094 | 6·76 | 456906 | 45 |
| 16 | 518468 | 6·03 | 974969 | ·74 | 543499 | 6·76 | 456501 | 44 |
| 17 | 518829 | 6·02 | 974925 | ·74 | 543905 | 6·75 | 456095 | 43 |
| 18 | 519190 | 6·01 | 974880 | ·74 | 544310 | 6·75 | 455690 | 42 |
| 19 | 519551 | 6·01 | 974836 | ·74 | 544715 | 6·74 | 455285 | 41 |
| 20 | 519911 | 6·00 | 974792 | ·74 | 545119 | 6·74 | 454881 | 40 |
| 21 | 9·520271 | 6·00 | 9·974748 | ·74 | 9·545524 | 6·73 | 10·454476 | 39 |
| 22 | 520631 | 5·99 | 974703 | ·74 | 545928 | 6·73 | 454072 | 38 |
| 23 | 520990 | 5·99 | 974659 | ·74 | 546331 | 6·72 | 453669 | 37 |
| 24 | 521349 | 5·98 | 974614 | ·74 | 546735 | 6·72 | 453265 | 36 |
| 25 | 521707 | 5·98 | 974570 | ·74 | 547138 | 6·71 | 452862 | 35 |
| 26 | 522066 | 5·97 | 974525 | ·74 | 547540 | 6·71 | 452460 | 34 |
| 27 | 522424 | 5·96 | 974481 | ·74 | 547943 | 6·70 | 452057 | 33 |
| 28 | 522781 | 5·96 | 974436 | ·74 | 548345 | 6·70 | 451655 | 32 |
| 29 | 523138 | 5·95 | 974391 | ·74 | 548747 | 6·69 | 451253 | 31 |
| 30 | 523495 | 5·95 | 974347 | ·75 | 549149 | 6·69 | 450851 | 30 |
| 31 | 9·523852 | 5·94 | 9·974302 | ·75 | 9·549550 | 6·68 | 10·450450 | 29 |
| 32 | 524208 | 5·94 | 974257 | ·75 | 549951 | 6·68 | 450049 | 28 |
| 33 | 524564 | 5·93 | 974212 | ·75 | 550352 | 6·67 | 449648 | 27 |
| 34 | 524920 | 5·93 | 974167 | ·75 | 550752 | 6·67 | 449248 | 26 |
| 35 | 525275 | 5·92 | 974122 | ·75 | 551152 | 6·66 | 448848 | 25 |
| 36 | 525630 | 5·91 | 974077 | ·75 | 551552 | 6·66 | 448448 | 24 |
| 37 | 525984 | 5·91 | 974032 | ·75 | 551952 | 6·65 | 448048 | 23 |
| 38 | 526339 | 5·90 | 973987 | ·75 | 552351 | 6·65 | 447649 | 22 |
| 39 | 526693 | 5·90 | 973942 | ·75 | 552750 | 6·65 | 447250 | 21 |
| 40 | 527046 | 5·89 | 973897 | ·75 | 553149 | 6·64 | 446851 | 20 |
| 41 | 9·527400 | 5·89 | 9·973852 | ·75 | 9·553548 | 6·64 | 10·446452 | 19 |
| 42 | 527753 | 5·88 | 973807 | ·75 | 553946 | 6·63 | 446054 | 18 |
| 43 | 528105 | 5·88 | 973761 | ·75 | 554344 | 6·63 | 445656 | 17 |
| 44 | 528458 | 5·87 | 973716 | ·76 | 554741 | 6·62 | 445259 | 16 |
| 45 | 528810 | 5·87 | 973671 | ·76 | 555139 | 6·62 | 444861 | 15 |
| 46 | 529161 | 5·86 | 973625 | ·76 | 555536 | 6·61 | 444464 | 14 |
| 47 | 529513 | 5·86 | 973580 | ·76 | 555933 | 6·61 | 444067 | 13 |
| 48 | 529864 | 5·85 | 973535 | ·76 | 556329 | 6·60 | 443671 | 12 |
| 49 | 530215 | 5·85 | 973489 | ·76 | 556725 | 6·60 | 443275 | 11 |
| 50 | 530565 | 5·84 | 973444 | ·76 | 557121 | 6·59 | 442879 | 10 |
| 51 | 9·530915 | 5·84 | 9·973398 | ·76 | 9·557517 | 6·59 | 10·442483 | 9 |
| 52 | 531265 | 5·83 | 973352 | ·76 | 557913 | 6·59 | 442087 | 8 |
| 53 | 531614 | 5·82 | 973307 | ·76 | 558308 | 6·58 | 441692 | 7 |
| 54 | 531963 | 5·82 | 973261 | ·76 | 558702 | 6·58 | 441298 | 6 |
| 55 | 532312 | 5·81 | 973215 | ·76 | 559097 | 6·57 | 440903 | 5 |
| 56 | 532661 | 5·81 | 973169 | ·76 | 559491 | 6·57 | 440509 | 4 |
| 57 | 533009 | 5·80 | 973124 | ·76 | 559885 | 6·56 | 440115 | 3 |
| 58 | 533357 | 5·80 | 973078 | ·76 | 560279 | 6·56 | 439721 | 2 |
| 59 | 533704 | 5·79 | 973032 | ·77 | 560673 | 6·55 | 439327 | 1 |
| 60 | 534052 | 5·78 | 972986 | ·77 | 561066 | 6·55 | 438934 | 0 |
| | Cosine | D. | Sine | 70° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·534052 | 5·78 | 9·972986 | ·77 | 9·561066 | 6·55 | 10·438934 | 60 |
| 1 | 534399 | 5·77 | 972940 | ·77 | 561459 | 6·54 | 438541 | 59 |
| 2 | 534745 | 5·77 | 972894 | ·77 | 561851 | 6·54 | 438149 | 58 |
| 3 | 535092 | 5·77 | 972848 | ·77 | 562244 | 6·53 | 437756 | 57 |
| 4 | 535438 | 5·76 | 972802 | ·77 | 562636 | 6·53 | 437364 | 56 |
| 5 | 535783 | 5·76 | 972755 | ·77 | 563028 | 6·53 | 436972 | 55 |
| 6 | 536129 | 5·75 | 972709 | ·77 | 563419 | 6·52 | 436581 | 54 |
| 7 | 536474 | 5·74 | 972663 | ·77 | 563811 | 6·52 | 436189 | 53 |
| 8 | 536818 | 5·74 | 972617 | ·77 | 564202 | 6·51 | 435798 | 52 |
| 9 | 537163 | 5·73 | 972570 | ·77 | 564592 | 6·51 | 435408 | 51 |
| 10 | 537507 | 5·73 | 972524 | ·77 | 564983 | 6·50 | 435017 | 50 |
| 11 | 9·537851 | 5·72 | 9·972478 | ·77 | 9·565373 | 6·50 | 10·434627 | 49 |
| 12 | 538194 | 5·72 | 972431 | ·78 | 565763 | 6·49 | 434237 | 48 |
| 13 | 538538 | 5·71 | 972385 | ·78 | 566153 | 6·49 | 433847 | 47 |
| 14 | 538880 | 5·71 | 972338 | ·78 | 566542 | 6·49 | 433458 | 46 |
| 15 | 539223 | 5·70 | 972291 | ·78 | 566932 | 6·48 | 433068 | 45 |
| 16 | 539565 | 5·70 | 972245 | ·78 | 567320 | 6·48 | 432680 | 44 |
| 17 | 539907 | 5·69 | 972198 | ·78 | 567709 | 6·47 | 432291 | 43 |
| 18 | 540249 | 5·69 | 972151 | ·78 | 568098 | 6·47 | 431902 | 42 |
| 19 | 540590 | 5·68 | 972105 | ·78 | 568486 | 6·46 | 431514 | 41 |
| 20 | 540931 | 5·68 | 972058 | ·78 | 568873 | 6·46 | 431127 | 40 |
| 21 | 9·541272 | 5·67 | 9·972011 | ·78 | 9·569261 | 6·45 | 10·430739 | 39 |
| 22 | 541613 | 5·67 | 971964 | ·78 | 569648 | 6·45 | 430352 | 38 |
| 23 | 541953 | 5·66 | 971917 | ·78 | 570035 | 6·45 | 429965 | 37 |
| 24 | 542293 | 5·66 | 971870 | ·78 | 570422 | 6·44 | 429578 | 36 |
| 25 | 542632 | 5·65 | 971823 | ·78 | 570809 | 6·44 | 429191 | 35 |
| 26 | 542971 | 5·65 | 971776 | ·78 | 571195 | 6·43 | 428805 | 34 |
| 27 | 543310 | 5·64 | 971729 | ·79 | 571581 | 6·43 | 428419 | 33 |
| 28 | 543649 | 5·64 | 971682 | ·79 | 571967 | 6·42 | 428033 | 32 |
| 29 | 543987 | 5·63 | 971635 | ·79 | 572352 | 6·42 | 427648 | 31 |
| 30 | 544325 | 5·63 | 971588 | ·79 | 572738 | 6·42 | 427262 | 30 |
| 31 | 9·544663 | 5·62 | 9·971540 | ·79 | 9·573123 | 6·41 | 10·426877 | 29 |
| 32 | 545000 | 5·62 | 971493 | ·79 | 573507 | 6·41 | 426493 | 28 |
| 33 | 545338 | 5·61 | 971446 | ·79 | 573892 | 6·40 | 426108 | 27 |
| 34 | 545674 | 5·61 | 971398 | ·79 | 574276 | 6·40 | 425724 | 26 |
| 35 | 546011 | 5·60 | 971351 | ·79 | 574660 | 6·39 | 425340 | 25 |
| 36 | 546347 | 5·60 | 971303 | ·79 | 575044 | 6·39 | 424956 | 24 |
| 37 | 546683 | 5·59 | 971256 | ·79 | 575427 | 6·39 | 424573 | 23 |
| 38 | 547019 | 5·59 | 971208 | ·79 | 575810 | 6·38 | 424190 | 22 |
| 39 | 547354 | 5·58 | 971161 | ·79 | 576193 | 6·38 | 423807 | 21 |
| 40 | 547689 | 5·58 | 971113 | ·79 | 576576 | 6·37 | 423424 | 20 |
| 41 | 9·548024 | 5·57 | 9·971066 | ·80 | 9·576958 | 6·37 | 10·423041 | 19 |
| 42 | 548359 | 5·57 | 971018 | ·80 | 577341 | 6·36 | 422659 | 18 |
| 43 | 548693 | 5·56 | 970970 | ·80 | 577723 | 6·36 | 422277 | 17 |
| 44 | 549027 | 5·56 | 970922 | ·80 | 578104 | 6·36 | 421896 | 16 |
| 45 | 549360 | 5·55 | 970874 | ·80 | 578486 | 6·35 | 421514 | 15 |
| 46 | 549693 | 5·55 | 970827 | ·80 | 578867 | 6·35 | 421133 | 14 |
| 47 | 550026 | 5·54 | 970779 | ·80 | 579248 | 6·34 | 420752 | 13 |
| 48 | 550359 | 5·54 | 970731 | ·80 | 579629 | 6·34 | 420371 | 12 |
| 49 | 550692 | 5·53 | 970683 | ·80 | 580009 | 6·34 | 419991 | 11 |
| 50 | 551024 | 5·53 | 970635 | ·80 | 580389 | 6·33 | 419611 | 10 |
| 51 | 9·551356 | 5·52 | 9·970586 | ·80 | 9·580769 | 6·33 | 10·419231 | 9 |
| 52 | 551687 | 5·52 | 970538 | ·80 | 581149 | 6·32 | 418851 | 8 |
| 53 | 552018 | 5·52 | 970490 | ·80 | 581528 | 6·32 | 418472 | 7 |
| 54 | 552349 | 5·51 | 970442 | ·80 | 581907 | 6·32 | 418093 | 6 |
| 55 | 552680 | 5·51 | 970394 | ·80 | 582286 | 6·31 | 417714 | 5 |
| 56 | 553010 | 5·50 | 970345 | ·81 | 582665 | 6·31 | 417335 | 4 |
| 57 | 553341 | 5·50 | 970297 | ·81 | 583043 | 6·30 | 416957 | 3 |
| 58 | 553670 | 5·49 | 970249 | ·81 | 583422 | 6·30 | 416578 | 2 |
| 59 | 554000 | 5·49 | 970200 | ·81 | 583800 | 6·29 | 416200 | 1 |
| 60 | 554329 | 5·48 | 970152 | ·81 | 584177 | 6·29 | 415823 | 0 |
| | Cosine | D. | Sine | 69° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·554329 | 5·48 | 9·970152 | ·81 | 9·584177 | 6·29 | 10·415823 | 60 |
| 1 | 554658 | 5·48 | 970103 | ·81 | 584555 | 6·29 | 415445 | 59 |
| 2 | 554987 | 5·47 | 970055 | ·81 | 584932 | 6·28 | 415068 | 58 |
| 3 | 555315 | 5·47 | 970006 | ·81 | 585309 | 6·28 | 414691 | 57 |
| 4 | 555643 | 5·46 | 969957 | ·81 | 585686 | 6·27 | 414314 | 56 |
| 5 | 555971 | 5·46 | 969909 | ·81 | 586062 | 6·27 | 413938 | 55 |
| 6 | 556299 | 5·45 | 969860 | ·81 | 586439 | 6·27 | 413561 | 54 |
| 7 | 556626 | 5·45 | 969811 | ·81 | 586815 | 6·26 | 413185 | 53 |
| 8 | 556953 | 5·44 | 969762 | ·81 | 587190 | 6·26 | 412810 | 52 |
| 9 | 557280 | 5·44 | 969714 | ·81 | 587566 | 6·25 | 412434 | 51 |
| 10 | 557606 | 5·43 | 969665 | ·81 | 587941 | 6·25 | 412059 | 50 |
| 11 | 9·557932 | 5·43 | 9·969616 | ·82 | 9·588316 | 6·25 | 10·411684 | 49 |
| 12 | 558258 | 5·43 | 969567 | ·82 | 588691 | 6·24 | 411309 | 48 |
| 13 | 558583 | 5·42 | 969518 | ·82 | 589066 | 6·24 | 410934 | 47 |
| 14 | 558909 | 5·42 | 969469 | ·82 | 589440 | 6·23 | 410560 | 46 |
| 15 | 559234 | 5·41 | 969420 | ·82 | 589814 | 6·23 | 410186 | 45 |
| 16 | 559558 | 5·41 | 969370 | ·82 | 590188 | 6·23 | 409812 | 44 |
| 17 | 559883 | 5·40 | 969321 | ·82 | 590562 | 6·22 | 409438 | 43 |
| 18 | 560207 | 5·40 | 969272 | ·82 | 590935 | 6·22 | 409065 | 42 |
| 19 | 560531 | 5·39 | 969223 | ·82 | 591308 | 6·22 | 408692 | 41 |
| 20 | 560855 | 5·39 | 969173 | ·82 | 591681 | 6·21 | 408319 | 40 |
| 21 | 9·561178 | 5·38 | 9·969124 | ·82 | 9·592054 | 6·21 | 10·407946 | 39 |
| 22 | 561501 | 5·38 | 969075 | ·82 | 592426 | 6·20 | 407574 | 38 |
| 23 | 561824 | 5·37 | 969025 | ·82 | 592798 | 6·20 | 407202 | 37 |
| 24 | 562146 | 5·37 | 968976 | ·82 | 593170 | 6·19 | 406829 | 36 |
| 25 | 562468 | 5·36 | 968926 | ·83 | 593542 | 6·19 | 406458 | 35 |
| 26 | 562790 | 5·36 | 968877 | ·83 | 593914 | 6·18 | 406086 | 34 |
| 27 | 563112 | 5·36 | 968827 | ·83 | 594285 | 6·18 | 405715 | 33 |
| 28 | 563433 | 5·35 | 968777 | ·83 | 594656 | 6·18 | 405344 | 32 |
| 29 | 563755 | 5·35 | 968728 | ·83 | 595027 | 6·17 | 404973 | 31 |
| 30 | 564075 | 5·34 | 968678 | ·83 | 595398 | 6·17 | 404602 | 30 |
| 31 | 9·564396 | 5·34 | 9·968628 | ·83 | 9·595768 | 6·17 | 10·404232 | 29 |
| 32 | 564716 | 5·33 | 968578 | ·83 | 596138 | 6·16 | 403862 | 28 |
| 33 | 565036 | 5·33 | 968528 | ·83 | 596508 | 6·16 | 403492 | 27 |
| 34 | 565356 | 5·32 | 968479 | ·83 | 596878 | 6·16 | 403122 | 26 |
| 35 | 565676 | 5·32 | 968429 | ·83 | 597247 | 6·15 | 402753 | 25 |
| 36 | 565995 | 5·31 | 968379 | ·83 | 597616 | 6·15 | 402384 | 24 |
| 37 | 566314 | 5·31 | 968329 | ·83 | 597985 | 6·15 | 402015 | 23 |
| 38 | 566632 | 5·31 | 968278 | ·83 | 598354 | 6·14 | 401646 | 22 |
| 39 | 566951 | 5·30 | 968228 | ·84 | 598722 | 6·14 | 401278 | 21 |
| 40 | 567269 | 5·30 | 968178 | ·84 | 599091 | 6·13 | 400909 | 20 |
| 41 | 9·567587 | 5·29 | 9·968128 | ·84 | 9·599459 | 6·13 | 10·400541 | 19 |
| 42 | 567904 | 5·29 | 968078 | ·84 | 599827 | 6·13 | 400173 | 18 |
| 43 | 568222 | 5·28 | 968027 | ·84 | 600194 | 6·12 | 399806 | 17 |
| 44 | 568539 | 5·28 | 967977 | ·84 | 600562 | 6·12 | 399438 | 16 |
| 45 | 568856 | 5·28 | 967927 | ·84 | 600929 | 6·11 | 399071 | 15 |
| 46 | 569172 | 5·27 | 967876 | ·84 | 601296 | 6·11 | 398704 | 14 |
| 47 | 569488 | 5·27 | 967826 | ·84 | 601662 | 6·11 | 398338 | 13 |
| 48 | 569804 | 5·26 | 967775 | ·84 | 602029 | 6·10 | 397971 | 12 |
| 49 | 570120 | 5·26 | 967725 | ·84 | 602395 | 6·10 | 397605 | 11 |
| 50 | 570435 | 5·25 | 967674 | ·84 | 602761 | 6·10 | 397239 | 10 |
| 51 | 9·570751 | 5·25 | 9·967624 | ·84 | 9·603127 | 6·09 | 10·396873 | 9 |
| 52 | 571066 | 5·24 | 967573 | ·84 | 603493 | 6·09 | 396507 | 8 |
| 53 | 571380 | 5·24 | 967522 | ·85 | 603858 | 6·09 | 396142 | 7 |
| 54 | 571695 | 5·23 | 967471 | ·85 | 604223 | 6·08 | 395777 | 6 |
| 55 | 572009 | 5·23 | 967421 | ·85 | 604588 | 6·08 | 395412 | 5 |
| 56 | 572323 | 5·23 | 967370 | ·85 | 604953 | 6·07 | 395047 | 4 |
| 57 | 572636 | 5·22 | 967319 | ·85 | 605317 | 6·07 | 394683 | 3 |
| 58 | 572950 | 5·22 | 967268 | ·85 | 605682 | 6·07 | 394318 | 2 |
| 59 | 573263 | 5·21 | 967217 | ·85 | 606046 | 6·06 | 393954 | 1 |
| 60 | 573575 | 5·21 | 967166 | ·85 | 606410 | 6·06 | 393590 | 0 |
| | Cosine | D. | Sine | 68° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·573575 | 5·21 | 9·967166 | ·85 | 9·606410 | 6·06 | 10·393590 | 60 |
| 1 | 573888 | 5·20 | 967115 | ·85 | 606773 | 6·06 | 393227 | 59 |
| 2 | 574200 | 5·20 | 967064 | ·85 | 607137 | 6·05 | 392863 | 58 |
| 3 | 574512 | 5·19 | 967013 | ·85 | 607500 | 6·05 | 392500 | 57 |
| 4 | 574824 | 5·19 | 966961 | ·85 | 607863 | 6·04 | 392137 | 56 |
| 5 | 575136 | 5·19 | 966910 | ·85 | 608225 | 6·04 | 391775 | 55 |
| 6 | 575447 | 5·18 | 966859 | ·85 | 608588 | 6·04 | 391412 | 54 |
| 7 | 575758 | 5·18 | 966808 | ·85 | 608950 | 6·03 | 391050 | 53 |
| 8 | 576069 | 5·17 | 966756 | ·86 | 609312 | 6·03 | 390688 | 52 |
| 9 | 576379 | 5·17 | 966705 | ·86 | 609674 | 6·03 | 390326 | 51 |
| 10 | 576689 | 5·16 | 966653 | ·86 | 610036 | 6·02 | 389964 | 50 |
| 11 | 9·576999 | 5·16 | 9·966602 | ·86 | 9·610397 | 6·02 | 10·389603 | 49 |
| 12 | 577309 | 5·16 | 966550 | ·86 | 610759 | 6·02 | 389241 | 48 |
| 13 | 577618 | 5·15 | 966499 | ·86 | 611120 | 6·01 | 388880 | 47 |
| 14 | 577927 | 5·15 | 966447 | ·86 | 611480 | 6·01 | 388520 | 46 |
| 15 | 578236 | 5·14 | 966395 | ·86 | 611841 | 6·01 | 388159 | 45 |
| 16 | 578545 | 5·14 | 966344 | ·86 | 612201 | 6·00 | 387799 | 44 |
| 17 | 578853 | 5·13 | 966292 | ·86 | 612561 | 6·00 | 387439 | 43 |
| 18 | 579162 | 5·13 | 966240 | ·86 | 612921 | 6·00 | 387079 | 42 |
| 19 | 579470 | 5·13 | 966188 | ·86 | 613281 | 5·99 | 386719 | 41 |
| 20 | 579777 | 5·12 | 966136 | ·86 | 613641 | 5·99 | 386359 | 40 |
| 21 | 9·580085 | 5·12 | 9·966085 | ·87 | 9·614000 | 5·98 | 10·386000 | 39 |
| 22 | 580392 | 5·11 | 966033 | ·87 | 614359 | 5·98 | 385641 | 38 |
| 23 | 580699 | 5·11 | 965981 | ·87 | 614718 | 5·98 | 385282 | 37 |
| 24 | 581005 | 5·11 | 965928 | ·87 | 615077 | 5·97 | 384923 | 36 |
| 25 | 581312 | 5·10 | 965876 | ·87 | 615435 | 5·97 | 384565 | 35 |
| 26 | 581618 | 5·10 | 965824 | ·87 | 615793 | 5·97 | 384207 | 34 |
| 27 | 581924 | 5·09 | 965772 | ·87 | 616151 | 5·96 | 383849 | 33 |
| 28 | 582229 | 5·09 | 965720 | ·87 | 616509 | 5·96 | 383491 | 32 |
| 29 | 582535 | 5·09 | 965668 | ·87 | 616867 | 5·96 | 383133 | 31 |
| 30 | 582840 | 5·08 | 965615 | ·87 | 617224 | 5·95 | 382776 | 30 |
| 31 | 9·583145 | 5·08 | 9·965563 | ·87 | 9·617582 | 5·95 | 10·382418 | 29 |
| 32 | 583449 | 5·07 | 965511 | ·87 | 617939 | 5·95 | 382061 | 28 |
| 33 | 583754 | 5·07 | 965458 | ·87 | 618295 | 5·94 | 381705 | 27 |
| 34 | 584058 | 5·06 | 965406 | ·87 | 618652 | 5·94 | 381348 | 26 |
| 35 | 584361 | 5·06 | 965353 | ·88 | 619008 | 5·94 | 380992 | 25 |
| 36 | 584665 | 5·06 | 965301 | ·88 | 619364 | 5·93 | 380636 | 24 |
| 37 | 584968 | 5·05 | 965248 | ·88 | 619721 | 5·93 | 380279 | 23 |
| 38 | 585272 | 5·05 | 965195 | ·88 | 620076 | 5·93 | 379924 | 22 |
| 39 | 585574 | 5·04 | 965143 | ·88 | 620432 | 5·92 | 379568 | 21 |
| 40 | 585877 | 5·04 | 965090 | ·88 | 620787 | 5·92 | 379213 | 20 |
| 41 | 9·586179 | 5·03 | 9·965037 | ·88 | 9·621142 | 5·92 | 10·378858 | 19 |
| 42 | 586482 | 5·03 | 964984 | ·88 | 621497 | 5·91 | 378503 | 18 |
| 43 | 586783 | 5·03 | 964931 | ·88 | 621852 | 5·91 | 378148 | 17 |
| 44 | 587085 | 5·02 | 964879 | ·88 | 622207 | 5·90 | 377793 | 16 |
| 45 | 587386 | 5·02 | 964826 | ·88 | 622561 | 5·90 | 377439 | 15 |
| 46 | 587688 | 5·01 | 964773 | ·88 | 622915 | 5·90 | 377085 | 14 |
| 47 | 587989 | 5·01 | 964719 | ·88 | 623269 | 5·89 | 376731 | 13 |
| 48 | 588289 | 5·01 | 964666 | ·89 | 623623 | 5·89 | 376377 | 12 |
| 49 | 588590 | 5·00 | 964613 | ·89 | 623976 | 5·89 | 376024 | 11 |
| 50 | 588890 | 5·00 | 964560 | ·89 | 624330 | 5·88 | 375670 | 10 |
| 51 | 9·589190 | 4·99 | 9·964507 | ·89 | 9·624683 | 5·88 | 10·375317 | 9 |
| 52 | 589489 | 4·99 | 964454 | ·89 | 625036 | 5·88 | 374964 | 8 |
| 53 | 589789 | 4·99 | 964400 | ·89 | 625388 | 5·87 | 374612 | 7 |
| 54 | 590088 | 4·98 | 964347 | ·89 | 625741 | 5·87 | 374259 | 6 |
| 55 | 590387 | 4·98 | 964294 | ·89 | 626093 | 5·87 | 373907 | 5 |
| 56 | 590686 | 4·97 | 964240 | ·89 | 626445 | 5·86 | 373555 | 4 |
| 57 | 590984 | 4·97 | 964187 | ·89 | 626797 | 5·86 | 373203 | 3 |
| 58 | 591282 | 4·97 | 964133 | ·89 | 627149 | 5·86 | 372851 | 2 |
| 59 | 591580 | 4·96 | 964080 | ·89 | 627501 | 5·85 | 372499 | 1 |
| 60 | 591878 | 4·96 | 964026 | ·89 | 627852 | 5·85 | 372148 | 0 |
| | Cosine | D. | Sine | 67° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·591878 | 4·96 | 9·964026 | ·89 | 9·627852 | 5·85 | 10·372148 | 60 |
| 1 | 592176 | 4·95 | 963972 | ·89 | 628203 | 5·85 | 371797 | 59 |
| 2 | 592473 | 4·95 | 963919 | ·89 | 628554 | 5·85 | 371446 | 58 |
| 3 | 592770 | 4·95 | 963865 | ·90 | 628905 | 5·84 | 371095 | 57 |
| 4 | 593067 | 4·94 | 963811 | ·90 | 629255 | 5·84 | 370745 | 56 |
| 5 | 593363 | 4·94 | 963757 | ·90 | 629606 | 5·83 | 370394 | 55 |
| 6 | 593659 | 4·93 | 963704 | ·90 | 629956 | 5·83 | 370044 | 54 |
| 7 | 593955 | 4·93 | 963650 | ·90 | 630306 | 5·83 | 369694 | 53 |
| 8 | 594251 | 4·93 | 963596 | ·90 | 630656 | 5·83 | 369344 | 52 |
| 9 | 594547 | 4·92 | 963542 | ·90 | 631005 | 5·82 | 368995 | 51 |
| 10 | 594842 | 4·92 | 963488 | ·90 | 631355 | 5·82 | 368645 | 50 |
| 11 | 9·595137 | 4·91 | 9·963434 | ·90 | 9·631704 | 5·82 | 10·368296 | 49 |
| 12 | 595432 | 4·91 | 963379 | ·90 | 632053 | 5·81 | 367947 | 48 |
| 13 | 595727 | 4·91 | 963325 | ·90 | 632401 | 5·81 | 367599 | 47 |
| 14 | 596021 | 4·90 | 963271 | ·90 | 632750 | 5·81 | 367250 | 46 |
| 15 | 596315 | 4·90 | 963217 | ·90 | 633098 | 5·80 | 366902 | 45 |
| 16 | 596609 | 4·89 | 963163 | ·90 | 633447 | 5·80 | 366553 | 44 |
| 17 | 596903 | 4·89 | 963108 | ·91 | 633795 | 5·80 | 366205 | 43 |
| 18 | 597196 | 4·89 | 963054 | ·91 | 634143 | 5·79 | 365857 | 42 |
| 19 | 597490 | 4·88 | 962999 | ·91 | 634490 | 5·79 | 365510 | 41 |
| 20 | 597783 | 4·88 | 962945 | ·91 | 634838 | 5·79 | 365162 | 40 |
| 21 | 9·598075 | 4·87 | 9·962890 | ·91 | 9·635185 | 5·78 | 10·364815 | 39 |
| 22 | 598368 | 4·87 | 962836 | ·91 | 635532 | 5·78 | 364468 | 38 |
| 23 | 598660 | 4·87 | 962781 | ·91 | 635879 | 5·78 | 364121 | 37 |
| 24 | 598952 | 4·86 | 962727 | ·91 | 636226 | 5·77 | 363774 | 36 |
| 25 | 599244 | 4·86 | 962672 | ·91 | 636572 | 5·77 | 363428 | 35 |
| 26 | 599536 | 4·85 | 962617 | ·91 | 636919 | 5·77 | 363081 | 34 |
| 27 | 599827 | 4·85 | 962562 | ·91 | 637265 | 5·77 | 362735 | 33 |
| 28 | 600118 | 4·85 | 962508 | ·91 | 637611 | 5·76 | 362389 | 32 |
| 29 | 600409 | 4·84 | 962453 | ·91 | 637956 | 5·76 | 362044 | 31 |
| 30 | 600700 | 4·84 | 962398 | ·92 | 638302 | 5·76 | 361698 | 30 |
| 31 | 9·600990 | 4·84 | 9·962343 | ·92 | 9·638647 | 5·75 | 10·361353 | 29 |
| 32 | 601280 | 4·83 | 962288 | ·92 | 638992 | 5·75 | 361008 | 28 |
| 33 | 601570 | 4·83 | 962233 | ·92 | 639337 | 5·75 | 360663 | 27 |
| 34 | 601860 | 4·82 | 962178 | ·92 | 639682 | 5·74 | 360318 | 26 |
| 35 | 602150 | 4·82 | 962123 | ·92 | 640027 | 5·74 | 359973 | 25 |
| 36 | 602439 | 4·82 | 962067 | ·92 | 640371 | 5·74 | 359629 | 24 |
| 37 | 602728 | 4·81 | 962012 | ·92 | 640716 | 5·73 | 359284 | 23 |
| 38 | 603017 | 4·81 | 961957 | ·92 | 641060 | 5·73 | 358940 | 22 |
| 39 | 603305 | 4·81 | 961902 | ·92 | 641404 | 5·73 | 358596 | 21 |
| 40 | 603594 | 4·80 | 961846 | ·92 | 641747 | 5·72 | 358253 | 20 |
| 41 | 9·603882 | 4·80 | 9·961791 | ·92 | 9·642091 | 5·72 | 10·357909 | 19 |
| 42 | 604170 | 4·79 | 961735 | ·92 | 642434 | 5·72 | 357566 | 18 |
| 43 | 604457 | 4·79 | 961680 | ·92 | 642777 | 5·72 | 357223 | 17 |
| 44 | 604745 | 4·79 | 961624 | ·93 | 643120 | 5·71 | 356880 | 16 |
| 45 | 605032 | 4·78 | 961569 | ·93 | 643463 | 5·71 | 356537 | 15 |
| 46 | 605319 | 4·78 | 961513 | ·93 | 643806 | 5·71 | 356194 | 14 |
| 47 | 605606 | 4·78 | 961458 | ·93 | 644148 | 5·70 | 355852 | 13 |
| 48 | 605892 | 4·77 | 961402 | ·93 | 644490 | 5·70 | 355510 | 12 |
| 49 | 606179 | 4·77 | 961346 | ·93 | 644832 | 5·70 | 355168 | 11 |
| 50 | 606465 | 4·76 | 961290 | ·93 | 645174 | 5·69 | 354826 | 10 |
| 51 | 9·606751 | 4·76 | 9·961235 | ·93 | 9·645516 | 5·69 | 10·354484 | 9 |
| 52 | 607036 | 4·76 | 961179 | ·93 | 645857 | 5·69 | 354143 | 8 |
| 53 | 607322 | 4·75 | 961123 | ·93 | 646199 | 5·69 | 353801 | 7 |
| 54 | 607607 | 4·75 | 961067 | ·93 | 646540 | 5·68 | 353460 | 6 |
| 55 | 607892 | 4·74 | 961011 | ·93 | 646881 | 5·68 | 353119 | 5 |
| 56 | 608177 | 4·74 | 960955 | ·93 | 647222 | 5·68 | 352778 | 4 |
| 57 | 608461 | 4·74 | 960899 | ·93 | 647562 | 5·67 | 352438 | 3 |
| 58 | 608745 | 4·73 | 960843 | ·94 | 647903 | 5·67 | 352097 | 2 |
| 59 | 609029 | 4·73 | 960786 | ·94 | 648243 | 5·67 | 351757 | 1 |
| 60 | 609313 | 4·73 | 960730 | ·94 | 648583 | 5·66 | 351417 | 0 |
| | **Cosine** | **D.** | **Sine** | **66°** | **Cotang.** | **D.** | **Tang.** | **M.** |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·609313 | 4·73 | 9·960730 | ·94 | 9·648583 | 5·66 | 10·351417 | 60 |
| 1 | 609597 | 4·72 | 960674 | ·94 | 648923 | 5·66 | 351077 | 59 |
| 2 | 609880 | 4·72 | 960618 | ·94 | 649263 | 5·66 | 350737 | 58 |
| 3 | 610164 | 4·72 | 960561 | ·94 | 649602 | 5·66 | 350398 | 57 |
| 4 | 610447 | 4·71 | 960505 | ·94 | 649942 | 5·65 | 350058 | 56 |
| 5 | 610729 | 4·71 | 960448 | ·94 | 650281 | 5·65 | 349719 | 55 |
| 6 | 611012 | 4·70 | 960392 | ·94 | 650620 | 5·65 | 349380 | 54 |
| 7 | 611294 | 4·70 | 960335 | ·94 | 650959 | 5·64 | 349041 | 53 |
| 8 | 611576 | 4·70 | 960279 | ·94 | 651297 | 5·64 | 348703 | 52 |
| 9 | 611858 | 4·69 | 960222 | ·94 | 651636 | 5·64 | 348364 | 51 |
| 10 | 612140 | 4·69 | 960165 | ·94 | 651974 | 5·63 | 348026 | 50 |
| 11 | 9·612421 | 4·69 | 9·960109 | ·95 | 9·652312 | 5·63 | 10·347688 | 49 |
| 12 | 612702 | 4·68 | 960052 | ·95 | 652650 | 5·63 | 347350 | 48 |
| 13 | 612983 | 4·68 | 959995 | ·95 | 652988 | 5·63 | 347012 | 47 |
| 14 | 613264 | 4·67 | 959938 | ·95 | 653326 | 5·62 | 346674 | 46 |
| 15 | 613545 | 4·67 | 959882 | ·95 | 653663 | 5·62 | 346337 | 45 |
| 16 | 613825 | 4·67 | 959825 | ·95 | 654000 | 5·62 | 346000 | 44 |
| 17 | 614105 | 4·66 | 959768 | ·95 | 654337 | 5·61 | 345663 | 43 |
| 18 | 614385 | 4·66 | 959711 | ·95 | 654674 | 5·61 | 345326 | 42 |
| 19 | 614665 | 4·66 | 959654 | ·95 | 655011 | 5·61 | 344989 | 41 |
| 20 | 614944 | 4·65 | 959596 | ·95 | 655348 | 5·61 | 344652 | 40 |
| 21 | 9·615223 | 4·65 | 9·959539 | ·95 | 9·655684 | 5·60 | 10·344316 | 39 |
| 22 | 615502 | 4·65 | 959482 | ·95 | 656020 | 5·60 | 343980 | 38 |
| 23 | 615781 | 4·64 | 959425 | ·95 | 656356 | 5·60 | 343644 | 37 |
| 24 | 616060 | 4·64 | 959368 | ·95 | 656692 | 5·59 | 343308 | 36 |
| 25 | 616338 | 4·64 | 959310 | ·96 | 657028 | 5·59 | 342972 | 35 |
| 26 | 616616 | 4·63 | 959253 | ·96 | 657364 | 5·59 | 342636 | 34 |
| 27 | 616894 | 4·63 | 959195 | ·96 | 657699 | 5·59 | 342301 | 33 |
| 28 | 617172 | 4·62 | 959138 | ·96 | 658034 | 5·58 | 341966 | 32 |
| 29 | 617450 | 4·62 | 959081 | ·96 | 658369 | 5·58 | 341631 | 31 |
| 30 | 617727 | 4·62 | 959023 | ·96 | 658704 | 5·58 | 341296 | 30 |
| 31 | 9·618004 | 4·61 | 9·958965 | ·96 | 9·659039 | 5·58 | 10·340961 | 29 |
| 32 | 618281 | 4·61 | 958908 | ·96 | 659373 | 5·57 | 340627 | 28 |
| 33 | 618558 | 4·61 | 958850 | ·96 | 659708 | 5·57 | 340292 | 27 |
| 34 | 618834 | 4·60 | 958792 | ·96 | 660042 | 5·57 | 339958 | 26 |
| 35 | 619110 | 4·60 | 958734 | ·96 | 660376 | 5·57 | 339624 | 25 |
| 36 | 619386 | 4·60 | 958677 | ·96 | 660710 | 5·56 | 339290 | 24 |
| 37 | 619662 | 4·59 | 958619 | ·96 | 661043 | 5·56 | 338957 | 23 |
| 38 | 619938 | 4·59 | 958561 | ·96 | 661377 | 5·56 | 338623 | 22 |
| 39 | 620213 | 4·59 | 958503 | ·97 | 661710 | 5·55 | 338290 | 21 |
| 40 | 620488 | 4·58 | 958445 | ·97 | 662043 | 5·55 | 337957 | 20 |
| 41 | 9·620763 | 4·58 | 9·958387 | ·97 | 9·662376 | 5·55 | 10·337624 | 19 |
| 42 | 621038 | 4·57 | 958329 | ·97 | 662709 | 5·54 | 337291 | 18 |
| 43 | 621313 | 4·57 | 958271 | ·97 | 663042 | 5·54 | 336958 | 17 |
| 44 | 621587 | 4·57 | 958213 | ·97 | 663375 | 5·54 | 336625 | 16 |
| 45 | 621861 | 4·56 | 958154 | ·97 | 663707 | 5·54 | 336293 | 15 |
| 46 | 622135 | 4·56 | 958096 | ·97 | 664039 | 5·53 | 335961 | 14 |
| 47 | 622409 | 4·56 | 958038 | ·97 | 664371 | 5·53 | 335629 | 13 |
| 48 | 622682 | 4·55 | 957979 | ·97 | 664703 | 5·53 | 335297 | 12 |
| 49 | 622956 | 4·55 | 957921 | ·97 | 665035 | 5·53 | 334965 | 11 |
| 50 | 623229 | 4·55 | 957863 | ·97 | 665366 | 5·52 | 334634 | 10 |
| 51 | 9·623502 | 4·54 | 9·957804 | ·97 | 9·665697 | 5·52 | 10·334303 | 9 |
| 52 | 623774 | 4·54 | 957746 | ·98 | 666029 | 5·52 | 333971 | 8 |
| 53 | 624047 | 4·54 | 957687 | ·98 | 666360 | 5·51 | 333640 | 7 |
| 54 | 624319 | 4·53 | 957628 | ·98 | 666691 | 5·51 | 333309 | 6 |
| 55 | 624591 | 4·53 | 957570 | ·98 | 667021 | 5·51 | 332979 | 5 |
| 56 | 624863 | 4·53 | 957511 | ·98 | 667352 | 5·51 | 332648 | 4 |
| 57 | 625135 | 4·52 | 957452 | ·98 | 667682 | 5·50 | 332318 | 3 |
| 58 | 625406 | 4·52 | 957393 | ·98 | 668013 | 5·50 | 331987 | 2 |
| 59 | 625677 | 4·52 | 957335 | ·98 | 668343 | 5·50 | 331657 | 1 |
| 60 | 625948 | 4·51 | 957276 | ·98 | 668672 | 5·50 | 331328 | 0 |
| | Cosine | D. | Sine | 65° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·625948 | 4·51 | 9·957276 | ·98 | 9·668673 | 5·50 | 10·331327 | 60 |
| 1 | 626219 | 4·51 | 957217 | ·98 | 669002 | 5·49 | 330998 | 59 |
| 2 | 626490 | 4·51 | 957158 | ·98 | 669332 | 5·49 | 330668 | 58 |
| 3 | 626760 | 4·50 | 957099 | ·98 | 669661 | 5·49 | 330339 | 57 |
| 4 | 627030 | 4·50 | 957040 | ·98 | 669991 | 5·48 | 330009 | 56 |
| 5 | 627300 | 4·50 | 956981 | ·98 | 670320 | 5·48 | 329680 | 55 |
| 6 | 627570 | 4·49 | 956921 | ·99 | 670649 | 5·48 | 329351 | 54 |
| 7 | 627840 | 4·49 | 956862 | ·99 | 670977 | 5·48 | 329023 | 53 |
| 8 | 628109 | 4·49 | 956803 | ·99 | 671306 | 5·47 | 328694 | 52 |
| 9 | 628378 | 4·48 | 956744 | ·99 | 671634 | 5·47 | 328366 | 51 |
| 10 | 628647 | 4·48 | 956684 | ·99 | 671963 | 5·47 | 328037 | 50 |
| 11 | 9·628916 | 4·47 | 9·956625 | ·99 | 9·672291 | 5·47 | 10·327709 | 49 |
| 12 | 629185 | 4·47 | 956566 | ·99 | 672619 | 5·46 | 327381 | 48 |
| 13 | 629453 | 4·47 | 956506 | ·99 | 672947 | 5·46 | 327053 | 47 |
| 14 | 629721 | 4·46 | 956447 | ·99 | 673274 | 5·46 | 326726 | 46 |
| 15 | 629989 | 4·46 | 956387 | ·99 | 673602 | 5·46 | 326398 | 45 |
| 16 | 630257 | 4·46 | 956327 | ·99 | 673929 | 5·45 | 326071 | 44 |
| 17 | 630524 | 4·46 | 956268 | ·99 | 674257 | 5·45 | 325743 | 43 |
| 18 | 630792 | 4·45 | 956208 | 1·00 | 674584 | 5·45 | 325416 | 42 |
| 19 | 631059 | 4·45 | 956148 | 1·00 | 674910 | 5·44 | 325090 | 41 |
| 20 | 631326 | 4·45 | 956089 | 1·00 | 675237 | 5·44 | 324763 | 40 |
| 21 | 9·631593 | 4·44 | 9·956029 | 1·00 | 9·675564 | 5·44 | 10·324436 | 39 |
| 22 | 631859 | 4·44 | 955969 | 1·00 | 675890 | 5·44 | 324110 | 38 |
| 23 | 632125 | 4·44 | 955909 | 1·00 | 676216 | 5·43 | 323784 | 37 |
| 24 | 632392 | 4·43 | 955849 | 1·00 | 676543 | 5·43 | 323457 | 36 |
| 25 | 632658 | 4·43 | 955789 | 1·00 | 676869 | 5·43 | 323131 | 35 |
| 26 | 632923 | 4·43 | 955729 | 1·00 | 677194 | 5·43 | 322806 | 34 |
| 27 | 633189 | 4·42 | 955669 | 1·00 | 677520 | 5·42 | 322480 | 33 |
| 28 | 633454 | 4·42 | 955609 | 1·00 | 677846 | 5·42 | 322154 | 32 |
| 29 | 633719 | 4·42 | 955548 | 1·00 | 678171 | 5·42 | 321829 | 31 |
| 30 | 633984 | 4·41 | 955488 | 1·00 | 678496 | 5·42 | 321504 | 30 |
| 31 | 9·634249 | 4·41 | 9·955428 | 1·01 | 9·678821 | 5·41 | 10·321179 | 29 |
| 32 | 634514 | 4·40 | 955368 | 1·01 | 679146 | 5·41 | 320854 | 28 |
| 33 | 634778 | 4·40 | 955307 | 1·01 | 679471 | 5·41 | 320529 | 27 |
| 34 | 635042 | 4·40 | 955247 | 1·01 | 679795 | 5·41 | 320205 | 26 |
| 35 | 635306 | 4·39 | 955186 | 1·01 | 680120 | 5·40 | 319880 | 25 |
| 36 | 635570 | 4·39 | 955126 | 1·01 | 680444 | 5·40 | 319556 | 24 |
| 37 | 635834 | 4·39 | 955065 | 1·01 | 680768 | 5·40 | 319232 | 23 |
| 38 | 636097 | 4·38 | 955005 | 1·01 | 681092 | 5·40 | 318908 | 22 |
| 39 | 636360 | 4·38 | 954944 | 1·01 | 681416 | 5·39 | 318584 | 21 |
| 40 | 636623 | 4·38 | 954883 | 1·01 | 681740 | 5·39 | 318260 | 20 |
| 41 | 9·636886 | 4·37 | 9·954823 | 1·01 | 9·682063 | 5·39 | 10·317937 | 19 |
| 42 | 637148 | 4·37 | 954762 | 1·01 | 682387 | 5·39 | 317613 | 18 |
| 43 | 637411 | 4·37 | 954701 | 1·01 | 682710 | 5·38 | 317290 | 17 |
| 44 | 637673 | 4·37 | 954640 | 1·01 | 683033 | 5·38 | 316967 | 16 |
| 45 | 637935 | 4·36 | 954579 | 1·01 | 683356 | 5·38 | 316644 | 15 |
| 46 | 638197 | 4·36 | 954518 | 1·02 | 683679 | 5·38 | 316321 | 14 |
| 47 | 638458 | 4·36 | 954457 | 1·02 | 684001 | 5·37 | 315999 | 13 |
| 48 | 638720 | 4·35 | 954396 | 1·02 | 684324 | 5·37 | 315676 | 12 |
| 49 | 638981 | 4·35 | 954335 | 1·02 | 684646 | 5·37 | 315354 | 11 |
| 50 | 639242 | 4·35 | 954274 | 1·02 | 684968 | 5·37 | 315032 | 10 |
| 51 | 9·639503 | 4·34 | 9·954213 | 1·02 | 9·685290 | 5·36 | 10·314710 | 9 |
| 52 | 639764 | 4·34 | 954152 | 1·02 | 685612 | 5·36 | 314388 | 8 |
| 53 | 640024 | 4·34 | 954090 | 1·02 | 685934 | 5·36 | 314066 | 7 |
| 54 | 640284 | 4·33 | 954029 | 1·02 | 686255 | 5·36 | 313745 | 6 |
| 55 | 640544 | 4·33 | 953968 | 1·02 | 686577 | 5·35 | 313423 | 5 |
| 56 | 640804 | 4·33 | 953906 | 1·02 | 686898 | 5·35 | 313102 | 4 |
| 57 | 641064 | 4·32 | 953845 | 1·02 | 687219 | 5·35 | 312781 | 3 |
| 58 | 641324 | 4·32 | 953783 | 1·02 | 687540 | 5·35 | 312460 | 2 |
| 59 | 641584 | 4·32 | 953722 | 1·03 | 687861 | 5·34 | 312139 | 1 |
| 60 | 641842 | 4·31 | 953660 | 1·03 | 688182 | 5·34 | 311818 | 0 |
| | Cosine | D. | Sine | 64° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·641842 | 4·31 | 9·953660 | 1·03 | 9·688182 | 5·34 | 10·311818 | 60 |
| 1 | 642101 | 4·31 | 953599 | 1·03 | 688502 | 5·34 | 311498 | 59 |
| 2 | 642360 | 4·31 | 953537 | 1·03 | 688823 | 5·34 | 311177 | 58 |
| 3 | 642618 | 4·30 | 953475 | 1·03 | 689143 | 5·33 | 310857 | 57 |
| 4 | 642877 | 4·30 | 953413 | 1·03 | 689463 | 5·33 | 310537 | 56 |
| 5 | 643135 | 4·30 | 953352 | 1·03 | 689783 | 5·33 | 310217 | 55 |
| 6 | 643393 | 4·30 | 953290 | 1·03 | 690103 | 5·33 | 309897 | 54 |
| 7 | 643650 | 4·29 | 953228 | 1·03 | 690423 | 5·33 | 309577 | 53 |
| 8 | 643908 | 4·29 | 953166 | 1·03 | 690742 | 5·32 | 309258 | 52 |
| 9 | 644165 | 4·29 | 953104 | 1·03 | 691062 | 5·32 | 308938 | 51 |
| 10 | 644423 | 4·28 | 953042 | 1·03 | 691381 | 5·32 | 308619 | 50 |
| 11 | 9·644680 | 4·28 | 9·952980 | 1·04 | 9·691700 | 5·31 | 10·308300 | 49 |
| 12 | 644936 | 4·28 | 952918 | 1·04 | 692019 | 5·31 | 307981 | 48 |
| 13 | 645193 | 4·27 | 952855 | 1·04 | 692338 | 5·31 | 307662 | 47 |
| 14 | 645450 | 4·27 | 952793 | 1·04 | 692656 | 5·31 | 307344 | 46 |
| 15 | 645706 | 4·27 | 952731 | 1·04 | 692975 | 5·31 | 307025 | 45 |
| 16 | 645962 | 4·26 | 952669 | 1·04 | 693293 | 5·30 | 306707 | 44 |
| 17 | 646218 | 4·26 | 952606 | 1·04 | 693612 | 5·30 | 306388 | 43 |
| 18 | 646474 | 4·26 | 952544 | 1·04 | 693930 | 5·30 | 306070 | 42 |
| 19 | 646729 | 4·25 | 952481 | 1·04 | 694248 | 5·30 | 305752 | 41 |
| 20 | 646984 | 4·25 | 952419 | 1·04 | 694566 | 5·29 | 305434 | 40 |
| 21 | 9·647240 | 4·25 | 9·952356 | 1·04 | 9·694883 | 5·29 | 10·305117 | 39 |
| 22 | 647494 | 4·24 | 952294 | 1·04 | 695201 | 5·29 | 304799 | 38 |
| 23 | 647749 | 4·24 | 952231 | 1·04 | 695518 | 5·29 | 304482 | 37 |
| 24 | 648004 | 4·24 | 952168 | 1·05 | 695836 | 5·29 | 304164 | 36 |
| 25 | 648258 | 4·24 | 952106 | 1·05 | 696153 | 5·28 | 303847 | 35 |
| 26 | 648512 | 4·23 | 952043 | 1·05 | 696470 | 5·28 | 303530 | 34 |
| 27 | 648766 | 4·23 | 951980 | 1·05 | 696787 | 5·28 | 303213 | 33 |
| 28 | 649020 | 4·23 | 951917 | 1·05 | 697103 | 5·28 | 302897 | 32 |
| 29 | 649274 | 4·22 | 951854 | 1·05 | 697420 | 5·27 | 302580 | 31 |
| 30 | 649527 | 4·22 | 951791 | 1·05 | 697736 | 5·27 | 302264 | 30 |
| 31 | 9·649781 | 4·22 | 9·951728 | 1·05 | 9·698053 | 5·27 | 10·301947 | 29 |
| 32 | 650034 | 4·22 | 951665 | 1·05 | 698369 | 5·27 | 301631 | 28 |
| 33 | 650287 | 4·21 | 951602 | 1·05 | 698685 | 5·26 | 301315 | 27 |
| 34 | 650539 | 4·21 | 951539 | 1·05 | 699001 | 5·26 | 300999 | 26 |
| 35 | 650792 | 4·21 | 951476 | 1·05 | 699316 | 5·26 | 300684 | 25 |
| 36 | 651044 | 4·20 | 951412 | 1·05 | 699632 | 5·26 | 300368 | 24 |
| 37 | 651297 | 4·20 | 951349 | 1·06 | 699947 | 5·26 | 300053 | 23 |
| 38 | 651549 | 4·20 | 951286 | 1·06 | 700263 | 5·25 | 299737 | 22 |
| 39 | 651800 | 4·19 | 951222 | 1·06 | 700578 | 5·25 | 299422 | 21 |
| 40 | 652052 | 4·19 | 951159 | 1·06 | 700893 | 5·25 | 299107 | 20 |
| 41 | 9 652304 | 4·19 | 9·951096 | 1·06 | 9·701208 | 5·24 | 10·298792 | 19 |
| 42 | 652555 | 4·18 | 951032 | 1·06 | 701523 | 5·24 | 298477 | 18 |
| 43 | 652806 | 4·18 | 950968 | 1·06 | 701837 | 5·24 | 298163 | 17 |
| 44 | 653057 | 4·18 | 950905 | 1·06 | 702152 | 5·24 | 297848 | 16 |
| 45 | 653308 | 4·18 | 950841 | 1·06 | 702466 | 5·24 | 297534 | 15 |
| 46 | 653558 | 4·17 | 950778 | 1·06 | 702780 | 5·23 | 297220 | 14 |
| 47 | 653808 | 4·17 | 950714 | 1·06 | 703095 | 5·23 | 296905 | 13 |
| 48 | 654059 | 4·17 | 950650 | 1·06 | 703409 | 5·23 | 296591 | 12 |
| 49 | 654309 | 4·16 | 950586 | 1·06 | 703723 | 5·23 | 296277 | 11 |
| 50 | 654558 | 4·16 | 950522 | 1·07 | 704036 | 5·22 | 295964 | 10 |
| 51 | 9·654808 | 4·16 | 9·950458 | 1·07 | 9·704350 | 5·22 | 10·295650 | 9 |
| 52 | 655058 | 4·16 | 950394 | 1·07 | 704663 | 5·22 | 295337 | 8 |
| 53 | 655307 | 4·15 | 950330 | 1·07 | 704977 | 5·22 | 295023 | 7 |
| 54 | 655556 | 4·15 | 950266 | 1·07 | 705290 | 5·22 | 294710 | 6 |
| 55 | 655805 | 4·15 | 950202 | 1·07 | 705603 | 5·21 | 294397 | 5 |
| 56 | 656054 | 4·14 | 950138 | 1·07 | 705916 | 5·21 | 294084 | 4 |
| 57 | 656302 | 4·14 | 950074 | 1·07 | 706228 | 5·21 | 293772 | 3 |
| 58 | 656551 | 4·14 | 950010 | 1·07 | 706541 | 5·21 | 293459 | 2 |
| 59 | 656799 | 4·13 | 949945 | 1·07 | 706854 | 5·21 | 293146 | 1 |
| 60 | 657047 | 4·13 | 949881 | 1·07 | 707166 | 5·20 | 292834 | 0 |
| | Cosine | D. | Sine | 63° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·657047 | 4·13 | 9·949881 | 1·07 | 9·707166 | 5·20 | 10·292834 | 60 |
| 1 | 657295 | 4·13 | 949816 | 1·07 | 707478 | 5·20 | 292522 | 59 |
| 2 | 657542 | 4·12 | 949752 | 1·07 | 707790 | 5·20 | 292210 | 58 |
| 3 | 657790 | 4·12 | 949688 | 1·08 | 708102 | 5·20 | 291898 | 57 |
| 4 | 658037 | 4·12 | 949623 | 1·08 | 708414 | 5·19 | 291586 | 56 |
| 5 | 658284 | 4·12 | 949558 | 1·08 | 708726 | 5·19 | 291274 | 55 |
| 6 | 658531 | 4·11 | 949494 | 1·08 | 709037 | 5·19 | 290963 | 54 |
| 7 | 658778 | 4·11 | 949429 | 1·08 | 709349 | 5·19 | 290651 | 53 |
| 8 | 659025 | 4·11 | 949364 | 1·08 | 709660 | 5·19 | 290340 | 52 |
| 9 | 659271 | 4·10 | 949300 | 1·08 | 709971 | 5·18 | 290029 | 51 |
| 10 | 659517 | 4·10 | 949235 | 1·08 | 710282 | 5·18 | 289718 | 50 |
| 11 | 9·659763 | 4·10 | 9·949170 | 1·08 | 9·710593 | 5·18 | 10·289407 | 49 |
| 12 | 660009 | 4·09 | 949105 | 1·08 | 710904 | 5·18 | 289096 | 48 |
| 13 | 660255 | 4·09 | 949040 | 1·08 | 711215 | 5·18 | 288785 | 47 |
| 14 | 660501 | 4·09 | 948975 | 1·08 | 711525 | 5·17 | 288475 | 46 |
| 15 | 660746 | 4·09 | 948910 | 1·08 | 711836 | 5·17 | 288164 | 45 |
| 16 | 660991 | 4·08 | 948845 | 1·08 | 712146 | 5·17 | 287854 | 44 |
| 17 | 661236 | 4·08 | 948780 | 1·09 | 712456 | 5·17 | 287544 | 43 |
| 18 | 661481 | 4·08 | 948715 | 1·09 | 712766 | 5·16 | 287234 | 42 |
| 19 | 661726 | 4·07 | 948650 | 1·09 | 713076 | 5·16 | 286924 | 41 |
| 20 | 661970 | 4·07 | 948584 | 1·09 | 713386 | 5·16 | 286614 | 40 |
| 21 | 9·662214 | 4·07 | 9·948519 | 1·09 | 9·713696 | 5·16 | 10·286304 | 39 |
| 22 | 662459 | 4·07 | 948454 | 1·09 | 714005 | 5·16 | 285995 | 38 |
| 23 | 662703 | 4·06 | 948388 | 1·09 | 714314 | 5·15 | 285686 | 37 |
| 24 | 662946 | 4·06 | 948323 | 1·09 | 714624 | 5·15 | 285376 | 36 |
| 25 | 663190 | 4·06 | 948257 | 1·09 | 714933 | 5·15 | 285067 | 35 |
| 26 | 663433 | 4·05 | 948192 | 1·09 | 715242 | 5·15 | 284758 | 34 |
| 27 | 663677 | 4·05 | 948126 | 1·09 | 715551 | 5·14 | 284449 | 33 |
| 28 | 663920 | 4·05 | 948060 | 1·09 | 715860 | 5·14 | 284140 | 32 |
| 29 | 664163 | 4·05 | 947995 | 1·10 | 716168 | 5·14 | 283832 | 31 |
| 30 | 664406 | 4·04 | 947929 | 1·10 | 716477 | 5·14 | 283523 | 30 |
| 31 | 9·664648 | 4·04 | 9·947863 | 1·10 | 9·716785 | 5·14 | 10·283215 | 29 |
| 32 | 664891 | 4·04 | 947797 | 1·10 | 717093 | 5·13 | 282907 | 28 |
| 33 | 665133 | 4·03 | 947731 | 1·10 | 717401 | 5·13 | 282599 | 27 |
| 34 | 665375 | 4·03 | 947665 | 1·10 | 717709 | 5·13 | 282291 | 26 |
| 35 | 665617 | 4·03 | 947600 | 1·10 | 718017 | 5·13 | 281983 | 25 |
| 36 | 665859 | 4·02 | 947533 | 1·10 | 718325 | 5·13 | 281670 | 24 |
| 37 | 666100 | 4·02 | 947467 | 1·10 | 718633 | 5·12 | 281367 | 23 |
| 38 | 666342 | 4·02 | 947401 | 1·10 | 718940 | 5·12 | 281060 | 22 |
| 39 | 666583 | 4·02 | 947335 | 1·10 | 719248 | 5·12 | 280752 | 21 |
| 40 | 666824 | 4·01 | 947269 | 1·10 | 719555 | 5·12 | 280445 | 20 |
| 41 | 9·667065 | 4·01 | 9·947203 | 1·10 | 9·719862 | 5·12 | 10·280138 | 19 |
| 42 | 667305 | 4·01 | 947136 | 1·11 | 720169 | 5·11 | 279831 | 18 |
| 43 | 667546 | 4·01 | 947070 | 1·11 | 720476 | 5·11 | 279524 | 17 |
| 44 | 667786 | 4·00 | 947004 | 1·11 | 720783 | 5·11 | 279217 | 16 |
| 45 | 668027 | 4·00 | 946937 | 1·11 | 721089 | 5·11 | 278911 | 15 |
| 46 | 668267 | 4·00 | 946871 | 1·11 | 721396 | 5·11 | 278604 | 14 |
| 47 | 668506 | 3·99 | 946804 | 1·11 | 721702 | 5·10 | 278298 | 13 |
| 48 | 668746 | 3·99 | 946738 | 1·11 | 722009 | 5·10 | 277991 | 12 |
| 49 | 668986 | 3·99 | 946671 | 1·11 | 722315 | 5·10 | 277685 | 11 |
| 50 | 669225 | 3·99 | 946604 | 1·11 | 722621 | 5·10 | 277379 | 10 |
| 51 | 9·669464 | 3·98 | 9 946538 | 1·11 | 9·722927 | 5·10 | 10·277073 | 9 |
| 52 | 669703 | 3·98 | 946471 | 1·11 | 723232 | 5·09 | 276768 | 8 |
| 53 | 669942 | 3·98 | 946404 | 1·11 | 723538 | 5·09 | 276462 | 7 |
| 54 | 670181 | 3·97 | 946337 | 1·11 | 723844 | 5·09 | 276156 | 6 |
| 55 | 670419 | 3·97 | 946270 | 1·12 | 724149 | 5·09 | 275851 | 5 |
| 56 | 670658 | 3·97 | 946203 | 1·12 | 724454 | 5·09 | 275546 | 4 |
| 57 | 670896 | 3·97 | 946136 | 1·12 | 724759 | 5·08 | 275241 | 3 |
| 58 | 671134 | 3·96 | 946069 | 1·12 | 725065 | 5·08 | 274935 | 2 |
| 59 | 671372 | 3·96 | 946002 | 1·12 | 725369 | 5·08 | 274631 | 1 |
| 60 | 671609 | 3·96 | 945935 | 1·12 | 725674 | 5·08 | 274326 | 0 |
| | Cosine | D. | Sine | 62° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·671609 | 3·96 | 9·945935 | 1·12 | 9·725674 | 5·08 | 10·274326 | 60 |
| 1 | 671847 | 3·95 | 945868 | 1·12 | 725979 | 5·08 | 274021 | 59 |
| 2 | 672084 | 3·95 | 945800 | 1·12 | 726284 | 5·07 | 273716 | 58 |
| 3 | 672321 | 3·95 | 945733 | 1·12 | 726588 | 5·07 | 273412 | 57 |
| 4 | 672558 | 3·95 | 945666 | 1·12 | 726892 | 5·07 | 273108 | 56 |
| 5 | 672795 | 3·94 | 945598 | 1·12 | 727197 | 5·07 | 272803 | 55 |
| 6 | 673032 | 3·94 | 945531 | 1·12 | 727501 | 5·07 | 272499 | 54 |
| 7 | 673268 | 3·94 | 945464 | 1·13 | 727805 | 5·06 | 272195 | 53 |
| 8 | 673505 | 3·94 | 945396 | 1·13 | 728109 | 5·06 | 271891 | 52 |
| 9 | 673741 | 3·93 | 945328 | 1·13 | 728412 | 5·06 | 271588 | 51 |
| 10 | 673977 | 3·93 | 945261 | 1·13 | 728716 | 5·06 | 271284 | 50 |
| 11 | 9·674213 | 3·93 | 9·945193 | 1·13 | 9·729020 | 5·06 | 10·270980 | 49 |
| 12 | 674448 | 3·92 | 945125 | 1·13 | 729323 | 5·05 | 270677 | 48 |
| 13 | 674684 | 3·92 | 945058 | 1·13 | 729626 | 5·05 | 270374 | 47 |
| 14 | 674919 | 3·92 | 944990 | 1·13 | 729929 | 5·05 | 270071 | 46 |
| 15 | 675155 | 3·92 | 944922 | 1·13 | 730233 | 5·05 | 269767 | 45 |
| 16 | 675390 | 3·91 | 944854 | 1·13 | 730535 | 5·05 | 269465 | 44 |
| 17 | 675624 | 3·91 | 944786 | 1·13 | 730838 | 5·04 | 269162 | 43 |
| 18 | 675859 | 3·91 | 944718 | 1·13 | 731141 | 5·04 | 268859 | 42 |
| 19 | 676094 | 3·91 | 944650 | 1·13 | 731444 | 5·04 | 268556 | 41 |
| 20 | 676328 | 3·90 | 944582 | 1·14 | 731746 | 5·04 | 268254 | 40 |
| 21 | 9·676562 | 3·90 | 9·944514 | 1·14 | 9·732048 | 5·04 | 10·267952 | 39 |
| 22 | 676796 | 3·90 | 944446 | 1·14 | 732351 | 5·03 | 267649 | 38 |
| 23 | 677030 | 3·90 | 944377 | 1·14 | 732653 | 5·03 | 267347 | 37 |
| 24 | 677264 | 3·89 | 944309 | 1·14 | 732955 | 5·03 | 267045 | 36 |
| 25 | 677498 | 3·89 | 944241 | 1·14 | 733257 | 5·03 | 266743 | 35 |
| 26 | 677731 | 3·89 | 944172 | 1·14 | 733558 | 5·03 | 266442 | 34 |
| 27 | 677964 | 3·88 | 944104 | 1·14 | 733860 | 5·02 | 266140 | 33 |
| 28 | 678197 | 3·88 | 944036 | 1·14 | 734162 | 5·02 | 265838 | 32 |
| 29 | 678430 | 3·88 | 943967 | 1·14 | 734463 | 5·02 | 265537 | 31 |
| 30 | 678663 | 3·88 | 943899 | 1·14 | 734764 | 5·02 | 265236 | 30 |
| 31 | 9·678895 | 3·87 | 9·943830 | 1·14 | 9·735066 | 5·02 | 10·264934 | 29 |
| 32 | 679128 | 3·87 | 943761 | 1·14 | 735367 | 5·02 | 264633 | 28 |
| 33 | 679360 | 3·87 | 943693 | 1·15 | 735668 | 5·01 | 264332 | 27 |
| 34 | 679592 | 3·87 | 943624 | 1·15 | 735969 | 5·01 | 264031 | 26 |
| 35 | 679824 | 3·86 | 943555 | 1·15 | 736269 | 5·01 | 263731 | 25 |
| 36 | 680056 | 3·86 | 943486 | 1·15 | 736570 | 5·01 | 263430 | 24 |
| 37 | 680288 | 3·86 | 943417 | 1·15 | 736871 | 5·01 | 263129 | 23 |
| 38 | 680519 | 3·85 | 943348 | 1·15 | 737171 | 5·00 | 262829 | 22 |
| 39 | 680750 | 3·85 | 943279 | 1·15 | 737471 | 5·00 | 262529 | 21 |
| 40 | 680982 | 3·85 | 943210 | 1·15 | 737771 | 5·00 | 262229 | 20 |
| 41 | 9·681213 | 3·85 | 9·943141 | 1·15 | 9·738071 | 5·00 | 10·261929 | 19 |
| 42 | 681443 | 3·84 | 943072 | 1·15 | 738371 | 5·00 | 261629 | 18 |
| 43 | 681674 | 3·84 | 943003 | 1·15 | 738671 | 4·99 | 261329 | 17 |
| 44 | 681905 | 3·84 | 942934 | 1·15 | 738971 | 4·99 | 261029 | 16 |
| 45 | 682135 | 3·84 | 942864 | 1·15 | 739271 | 4·99 | 260729 | 15 |
| 46 | 682365 | 3·83 | 942795 | 1·16 | 739570 | 4·99 | 260430 | 14 |
| 47 | 682595 | 3·83 | 942726 | 1·16 | 739870 | 4·99 | 260130 | 13 |
| 48 | 682825 | 3·83 | 942656 | 1·16 | 740169 | 4·99 | 259831 | 12 |
| 49 | 683055 | 3·83 | 942587 | 1·16 | 740468 | 4·98 | 259532 | 11 |
| 50 | 683284 | 3·82 | 942517 | 1·16 | 740767 | 4·98 | 259233 | 10 |
| 51 | 9·683514 | 3·82 | 9·942448 | 1·16 | 9·741066 | 4·98 | 10·258934 | 9 |
| 52 | 683743 | 3·82 | 942378 | 1·16 | 741365 | 4·98 | 258635 | 8 |
| 53 | 683972 | 3·82 | 942308 | 1·16 | 741664 | 4·98 | 258336 | 7 |
| 54 | 684201 | 3·81 | 942239 | 1·16 | 741962 | 4·97 | 258038 | 6 |
| 55 | 684430 | 3·81 | 942169 | 1·16 | 742261 | 4·97 | 257739 | 5 |
| 56 | 684658 | 3·81 | 942099 | 1·16 | 742559 | 4·97 | 257441 | 4 |
| 57 | 684887 | 3·80 | 942029 | 1·16 | 742858 | 4·97 | 257142 | 3 |
| 58 | 685115 | 3·80 | 941959 | 1·16 | 743156 | 4·97 | 256844 | 2 |
| 59 | 685343 | 3·80 | 941889 | 1·17 | 743454 | 4·97 | 256546 | 1 |
| 60 | 685571 | 3·80 | 941819 | 1·17 | 743752 | 4·96 | 256248 | 0 |
| | Cosine | D. | Sine | 61° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·685571 | 3·80 | 9·941819 | 1·17 | 9·743752 | 4·96 | 10·256248 | 60 |
| 1 | 685799 | 3·79 | 941749 | 1·17 | 744050 | 4·96 | 255950 | 59 |
| 2 | 686027 | 3·79 | 941679 | 1·17 | 744348 | 4·96 | 255652 | 58 |
| 3 | 686254 | 3·79 | 941609 | 1·17 | 744645 | 4·96 | 255355 | 57 |
| 4 | 686482 | 3·79 | 941539 | 1·17 | 744943 | 4·96 | 255057 | 56 |
| 5 | 686709 | 3·78 | 941469 | 1·17 | 745240 | 4·96 | 254760 | 55 |
| 6 | 686936 | 3·78 | 941398 | 1·17 | 745538 | 4·95 | 254462 | 54 |
| 7 | 687163 | 3·78 | 941328 | 1·17 | 745835 | 4·95 | 254165 | 53 |
| 8 | 687389 | 3·78 | 941258 | 1·17 | 746132 | 4·95 | 253868 | 52 |
| 9 | 687616 | 3·77 | 941187 | 1·17 | 746429 | 4·95 | 253571 | 51 |
| 10 | 687843 | 3·77 | 941117 | 1·17 | 746726 | 4·95 | 253274 | 50 |
| 11 | 9·688069 | 3·77 | 9·941046 | 1·18 | 9·747023 | 4·94 | 10·252977 | 49 |
| 12 | 688295 | 3·77 | 940975 | 1·18 | 747319 | 4·94 | 252681 | 48 |
| 13 | 688521 | 3·76 | 940005 | 1·18 | 747616 | 4·94 | 252384 | 47 |
| 14 | 688747 | 3·76 | 940834 | 1·18 | 747913 | 4·94 | 252087 | 46 |
| 15 | 688972 | 3·76 | 940763 | 1·18 | 748209 | 4·94 | 251791 | 45 |
| 16 | 689198 | 3·76 | 940693 | 1·18 | 748505 | 4·93 | 251495 | 44 |
| 17 | 689423 | 3·75 | 940622 | 1·18 | 748801 | 4·93 | 251199 | 43 |
| 18 | 689648 | 3·75 | 940551 | 1·18 | 749097 | 4·93 | 250903 | 42 |
| 19 | 689873 | 3·75 | 940480 | 1·18 | 749393 | 4·93 | 250607 | 41 |
| 20 | 690098 | 3·75 | 940409 | 1·18 | 749689 | 4·93 | 250311 | 40 |
| 21 | 9·690323 | 3·74 | 9·940338 | 1·18 | 9·749985 | 4·93 | 10·250015 | 39 |
| 22 | 690548 | 3·74 | 940267 | 1·18 | 750281 | 4·92 | 249719 | 38 |
| 23 | 690772 | 3·74 | 940196 | 1·18 | 750576 | 4·92 | 249424 | 37 |
| 24 | 690996 | 3·74 | 940125 | 1·19 | 750872 | 4·92 | 249128 | 36 |
| 25 | 691220 | 3·73 | 940054 | 1·19 | 751167 | 4·92 | 248833 | 35 |
| 26 | 691444 | 3·73 | 939982 | 1·19 | 751462 | 4·92 | 248538 | 34 |
| 27 | 691668 | 3·73 | 939911 | 1·19 | 751757 | 4·92 | 248243 | 33 |
| 28 | 691892 | 3·73 | 939840 | 1·19 | 752052 | 4·91 | 247948 | 32 |
| 29 | 692115 | 3·72 | 939768 | 1·19 | 752347 | 4·91 | 247653 | 31 |
| 30 | 692339 | 3·72 | 939697 | 1·19 | 752642 | 4·91 | 247358 | 30 |
| 31 | 9·692562 | 3·72 | 9·939625 | 1·19 | 9·752937 | 4·91 | 10·247063 | 29 |
| 32 | 692785 | 3·71 | 939554 | 1·19 | 753231 | 4·91 | 246769 | 28 |
| 33 | 693008 | 3·71 | 939482 | 1·19 | 753526 | 4·91 | 246474 | 27 |
| 34 | 693231 | 3·71 | 939410 | 1·19 | 753820 | 4·90 | 246180 | 26 |
| 35 | 693453 | 3·71 | 939339 | 1·19 | 754115 | 4·90 | 245885 | 25 |
| 36 | 693676 | 3·70 | 939267 | 1·20 | 754409 | 4·90 | 245591 | 24 |
| 37 | 693898 | 3·70 | 939195 | 1·20 | 754703 | 4·90 | 245297 | 23 |
| 38 | 694120 | 3·70 | 939123 | 1·20 | 754997 | 4·90 | 245003 | 22 |
| 39 | 694342 | 3·70 | 939052 | 1·20 | 755291 | 4·90 | 244709 | 21 |
| 40 | 694564 | 3·69 | 938980 | 1·20 | 755585 | 4·89 | 244415 | 20 |
| 41 | 9·694786 | 3·69 | 9·938908 | 1·20 | 9·755878 | 4·89 | 10·244122 | 19 |
| 42 | 695007 | 3·69 | 938836 | 1·20 | 756172 | 4·89 | 243828 | 18 |
| 43 | 695229 | 3·69 | 938763 | 1·20 | 756465 | 4·89 | 243535 | 17 |
| 44 | 695450 | 3·68 | 938691 | 1·20 | 756759 | 4·89 | 243241 | 16 |
| 45 | 695671 | 3·68 | 938619 | 1·20 | 757052 | 4·89 | 242948 | 15 |
| 46 | 695892 | 3·68 | 938547 | 1·20 | 757345 | 4·88 | 242655 | 14 |
| 47 | 696113 | 3·68 | 938475 | 1·20 | 757638 | 4·88 | 242362 | 13 |
| 48 | 696334 | 3·67 | 938402 | 1·21 | 757931 | 4·88 | 242069 | 12 |
| 49 | 696554 | 3·67 | 938330 | 1·21 | 758224 | 4·88 | 241776 | 11 |
| 50 | 696775 | 3·67 | 938258 | 1·21 | 758517 | 4·88 | 241483 | 10 |
| 51 | 9·696995 | 3·67 | 9·938185 | 1·21 | 9·758810 | 4·88 | 10·241190 | 9 |
| 52 | 697215 | 3·66 | 938113 | 1·21 | 759102 | 4·87 | 240898 | 8 |
| 53 | 697435 | 3·66 | 938040 | 1·21 | 759395 | 4·87 | 240605 | 7 |
| 54 | 697654 | 3·66 | 937967 | 1·21 | 759687 | 4·87 | 240313 | 6 |
| 55 | 697874 | 3·66 | 937895 | 1·21 | 759979 | 4·87 | 240021 | 5 |
| 56 | 698094 | 3·65 | 937822 | 1·21 | 760272 | 4·87 | 239728 | 4 |
| 57 | 698313 | 3·65 | 937749 | 1·21 | 760564 | 4·87 | 239436 | 3 |
| 58 | 698532 | 3·65 | 937676 | 1·21 | 760856 | 4·86 | 239144 | 2 |
| 59 | 698751 | 3·65 | 937604 | 1·21 | 761148 | 4·86 | 238852 | 1 |
| 60 | 698970 | 3·64 | 937531 | 1·21 | 761439 | 4·86 | 238561 | 0 |
| | Cosine | D. | Sine | 60° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·698970 | 3·64 | 9·937531 | 1·21 | 9·761439 | 4·86 | 10·238561 | 60 |
| 1 | 699189 | 3·64 | 937458 | 1·22 | 761731 | 4·86 | 238269 | 59 |
| 2 | 699407 | 3·64 | 937385 | 1·22 | 762023 | 4·86 | 237977 | 58 |
| 3 | 699626 | 3·64 | 937312 | 1·22 | 762314 | 4·86 | 237686 | 57 |
| 4 | 699844 | 3·63 | 937238 | 1·22 | 762606 | 4·85 | 237394 | 56 |
| 5 | 700062 | 3·63 | 937165 | 1·22 | 762897 | 4·85 | 237103 | 55 |
| 6 | 700280 | 3·63 | 937092 | 1·22 | 763188 | 4·85 | 236812 | 54 |
| 7 | 700498 | 3·63 | 937019 | 1·22 | 763479 | 4·85 | 236521 | 53 |
| 8 | 700716 | 3·63 | 936946 | 1·22 | 763770 | 4·85 | 236230 | 52 |
| 9 | 700933 | 3·62 | 936872 | 1·22 | 764061 | 4·85 | 235939 | 51 |
| 10 | 701151 | 3·62 | 936799 | 1·22 | 764352 | 4·84 | 235648 | 50 |
| 11 | 9·701368 | 3·62 | 9·936725 | 1·22 | 9·764643 | 4·84 | 10·235357 | 49 |
| 12 | 701585 | 3·62 | 936652 | 1·23 | 764933 | 4·84 | 235067 | 48 |
| 13 | 701802 | 3·61 | 936578 | 1·23 | 765224 | 4·84 | 234776 | 47 |
| 14 | 702019 | 3·61 | 936505 | 1·23 | 765514 | 4·84 | 234486 | 46 |
| 15 | 702236 | 3·61 | 936431 | 1·23 | 765805 | 4·84 | 234195 | 45 |
| 16 | 702452 | 3·61 | 936357 | 1·23 | 766095 | 4·84 | 233905 | 44 |
| 17 | 702669 | 3·60 | 936284 | 1·23 | 766385 | 4·83 | 233615 | 43 |
| 18 | 702885 | 3·60 | 936210 | 1·23 | 766675 | 4·83 | 233325 | 42 |
| 19 | 703101 | 3·60 | 936136 | 1·23 | 766965 | 4·83 | 233035 | 41 |
| 20 | 703317 | 3·60 | 936062 | 1·23 | 767255 | 4·83 | 232745 | 40 |
| 21 | 9·703533 | 3·59 | 9·935988 | 1·23 | 9·767545 | 4·83 | 10·232455 | 39 |
| 22 | 703749 | 3·59 | 935914 | 1·23 | 767834 | 4·83 | 232166 | 38 |
| 23 | 703964 | 3·59 | 935840 | 1·23 | 768124 | 4·82 | 231876 | 37 |
| 24 | 704179 | 3·59 | 935766 | 1·24 | 768413 | 4·82 | 231587 | 36 |
| 25 | 704395 | 3·59 | 935692 | 1·24 | 768703 | 4·82 | 231297 | 35 |
| 26 | 704610 | 3·58 | 935618 | 1·24 | 768992 | 4·82 | 231008 | 34 |
| 27 | 704825 | 3·58 | 935543 | 1·24 | 769281 | 4·82 | 230719 | 33 |
| 28 | 705040 | 3·58 | 935469 | 1·24 | 769570 | 4·82 | 230430 | 32 |
| 29 | 705254 | 3·58 | 935395 | 1·24 | 769860 | 4·81 | 230140 | 31 |
| 30 | 705469 | 3·57 | 935320 | 1·24 | 770148 | 4·81 | 229852 | 30 |
| 31 | 9·705683 | 3·57 | 9·935246 | 1·24 | 9·770437 | 4·81 | 10·229563 | 29 |
| 32 | 705898 | 3·57 | 935171 | 1·24 | 770726 | 4·81 | 229274 | 28 |
| 33 | 706112 | 3·57 | 935097 | 1·24 | 771015 | 4·81 | 228985 | 27 |
| 34 | 706326 | 3·56 | 935022 | 1·24 | 771303 | 4·81 | 228697 | 26 |
| 35 | 706539 | 3·56 | 934948 | 1·24 | 771592 | 4·81 | 228408 | 25 |
| 36 | 706753 | 3·56 | 934873 | 1·24 | 771880 | 4·80 | 228120 | 24 |
| 37 | 706967 | 3·56 | 934798 | 1·25 | 772168 | 4·80 | 227832 | 23 |
| 38 | 707180 | 3·55 | 934723 | 1·25 | 772457 | 4·80 | 227543 | 22 |
| 39 | 707393 | 3·55 | 934649 | 1·25 | 772745 | 4·80 | 227255 | 21 |
| 40 | 707606 | 3·55 | 934574 | 1·25 | 773033 | 4·80 | 226967 | 20 |
| 41 | 9·707819 | 3·55 | 9·934499 | 1·25 | 9·773321 | 4·80 | 10·226679 | 19 |
| 42 | 708032 | 3·54 | 934424 | 1·25 | 773608 | 4·79 | 226392 | 18 |
| 43 | 708245 | 3·54 | 934349 | 1·25 | 773896 | 4·79 | 226104 | 17 |
| 44 | 708458 | 3·54 | 934274 | 1·25 | 774184 | 4·79 | 225816 | 16 |
| 45 | 708670 | 3·54 | 934199 | 1·25 | 774471 | 4·79 | 225529 | 15 |
| 46 | 708882 | 3·53 | 934123 | 1·25 | 774759 | 4·79 | 225241 | 14 |
| 47 | 709094 | 3·53 | 934048 | 1·25 | 775046 | 4·79 | 224954 | 13 |
| 48 | 709306 | 3·53 | 933973 | 1·25 | 775333 | 4·79 | 224667 | 12 |
| 49 | 709518 | 3·53 | 933898 | 1·26 | 775621 | 4·78 | 224379 | 11 |
| 50 | 709730 | 3·53 | 933822 | 1·26 | 775908 | 4·78 | 224092 | 10 |
| 51 | 9·709941 | 3·52 | 9·933747 | 1·26 | 9·776195 | 4·78 | 10·223805 | 9 |
| 52 | 710153 | 3·52 | 933671 | 1·26 | 776482 | 4·78 | 223518 | 8 |
| 53 | 710364 | 3·52 | 933596 | 1·26 | 776769 | 4·78 | 223231 | 7 |
| 54 | 710575 | 3·52 | 933520 | 1·26 | 777055 | 4·78 | 222945 | 6 |
| 55 | 710786 | 3·51 | 933445 | 1·26 | 777342 | 4·78 | 222658 | 5 |
| 56 | 710997 | 3·51 | 933369 | 1·26 | 777628 | 4·77 | 222372 | 4 |
| 57 | 711208 | 3·51 | 933293 | 1·26 | 777915 | 4·77 | 222085 | 3 |
| 58 | 711419 | 3·51 | 933217 | 1·26 | 778201 | 4·77 | 221799 | 2 |
| 59 | 711629 | 3·50 | 933141 | 1·26 | 778487 | 4·77 | 221512 | 1 |
| 60 | 711839 | 3·50 | 933066 | 1·26 | 778774 | 4·77 | 221226 | 0 |
| | Cosine | D. | Sine | 59° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·711839 | 3·50 | 9·933066 | 1·26 | 9·778774 | 4·77 | 10·221226 | 60 |
| 1 | 712050 | 3·50 | 932990 | 1·27 | 779060 | 4·77 | 220940 | 59 |
| 2 | 712260 | 3·50 | 932914 | 1·27 | 779346 | 4·76 | 220654 | 58 |
| 3 | 712469 | 3·49 | 932838 | 1·27 | 779632 | 4·76 | 220368 | 57 |
| 4 | 712679 | 3·49 | 932762 | 1·27 | 779918 | 4·76 | 220082 | 56 |
| 5 | 712889 | 3·49 | 932685 | 1·27 | 780203 | 4·76 | 219797 | 55 |
| 6 | 713098 | 3·49 | 932609 | 1·27 | 780489 | 4·76 | 219511 | 54 |
| 7 | 713308 | 3·49 | 932533 | 1·27 | 780775 | 4·76 | 219225 | 53 |
| 8 | 713517 | 3·48 | 932457 | 1·27 | 781060 | 4·76 | 218940 | 52 |
| 9 | 713726 | 3·48 | 932380 | 1·27 | 781346 | 4·75 | 218654 | 51 |
| 10 | 713935 | 3·48 | 932304 | 1·27 | 781631 | 4·75 | 218369 | 50 |
| 11 | 9·714144 | 3·48 | 9·932228 | 1·27 | 9·781916 | 4·75 | 10·218084 | 49 |
| 12 | 714352 | 3·47 | 932151 | 1·27 | 782201 | 4·75 | 217799 | 48 |
| 13 | 714561 | 3·47 | 932075 | 1·28 | 782486 | 4·75 | 217514 | 47 |
| 14 | 714769 | 3·47 | 931998 | 1·28 | 782771 | 4·75 | 217229 | 46 |
| 15 | 714978 | 3·47 | 931921 | 1·28 | 783056 | 4·75 | 216944 | 45 |
| 16 | 715186 | 3·47 | 931845 | 1·28 | 783341 | 4·75 | 216659 | 44 |
| 17 | 715394 | 3·46 | 931768 | 1·28 | 783626 | 4·74 | 216374 | 43 |
| 18 | 715602 | 3·46 | 931691 | 1·28 | 783910 | 4·74 | 216090 | 42 |
| 19 | 715809 | 3·46 | 931614 | 1·28 | 784195 | 4·74 | 215805 | 41 |
| 20 | 716017 | 3·46 | 931537 | 1·28 | 784479 | 4·74 | 215521 | 40 |
| 21 | 9·716224 | 3·45 | 9·931460 | 1·28 | 9·784764 | 4·74 | 10·215236 | 39 |
| 22 | 716432 | 3·45 | 931383 | 1·28 | 785048 | 4·74 | 214952 | 38 |
| 23 | 716639 | 3·45 | 931306 | 1·28 | 785332 | 4·73 | 214668 | 37 |
| 24 | 716846 | 3·45 | 931229 | 1·29 | 785616 | 4·73 | 214384 | 36 |
| 25 | 717053 | 3·45 | 931152 | 1·29 | 785900 | 4·73 | 214100 | 35 |
| 26 | 717259 | 3·44 | 931075 | 1·29 | 786184 | 4·73 | 213816 | 34 |
| 27 | 717466 | 3·44 | 930998 | 1·29 | 786468 | 4·73 | 213532 | 33 |
| 28 | 717673 | 3·44 | 930921 | 1·29 | 786752 | 4·73 | 213248 | 32 |
| 29 | 717879 | 3·44 | 930843 | 1·29 | 787036 | 4·73 | 212964 | 31 |
| 30 | 718085 | 3·43 | 930766 | 1·29 | 787319 | 4·72 | 212681 | 30 |
| 31 | 9·718291 | 3·43 | 9·930688 | 1·29 | 9·787603 | 4·72 | 10·212397 | 29 |
| 32 | 718497 | 3·43 | 930611 | 1·29 | 787886 | 4·72 | 212114 | 28 |
| 33 | 718703 | 3·43 | 930533 | 1·29 | 788170 | 4·72 | 211830 | 27 |
| 34 | 718909 | 3·43 | 930456 | 1·29 | 788453 | 4·72 | 211547 | 26 |
| 35 | 719114 | 3·42 | 930378 | 1·29 | 788736 | 4·72 | 211264 | 25 |
| 36 | 719320 | 3·42 | 930300 | 1·30 | 789019 | 4·72 | 210981 | 24 |
| 37 | 719525 | 3·42 | 930223 | 1·30 | 789302 | 4·71 | 210698 | 23 |
| 38 | 719730 | 3·42 | 930145 | 1·30 | 789585 | 4·71 | 210415 | 22 |
| 39 | 719935 | 3·41 | 930067 | 1·30 | 789868 | 4·71 | 210132 | 21 |
| 40 | 720140 | 3·41 | 929989 | 1·30 | 790151 | 4·71 | 209849 | 20 |
| 41 | 9·720345 | 3·41 | 9·929911 | 1·30 | 9·790433 | 4·71 | 10·209567 | 19 |
| 42 | 720549 | 3·41 | 929833 | 1·30 | 790716 | 4·71 | 209284 | 18 |
| 43 | 720754 | 3·40 | 929755 | 1·30 | 790999 | 4·71 | 209001 | 17 |
| 44 | 720958 | 3·40 | 929677 | 1·30 | 791281 | 4·71 | 208719 | 16 |
| 45 | 721162 | 3·40 | 929599 | 1·30 | 791563 | 4·70 | 208437 | 15 |
| 46 | 721366 | 3·40 | 929521 | 1·30 | 791846 | 4·70 | 208154 | 14 |
| 47 | 721570 | 3·40 | 929442 | 1·30 | 792128 | 4·70 | 207872 | 13 |
| 48 | 721774 | 3·39 | 929364 | 1·31 | 792410 | 4·70 | 207590 | 12 |
| 49 | 721978 | 3·39 | 929286 | 1·31 | 792692 | 4·70 | 207308 | 11 |
| 50 | 722181 | 3·39 | 929207 | 1·31 | 792974 | 4·70 | 207026 | 10 |
| 51 | 9·722385 | 3·39 | 9·929129 | 1·31 | 9·793256 | 4·70 | 10·206744 | 9 |
| 52 | 722588 | 3·39 | 929050 | 1·31 | 793538 | 4·69 | 206462 | 8 |
| 53 | 722791 | 3·38 | 928972 | 1·31 | 793819 | 4·69 | 206181 | 7 |
| 54 | 722994 | 3·38 | 928893 | 1·31 | 794101 | 4·69 | 205899 | 6 |
| 55 | 723197 | 3·38 | 928815 | 1·31 | 794383 | 4·69 | 205617 | 5 |
| 56 | 723400 | 3·38 | 928736 | 1·31 | 794664 | 4·69 | 205336 | 4 |
| 57 | 723603 | 3·37 | 928657 | 1·31 | 794945 | 4·69 | 205055 | 3 |
| 58 | 723805 | 3·37 | 928578 | 1·31 | 795227 | 4·69 | 204773 | 2 |
| 59 | 724007 | 3·37 | 928499 | 1·31 | 795508 | 4·68 | 204492 | 1 |
| 60 | 724210 | 3·37 | 928420 | 1·31 | 795789 | 4·68 | 204211 | 0 |
| | Cosine | D. | Sine | 58° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·724210 | 3·37 | 9·928420 | 1·32 | 9·795789 | 4·68 | 10·204211 | 60 |
| 1 | 724412 | 3·37 | 928342 | 1·32 | 796070 | 4·68 | 203930 | 59 |
| 2 | 724614 | 3·36 | 928263 | 1·32 | 796351 | 4·68 | 203649 | 58 |
| 3 | 724816 | 3·36 | 928183 | 1·32 | 796632 | 4·68 | 203368 | 57 |
| 4 | 725017 | 3·36 | 928104 | 1·32 | 796913 | 4·68 | 203087 | 56 |
| 5 | 725219 | 3·36 | 928025 | 1·32 | 797194 | 4·68 | 202806 | 55 |
| 6 | 725420 | 3·35 | 927946 | 1·32 | 797475 | 4·68 | 202525 | 54 |
| 7 | 725622 | 3·35 | 927867 | 1·32 | 797755 | 4·68 | 202245 | 53 |
| 8 | 725823 | 3·35 | 927787 | 1·32 | 798036 | 4·67 | 201964 | 52 |
| 9 | 726024 | 3·35 | 927708 | 1·32 | 798316 | 4·67 | 201684 | 51 |
| 10 | 726225 | 3·35 | 927629 | 1·32 | 798596 | 4·67 | 201404 | 50 |
| 11 | 9·726426 | 3·34 | 9·927549 | 1·32 | 9·798877 | 4·67 | 10·201123 | 49 |
| 12 | 726626 | 3·34 | 927470 | 1·33 | 799157 | 4·67 | 200843 | 48 |
| 13 | 726827 | 3·34 | 927390 | 1·33 | 799437 | 4·67 | 200563 | 47 |
| 14 | 727027 | 3·34 | 927310 | 1·33 | 799717 | 4·67 | 200283 | 46 |
| 15 | 727228 | 3·34 | 927231 | 1·33 | 799997 | 4·66 | 200003 | 45 |
| 16 | 727428 | 3·33 | 927151 | 1·33 | 800277 | 4·66 | 199723 | 44 |
| 17 | 727628 | 3·33 | 927071 | 1·33 | 800557 | 4·66 | 199443 | 43 |
| 18 | 727828 | 3·33 | 926991 | 1·33 | 800836 | 4·66 | 199164 | 42 |
| 19 | 728027 | 3·33 | 926911 | 1·33 | 801116 | 4·66 | 198884 | 41 |
| 20 | 728227 | 3·33 | 926831 | 1·33 | 801396 | 4·66 | 198604 | 40 |
| 21 | 9·728427 | 3·32 | 9·926751 | 1·33 | 9·801675 | 4·66 | 10·198325 | 39 |
| 22 | 728626 | 3·32 | 926671 | 1·33 | 801955 | 4·66 | 198045 | 38 |
| 23 | 728825 | 3·32 | 926591 | 1·33 | 802234 | 4·65 | 197766 | 37 |
| 24 | 729024 | 3·32 | 926511 | 1·34 | 802513 | 4·65 | 197487 | 36 |
| 25 | 729223 | 3·31 | 926431 | 1·34 | 802792 | 4·65 | 197208 | 35 |
| 26 | 729422 | 3·31 | 926351 | 1·34 | 803072 | 4·65 | 196928 | 34 |
| 27 | 729621 | 3·31 | 926270 | 1·34 | 803351 | 4·65 | 196649 | 33 |
| 28 | 729820 | 3·31 | 926190 | 1·34 | 803630 | 4·65 | 196370 | 32 |
| 29 | 730018 | 3·30 | 926110 | 1·34 | 803908 | 4·65 | 196092 | 31 |
| 30 | 730216 | 3·30 | 926029 | 1·34 | 804187 | 4·65 | 195813 | 30 |
| 31 | 9·730415 | 3·30 | 9·925949 | 1·34 | 9·804466 | 4·64 | 10·195534 | 29 |
| 32 | 730613 | 3·30 | 925868 | 1·34 | 804745 | 4·64 | 195255 | 28 |
| 33 | 730811 | 3·30 | 925788 | 1·34 | 805023 | 4·64 | 194977 | 27 |
| 34 | 731009 | 3·29 | 925707 | 1·34 | 805302 | 4·64 | 194698 | 26 |
| 35 | 731206 | 3·29 | 925626 | 1·34 | 805580 | 4·64 | 194420 | 25 |
| 36 | 731404 | 3·29 | 925545 | 1·35 | 805859 | 4·64 | 194141 | 24 |
| 37 | 731602 | 3·29 | 925465 | 1·35 | 806137 | 4·64 | 193863 | 23 |
| 38 | 731799 | 3·29 | 925384 | 1·35 | 806415 | 4·63 | 193585 | 22 |
| 39 | 731996 | 3·28 | 925303 | 1·35 | 806693 | 4·63 | 193307 | 21 |
| 40 | 732193 | 3·28 | 925222 | 1·35 | 806971 | 4·63 | 193029 | 20 |
| 41 | 9·732390 | 3·28 | 9·925141 | 1·35 | 9·807249 | 4·63 | 10·192751 | 19 |
| 42 | 732587 | 3·28 | 925060 | 1·35 | 807527 | 4·63 | 192473 | 18 |
| 43 | 732784 | 3·28 | 924979 | 1·35 | 807805 | 4·63 | 192195 | 17 |
| 44 | 732980 | 3·27 | 924897 | 1·35 | 808083 | 4·63 | 191917 | 16 |
| 45 | 733177 | 3·27 | 924816 | 1·35 | 808361 | 4·63 | 191639 | 15 |
| 46 | 733373 | 3·27 | 924735 | 1·36 | 808638 | 4·62 | 191362 | 14 |
| 47 | 733569 | 3·27 | 924654 | 1·36 | 808916 | 4·62 | 191084 | 13 |
| 48 | 733765 | 3·27 | 924572 | 1·36 | 809193 | 4·62 | 190807 | 12 |
| 49 | 733961 | 3·26 | 924491 | 1·36 | 809471 | 4·62 | 190529 | 11 |
| 50 | 734157 | 3·26 | 924409 | 1·36 | 809748 | 4·62 | 190252 | 10 |
| 51 | 9·734353 | 3·26 | 9·924328 | 1·36 | 9·810025 | 4·62 | 10·189975 | 9 |
| 52 | 734549 | 3·26 | 924246 | 1·36 | 810302 | 4·62 | 189698 | 8 |
| 53 | 734744 | 3·25 | 924164 | 1·36 | 810580 | 4·62 | 189420 | 7 |
| 54 | 734939 | 3·25 | 924083 | 1·36 | 810857 | 4·62 | 189143 | 6 |
| 55 | 735135 | 3·25 | 924001 | 1·36 | 811134 | 4·61 | 188866 | 5 |
| 56 | 735330 | 3·25 | 923919 | 1·36 | 811410 | 4·61 | 188590 | 4 |
| 57 | 735525 | 3·25 | 923837 | 1·36 | 811687 | 4·61 | 188313 | 3 |
| 58 | 735719 | 3·24 | 923755 | 1·37 | 811964 | 4·61 | 188036 | 2 |
| 59 | 735914 | 3·24 | 923673 | 1·37 | 812241 | 4·61 | 187759 | 1 |
| 60 | 736109 | 3·24 | 923591 | 1·37 | 812517 | 4·61 | 187483 | 0 |
| | Cosine | D. | Sine | 57° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·736109 | 3·24 | 9·923591 | 1·37 | 9·812517 | 4·61 | 10·187483 | 60 |
| 1 | 736303 | 3·24 | 923509 | 1·37 | 812794 | 4·61 | 187206 | 59 |
| 2 | 736498 | 3·24 | 923427 | 1·37 | 813070 | 4·61 | 186930 | 58 |
| 3 | 736692 | 3·23 | 923345 | 1·37 | 813347 | 4·60 | 186653 | 57 |
| 4 | 736886 | 3·23 | 923263 | 1·37 | 813623 | 4·60 | 186377 | 56 |
| 5 | 737080 | 3·23 | 923181 | 1·37 | 813899 | 4·60 | 186101 | 55 |
| 6 | 737274 | 3·23 | 923098 | 1·37 | 814175 | 4·60 | 185825 | 54 |
| 7 | 737467 | 3·23 | 923016 | 1·37 | 814452 | 4·60 | 185548 | 53 |
| 8 | 737661 | 3·22 | 922933 | 1·37 | 814728 | 4·60 | 185272 | 52 |
| 9 | 737855 | 3·22 | 922851 | 1·37 | 815004 | 4·60 | 184996 | 51 |
| 10 | 738048 | 3·22 | 922768 | 1·38 | 815279 | 4·60 | 184721 | 50 |
| 11 | 9·738241 | 3·22 | 9·922686 | 1·38 | 9·815555 | 4·59 | 10·184445 | 49 |
| 12 | 738434 | 3·22 | 922603 | 1·38 | 815831 | 4·59 | 184169 | 48 |
| 13 | 738627 | 8·21 | 922520 | 1·38 | 816107 | 4·59 | 183893 | 47 |
| 14 | 738820 | 3·21 | 922438 | 1·38 | 816382 | 4·59 | 183618 | 46 |
| 15 | 739013 | 3·21 | 922355 | 1·38 | 816658 | 4·59 | 183342 | 45 |
| 16 | 739206 | 3·21 | 922272 | 1·38 | 816933 | 4·59 | 183067 | 44 |
| 17 | 739398 | 3·21 | 922189 | 1·38 | 817209 | 4·59 | 182791 | 43 |
| 18 | 739590 | 3·20 | 922106 | 1·38 | 817484 | 4·59 | 182516 | 42 |
| 19 | 739783 | 3·20 | 922023 | 1·38 | 817759 | 4·59 | 182241 | 41 |
| 20 | 739975 | 3·20 | 921940 | 1·38 | 818035 | 4·58 | 181965 | 40 |
| 21 | 9·740167 | 3·20 | 9·921857 | 1·39 | 9·818310 | 4·58 | 10·181690 | 39 |
| 22 | 740359 | 3·20 | 921774 | 1·39 | 818585 | 4·58 | 181415 | 38 |
| 23 | 740550 | 3·19 | 921691 | 1·39 | 818860 | 4·58 | 181140 | 37 |
| 24 | 740742 | 3·19 | 921607 | 1·39 | 819135 | 4·58 | 180865 | 36 |
| 25 | 740934 | 3·19 | 921524 | 1·39 | 819410 | 4·58 | 180590 | 35 |
| 26 | 741125 | 3·19 | 921441 | 1·39 | 819684 | 4·58 | 180316 | 34 |
| 27 | 741316 | 3·19 | 921357 | 1·39 | 819959 | 4·58 | 180041 | 33 |
| 28 | 741508 | 3·18 | 921274 | 1·39 | 820234 | 4·58 | 179766 | 32 |
| 29 | 741699 | 3·18 | 921190 | 1·39 | 820508 | 4·57 | 179492 | 31 |
| 30 | 741889 | 3·18 | 921107 | 1·39 | 820783 | 4·57 | 179217 | 30 |
| 31 | 9·742080 | 3·18 | 9·921023 | 1·39 | 9·821057 | 4·57 | 10·178943 | 29 |
| 32 | 742271 | 3·18 | 920939 | 1·40 | 821332 | 4·57 | 178668 | 28 |
| 33 | 742462 | 3·17 | 920856 | 1·40 | 821606 | 4·57 | 178394 | 27 |
| 34 | 742652 | 3·17 | 920772 | 1·40 | 821880 | 4·57 | 178120 | 26 |
| 35 | 742842 | 3·17 | 920688 | 1·40 | 822154 | 4·57 | 177846 | 25 |
| 36 | 743033 | 3·17 | 920604 | 1·40 | 822429 | 4·57 | 177571 | 24 |
| 37 | 743223 | 3·17 | 920520 | 1·40 | 822703 | 4·57 | 177297 | 23 |
| 38 | 743413 | 3·16 | 920436 | 1·40 | 822977 | 4·56 | 177023 | 22 |
| 39 | 743602 | 3·16 | 920352 | 1·40 | 823250 | 4·56 | 176750 | 21 |
| 40 | 743792 | 3·16 | 920268 | 1·40 | 823524 | 4·56 | 176476 | 20 |
| 41 | 9·743982 | 3·16 | 9·920184 | 1·40 | 9·823798 | 4·56 | 10·176202 | 19 |
| 42 | 744171 | 3·16 | 920099 | 1·40 | 824072 | 4·56 | 175928 | 18 |
| 43 | 744361 | 3·15 | 920015 | 1·40 | 824345 | 4·56 | 175655 | 17 |
| 44 | 744550 | 3·15 | 919931 | 1·41 | 824619 | 4·56 | 175381 | 16 |
| 45 | 744739 | 3·15 | 919846 | 1·41 | 824893 | 4·56 | 175107 | 15 |
| 46 | 744928 | 3·15 | 919762 | 1·41 | 825166 | 4·56 | 174834 | 14 |
| 47 | 745117 | 3·15 | 919677 | 1·41 | 825439 | 4·55 | 174561 | 13 |
| 48 | 745306 | 3·14 | 919593 | 1·41 | 825713 | 4·55 | 174287 | 12 |
| 49 | 745494 | 3·14 | 919508 | 1·41 | 825986 | 4·55 | 174014 | 11 |
| 50 | 745683 | 3·14 | 919424 | 1·41 | 826259 | 4·55 | 173741 | 10 |
| 51 | 9·745871 | 3·14 | 9·919339 | 1·41 | 9·826532 | 4·55 | 10·173468 | 9 |
| 52 | 746059 | 3·14 | 919254 | 1·41 | 826805 | 4·55 | 173195 | 8 |
| 53 | 746248 | 3·13 | 919169 | 1·41 | 827078 | 4·55 | 172922 | 7 |
| 54 | 746436 | 4·13 | 919085 | 1·41 | 827351 | 4·55 | 172649 | 6 |
| 55 | 746624 | 3·13 | 919000 | 1·41 | 827624 | 4·55 | 172376 | 5 |
| 56 | 746812 | 3·13 | 918915 | 1·42 | 827897 | 4·54 | 172103 | 4 |
| 57 | 746999 | 3·13 | 918830 | 1·42 | 828170 | 4·54 | 171830 | 3 |
| 58 | 747187 | 3·12 | 918745 | 1·42 | 828442 | 4·54 | 171558 | 2 |
| 59 | 747374 | 3·12 | 918659 | 1·42 | 828715 | 4·54 | 171285 | 1 |
| 60 | 747562 | 3·12 | 918574 | 1·42 | 828987 | 4·54 | 171013 | 0 |
| | Cosine | D. | Sine | 56° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·747562 | 3·12 | 9·918574 | 1·42 | 9·828987 | 4·54 | 10·171013 | 60 |
| 1 | 747749 | 3·12 | 918489 | 1·42 | 829260 | 4·54 | 170740 | 59 |
| 2 | 747936 | 3·12 | 918404 | 1·42 | 829532 | 4·54 | 170468 | 58 |
| 3 | 748123 | 3·11 | 918318 | 1·42 | 829805 | 4·54 | 170195 | 57 |
| 4 | 748310 | 3·11 | 918233 | 1·42 | 830077 | 4·54 | 169923 | 56 |
| 5 | 748497 | 3·11 | 918147 | 1·42 | 830349 | 4·53 | 169651 | 55 |
| 6 | 748683 | 3·11 | 918062 | 1·42 | 830621 | 4·53 | 169379 | 54 |
| 7 | 748870 | 3·11 | 917976 | 1·43 | 830893 | 4·53 | 169107 | 53 |
| 8 | 749056 | 3·10 | 917891 | 1·43 | 831165 | 4·53 | 168835 | 52 |
| 9 | 749243 | 3·10 | 917805 | 1·43 | 831437 | 4·53 | 168563 | 51 |
| 10 | 749429 | 3·10 | 917719 | 1·43 | 831709 | 4·53 | 168291 | 50 |
| 11 | 9·749615 | 3·10 | 9·917634 | 1·43 | 9·831981 | 4·53 | 10·168019 | 49 |
| 12 | 749801 | 3·10 | 917548 | 1·43 | 832253 | 4·53 | 167747 | 48 |
| 13 | 749987 | 3·09 | 917462 | 1·43 | 832525 | 4·53 | 167475 | 47 |
| 14 | 750172 | 3·09 | 917376 | 1·43 | 832796 | 4·53 | 167204 | 46 |
| 15 | 750358 | 3·09 | 917290 | 1·43 | 833068 | 4·52 | 166932 | 45 |
| 16 | 750543 | 3·09 | 917204 | 1·43 | 833339 | 4·52 | 166661 | 44 |
| 17 | 750729 | 3·09 | 917118 | 1·44 | 833611 | 4·52 | 166389 | 43 |
| 18 | 750914 | 3·08 | 917032 | 1·44 | 833882 | 4·52 | 166118 | 42 |
| 19 | 751099 | 3·08 | 916946 | 1·44 | 834154 | 4·52 | 165846 | 41 |
| 20 | 751284 | 3·08 | 916859 | 1·44 | 834425 | 4·52 | 165575 | 40 |
| 21 | 9·751469 | 3·08 | 9·916773 | 1·44 | 9·834696 | 4·52 | 10·165304 | 39 |
| 22 | 751654 | 3·08 | 916687 | 1·44 | 834967 | 4·52 | 165033 | 38 |
| 23 | 751839 | 3·08 | 916600 | 1·44 | 835238 | 4·52 | 164762 | 37 |
| 24 | 752023 | 3·07 | 916514 | 1·44 | 835509 | 4·52 | 164491 | 36 |
| 25 | 752208 | 3·07 | 916427 | 1·44 | 835780 | 4·51 | 164220 | 35 |
| 26 | 752392 | 3·07 | 916341 | 1·44 | 836051 | 4·51 | 163949 | 34 |
| 27 | 752576 | 3·07 | 916254 | 1·44 | 836322 | 4·51 | 163678 | 33 |
| 28 | 752760 | 3·07 | 916167 | 1·45 | 836593 | 4·51 | 163407 | 32 |
| 29 | 752944 | 3·06 | 916081 | 1·45 | 836864 | 4·51 | 163136 | 31 |
| 30 | 753128 | 3·06 | 915994 | 1·45 | 837134 | 4·51 | 162866 | 30 |
| 31 | 9·753312 | 3·06 | 9·915907 | 1·45 | 9·837405 | 4·51 | 10·162595 | 29 |
| 32 | 753495 | 3·06 | 915820 | 1·45 | 837675 | 4·51 | 162325 | 28 |
| 33 | 753679 | 3·06 | 915733 | 1·45 | 837946 | 4·51 | 162054 | 27 |
| 34 | 753862 | 3·05 | 915646 | 1·45 | 838216 | 4·51 | 161784 | 26 |
| 35 | 754046 | 3·05 | 915559 | 1·45 | 838487 | 4·50 | 161513 | 25 |
| 36 | 754229 | 3·05 | 915472 | 1·45 | 838757 | 4·50 | 161243 | 24 |
| 37 | 754412 | 3·05 | 915385 | 1·45 | 839027 | 4·50 | 160973 | 23 |
| 38 | 754595 | 3·05 | 915297 | 1·45 | 839297 | 4·50 | 160703 | 22 |
| 39 | 754778 | 3·04 | 915210 | 1·45 | 839568 | 4·50 | 160432 | 21 |
| 40 | 754960 | 3·04 | 915123 | 1·46 | 839838 | 4·50 | 160162 | 20 |
| 41 | 9·755143 | 3·04 | 9·915035 | 1·46 | 9·840108 | 4·50 | 10·159892 | 19 |
| 42 | 755326 | 3·04 | 914948 | 1·46 | 840378 | 4·50 | 159622 | 18 |
| 43 | 755508 | 3·04 | 914860 | 1·46 | 840647 | 4·50 | 159353 | 17 |
| 44 | 755690 | 3·04 | 914773 | 1·46 | 840917 | 4·49 | 159083 | 16 |
| 45 | 755872 | 3·03 | 914685 | 1·46 | 841187 | 4·49 | 158813 | 15 |
| 46 | 756054 | 3·03 | 914598 | 1·46 | 841457 | 4·49 | 158543 | 14 |
| 47 | 756236 | 3·03 | 914510 | 1·46 | 841726 | 4·49 | 158274 | 13 |
| 48 | 756418 | 3·03 | 914422 | 1·46 | 841996 | 4·49 | 158004 | 12 |
| 49 | 756600 | 3·03 | 914334 | 1·46 | 842266 | 4·49 | 157734 | 11 |
| 50 | 756782 | 3·02 | 914246 | 1·47 | 842535 | 4·49 | 157465 | 10 |
| 51 | 9·756963 | 3·02 | 9·914158 | 1·47 | 9·842805 | 4·49 | 10·157195 | 9 |
| 52 | 757144 | 3·02 | 914070 | 1·47 | 843074 | 4·49 | 156926 | 8 |
| 53 | 757326 | 3·02 | 913982 | 1·47 | 843343 | 4·49 | 156657 | 7 |
| 54 | 757507 | 3·02 | 913894 | 1·47 | 843612 | 4·49 | 156388 | 6 |
| 55 | 757688 | 3·01 | 913806 | 1·47 | 843882 | 4·48 | 156118 | 5 |
| 56 | 757869 | 3·01 | 913718 | 1·47 | 844151 | 4·48 | 155849 | 4 |
| 57 | 758050 | 3·01 | 913630 | 1·47 | 844420 | 4·48 | 155580 | 3 |
| 58 | 758230 | 3·01 | 913541 | 1·47 | 844689 | 4·48 | 155311 | 2 |
| 59 | 758411 | 3·01 | 913453 | 1·47 | 844958 | 4·48 | 155042 | 1 |
| 60 | 758591 | 3·01 | 913365 | 1·47 | 845227 | 4·48 | 154773 | 0 |
| | Cosine | D. | Sine | 55° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·758591 | 3·01 | 9·913365 | 1·47 | 9·845227 | 4·48 | 10·154773 | 60 |
| 1 | 758772 | 3·00 | 913276 | 1·47 | 845496 | 4·48 | 154504 | 59 |
| 2 | 758952 | 3·00 | 913187 | 1·48 | 845764 | 4·48 | 154236 | 58 |
| 3 | 759132 | 3·00 | 913099 | 1·48 | 846033 | 4·48 | 153967 | 57 |
| 4 | 759312 | 3·00 | 913010 | 1·48 | 846302 | 4·48 | 153698 | 56 |
| 5 | 759492 | 3·00 | 912922 | 1·48 | 846570 | 4·47 | 153430 | 55 |
| 6 | 759672 | 2·99 | 912833 | 1·48 | 846839 | 4·47 | 153161 | 54 |
| 7 | 759852 | 2·99 | 912744 | 1·48 | 847107 | 4·47 | 152893 | 53 |
| 8 | 760031 | 2·99 | 912655 | 1·48 | 847376 | 4·47 | 152624 | 52 |
| 9 | 760211 | 2·99 | 912566 | 1·48 | 847644 | 4·47 | 152356 | 51 |
| 10 | 760390 | 2·99 | 912477 | 1·48 | 847913 | 4·47 | 152087 | 50 |
| 11 | 9·760569 | 2·98 | 9·912388 | 1·48 | 9·848181 | 4·47 | 10·151819 | 49 |
| 12 | 760748 | 2·98 | 912299 | 1·49 | 848449 | 4·47 | 151551 | 48 |
| 13 | 760927 | 2·98 | 912210 | 1·49 | 848717 | 4·47 | 151283 | 47 |
| 14 | 761106 | 2·98 | 912121 | 1·49 | 848986 | 4·47 | 151014 | 46 |
| 15 | 761285 | 2·98 | 912031 | 1·49 | 849254 | 4·47 | 150746 | 45 |
| 16 | 761464 | 2·98 | 911942 | 1·49 | 849522 | 4·47 | 150478 | 44 |
| 17 | 761642 | 2·97 | 911853 | 1·49 | 849790 | 4·46 | 150210 | 43 |
| 18 | 761821 | 2·97 | 911763 | 1·49 | 850058 | 4·46 | 149942 | 42 |
| 19 | 761999 | 2·97 | 911674 | 1·49 | 850325 | 4·46 | 149675 | 41 |
| 20 | 762177 | 2·97 | 911584 | 1·49 | 850593 | 4·46 | 149407 | 40 |
| 21 | 9·762356 | 2·97 | 9·911495 | 1·49 | 9·850861 | 4·46 | 10·149139 | 39 |
| 22 | 762534 | 2·96 | 911405 | 1·49 | 851129 | 4·46 | 148871 | 38 |
| 23 | 762712 | 2·96 | 911315 | 1·50 | 851396 | 4·46 | 148604 | 37 |
| 24 | 762889 | 2·96 | 911226 | 1·50 | 851664 | 4·46 | 148336 | 36 |
| 25 | 763067 | 2·96 | 911136 | 1·50 | 851931 | 4·46 | 148069 | 35 |
| 26 | 763245 | 2·96 | 911046 | 1·50 | 852199 | 4·46 | 147801 | 34 |
| 27 | 763422 | 2·96 | 910956 | 1·50 | 852466 | 4·46 | 147534 | 33 |
| 28 | 763600 | 2·95 | 910866 | 1·50 | 852733 | 4·45 | 147267 | 32 |
| 29 | 763777 | 2·95 | 910776 | 1·50 | 853001 | 4·45 | 146999 | 31 |
| 30 | 763954 | 2·95 | 910686 | 1·50 | 853268 | 4·45 | 146732 | 30 |
| 31 | 9·764131 | 2·95 | 9·910596 | 1·50 | 9·853535 | 4·45 | 10·146465 | 29 |
| 32 | 764308 | 2·95 | 910506 | 1·50 | 853802 | 4·45 | 146198 | 28 |
| 33 | 764485 | 2·94 | 910415 | 1·50 | 854069 | 4·45 | 145931 | 27 |
| 34 | 764662 | 2·94 | 910325 | 1·51 | 854336 | 4·45 | 145664 | 26 |
| 35 | 764838 | 2·94 | 910235 | 1·51 | 854603 | 4·45 | 145397 | 25 |
| 36 | 765015 | 2·94 | 910144 | 1·51 | 854870 | 4·45 | 145130 | 24 |
| 37 | 765191 | 2·94 | 910054 | 1·51 | 855137 | 4·45 | 144863 | 23 |
| 38 | 765367 | 2·94 | 909963 | 1·51 | 855404 | 4·45 | 144596 | 22 |
| 39 | 765544 | 2·93 | 909873 | 1·51 | 855671 | 4·44 | 144329 | 21 |
| 40 | 765720 | 2·93 | 909782 | 1·51 | 855938 | 4·44 | 144062 | 20 |
| 41 | 9·765896 | 2·93 | 9·909691 | 1·51 | 9·856204 | 4·44 | 10·143796 | 19 |
| 42 | 766072 | 2·93 | 909601 | 1·51 | 856471 | 4·44 | 143529 | 18 |
| 43 | 766247 | 2·93 | 909510 | 1·51 | 856737 | 4·44 | 143263 | 17 |
| 44 | 766423 | 2·93 | 909419 | 1·51 | 857004 | 4·44 | 142996 | 16 |
| 45 | 766598 | 2·92 | 909328 | 1·52 | 857270 | 4·44 | 142730 | 15 |
| 46 | 766774 | 2·92. | 909237 | 1·52 | 857537 | 4·44 | 142463 | 14 |
| 47 | 766949 | 2·92 | 909146 | 1·52 | 857803 | 4·44 | 142197 | 13 |
| 48 | 767124 | 2·92 | 909055 | 1·52 | 858069 | 4·44 | 141931 | 12 |
| 49 | 767300 | 2·92 | 908964 | 1·52 | 858336 | 4·44 | 141664 | 11 |
| 50 | 767475 | 2·91 | 908873 | 1·52 | 858602 | 4·43 | 141398 | 10 |
| 51 | 9·767649 | 2·91 | 9·908781 | 1·52 | 9·858868 | 4·43 | 10·141132 | 9 |
| 52 | 767824 | 2·91 | 908690 | 1·52 | 859134 | 4·43 | 140866 | 8 |
| 53 | 767999 | 2·91 | 908599 | 1·52 | 859400 | 4·43 | 140600 | 7 |
| 54 | 768173 | 2·91 | 908507 | 1·52 | 859666 | 4·43 | 140334 | 6 |
| 55 | 768348 | 2·90 | 908416 | 1·53 | 859932 | 4·43 | 140068 | 5 |
| 56 | 768522 | 2·90 | 908324 | 1·53 | 860198 | 4·43 | 139802 | 4 |
| 57 | 768697 | 2·90 | 908233 | 1·53 | 860464 | 4·43 | 139536 | 3 |
| 58 | 768871 | 2·90 | 908141 | 1·53 | 860730 | 4·43 | 139270 | 2 |
| 59 | 769045 | 2·90 | 908049 | 1·53 | 860995 | 4·43 | 139005 | 1 |
| 60 | 769219 | 2·90 | 907958 | 1·53 | 861261 | 4·43 | 138739 | 0 |
| | Cosine | D. | Sine | 54° | Cotang. | D. | Tang. | M. |

1956 ;
14. 5 /

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·769219 | 2·90 | 9·907958 | 1·53 | 9·861261 | 4·43 | 10·138739 | 60 |
| 1 | 769393 | 2·89 | 907866 | 1·53 | 861527 | 4·43 | 138473 | 59 |
| 2 | 769566 | 2·89 | 907774 | 1·53 | 861792 | 4·42 | 138208 | 58 |
| 3 | 769740 | 2·89 | 907682 | 1·53 | 862058 | 4·42 | 137942 | 57 |
| 4 | 769913 | 2·89 | 907590 | 1·53 | 862323 | 4·42 | 137677 | 56 |
| 5 | 770087 | 2·89 | 907498 | 1·53 | 862589 | 4·42 | 137411 | 55 |
| 6 | 770260 | 2·88 | 907406 | 1·53 | 862854 | 4·42 | 137146 | 54 |
| 7 | 770433 | 2·88 | 907314 | 1·54 | 863119 | 4·42 | 136881 | 53 |
| 8 | 770606 | 2·88 | 907222 | 1·54 | 863385 | 4·42 | 136615 | 52 |
| 9 | 770779 | 2·88 | 907129 | 1·54 | 863650 | 4·42 | 136350 | 51 |
| 10 | 770952 | 2·88 | 907037 | 1·54 | 863915 | 4·42 | 136085 | 50 |
| 11 | 9·771125 | 2·88 | 9·906945 | 1·54 | 9·864180 | 4·42 | 10·135820 | 49 |
| 12 | 771298 | 2·87 | 906852 | 1·54 | 864445 | 4·42 | 135555 | 48 |
| 13 | 771470 | 2·87 | 906760 | 1·54 | 864710 | 4·42 | 135290 | 47 |
| 14 | 771643 | 2·87 | 906667 | 1·54 | 864975 | 4·41 | 135025 | 46 |
| 15 | 771815 | 2·87 | 906575 | 1·54 | 865240 | 4·41 | 134760 | 45 |
| 16 | 771987 | 2·87 | 906482 | 1·54 | 865505 | 4·41 | 134495 | 44 |
| 17 | 772159 | 2·87 | 906389 | 1·55 | 865770 | 4·41 | 134230 | 43 |
| 18 | 772331 | 2·86 | 906296 | 1·55 | 866035 | 4·41 | 133965 | 42 |
| 19 | 772503 | 2·86 | 906204 | 1·55 | 866300 | 4·41 | 133700 | 41 |
| 20 | 772675 | 2·86 | 906111 | 1·55 | 866564 | 4·41 | 133436 | 40 |
| 21 | 9·772847 | 2·86 | 9·906018 | 1·55 | 9·866829 | 4·41 | 10·133171 | 39 |
| 22 | 773018 | 2·86 | 905925 | 1·55 | 867094 | 4·41 | 132906 | 38 |
| 23 | 773190 | 2·86 | 905832 | 1·55 | 867358 | 4·41 | 132642 | 37 |
| 24 | 773361 | 2·85 | 905739 | 1·55 | 867623 | 4·41 | 132377 | 36 |
| 25 | 773533 | 2·85 | 905645 | 1·55 | 867887 | 4·41 | 132113 | 35 |
| 26 | 773704 | 2·85 | 905552 | 1·55 | 868152 | 4·40 | 131848 | 34 |
| 27 | 773875 | 2·85 | 905459 | 1·55 | 868416 | 4·40 | 131584 | 33 |
| 28 | 774046 | 2·85 | 905366 | 1·56 | 868680 | 4·40 | 131320 | 32 |
| 29 | 774217 | 2·85 | 905272 | 1·56 | 868945 | 4·40 | 131055 | 31 |
| 30 | 774388 | 2·84 | 905179 | 1·56 | 869209 | 4·40 | 130794 | 30 |
| 31 | 9·774558 | 2·84 | 9·905085 | 1·56 | 9·869473 | 4·40 | 10·130527 | 29 |
| 32 | 774729 | 2·84 | 904992 | 1·56 | 869737 | 4·40 | 130263 | 28 |
| 33 | 774899 | 2·84 | 904898 | 1·56 | 870001 | 4·40 | 129999 | 27 |
| 34 | 775070 | 2·84 | 904804 | 1·56 | 870265 | 4·40 | 129735 | 26 |
| 35 | 775240 | 2·84 | 904711 | 1·56 | 870529 | 4·40 | 129471 | 25 |
| 36 | 775410 | 2·83 | 904617 | 1·56 | 870793 | 4·40 | 129207 | 24 |
| 37 | 775580 | 2·83 | 904523 | 1·56 | 871057 | 4·40 | 128943 | 23 |
| 38 | 775750 | 2·83 | 904429 | 1·57 | 871321 | 4·40 | 128679 | 22 |
| 39 | 775920 | 2·83 | 904335 | 1·57 | 871585 | 4·40 | 128415 | 21 |
| 40 | 776090 | 2·83 | 904241 | 1·57 | 871849 | 4·39 | 128151 | 20 |
| 41 | 9·776259 | 2·83 | 9·904147 | 1·57 | 9·872112 | 4·39 | 10·127888 | 19 |
| 42 | 776429 | 2·82 | 904053 | 1·57 | 872376 | 4·39 | 127624 | 18 |
| 43 | 776598 | 2·82 | 903959 | 1·57 | 872640 | 4·39 | 127360 | 17 |
| 44 | 776768 | 2·82 | 903864 | 1·57 | 872903 | 4·39 | 127097 | 16 |
| 45 | 776937 | 2·82 | 903770 | 1·57 | 873167 | 4·39 | 126833 | 15 |
| 46 | 777106 | 2·82 | 903676 | 1·57 | 873430 | 4·39 | 126570 | 14 |
| 47 | 777275 | 2·81 | 903581 | 1·57 | 873694 | 4·39 | 126306 | 13 |
| 48 | 777444 | 2·81 | 903487 | 1·57 | 873957 | 4·39 | 126043 | 12 |
| 49 | 777613 | 2·81 | 903392 | 1·58 | 874220 | 4·39 | 125780 | 11 |
| 50 | 777781 | 2·81 | 903298 | 1·58 | 874484 | 4·39 | 125516 | 10 |
| 51 | 9·777950 | 2·81 | 9·903203 | 1·58 | 9·874747 | 4·39 | 10·125253 | 9 |
| 52 | 778119 | 2·81 | 903108 | 1·58 | 875010 | 4·39 | 124990 | 8 |
| 53 | 778287 | 2·80 | 903014 | 1·58 | 875273 | 4·38 | 124727 | 7 |
| 54 | 778455 | 2·80 | 902919 | 1·58 | 875536 | 4·38 | 124464 | 6 |
| 55 | 778624 | 2·80 | 902824 | 1·58 | 875800 | 4·38 | 124200 | 5 |
| 56 | 778792 | 2·80 | 902729 | 1·58 | 876063 | 4·38 | 123937 | 4 |
| 57 | 778960 | 2·80 | 902634 | 1·58 | 876326 | 4·38 | 123674 | 3 |
| 58 | 779126 | 2·80 | 902539 | 1·59 | 876589 | 4·38 | 123411 | 2 |
| 59 | 779295 | 2·79 | 902444 | 1·59 | 876851 | 4·38 | 123149 | 1 |
| 60 | 779463 | 2·79 | 902349 | 1·59 | 877114 | 4·38 | 122886 | 0 |
| | Cosine | D. | Sine | 53° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·779463 | 2·79 | 9·902349 | 1·59 | 9·877114 | 4·38 | 10·122886 | 60 |
| 1 | 779631 | 2·79 | 902253 | 1·59 | 877377 | 4·38 | 122623 | 59 |
| 2 | 779798 | 2·79 | 902158 | 1·59 | 877640 | 4·38 | 122360 | 58 |
| 3 | 779966 | 2·79 | 902063 | 1·59 | 877903 | 4·38 | 122097 | 57 |
| 4 | 780133 | 2·79 | 901967 | 1·59 | 878165 | 4·38 | 121835 | 56 |
| 5 | 780300 | 2·78 | 901872 | 1·59 | 878428 | 4·38 | 121572 | 55 |
| 6 | 780467 | 2·78 | 901776 | 1·59 | 878691 | 4·38 | 121309 | 54 |
| 7 | 780634 | 2·78 | 901681 | 1·59 | 878953 | 4·37 | 121047 | 53 |
| 8 | 780801 | 2·78 | 901585 | 1·59 | 879216 | 4·37 | 120784 | 52 |
| 9 | 780968 | 2·78 | 901490 | 1·59 | 879478 | 4·37 | 120522 | 51 |
| 10 | 781134 | 2·78 | 901394 | 1·60 | 879741 | 4·37 | 120259 | 50 |
| 11 | 9·781301 | 2·77 | 9·901298 | 1·60 | 9·880003 | 4·37 | 10·119997 | 49 |
| 12 | 781468 | 2·77 | 901202 | 1·60 | 880265 | 4·37 | 119735 | 48 |
| 13 | 781634 | 2·77 | 901106 | 1·60 | 880528 | 4·37 | 119472 | 47 |
| 14 | 781800 | 2·77 | 901010 | 1·60 | 880790 | 4·37 | 119210 | 46 |
| 15 | 781966 | 2·77 | 900914 | 1·60 | 881052 | 4·37 | 118948 | 45 |
| 16 | 782132 | 2·77 | 900818 | 1·60 | 881314 | 4·37 | 118686 | 44 |
| 17 | 782298 | 2·76 | 900722 | 1·60 | 881576 | 4·37 | 118424 | 43 |
| 18 | 782464 | 2·76 | 900626 | 1·60 | 881839 | 4·37 | 118161 | 42 |
| 19 | 782630 | 2·76 | 900529 | 1·60 | 882101 | 4·37 | 117899 | 41 |
| 20 | 782796 | 2·76 | 900433 | 1·61 | 882363 | 4·36 | 117637 | 40 |
| 21 | 9·782961 | 2·76 | 9·900337 | 1·61 | 9·882625 | 4·36 | 10·117375 | 39 |
| 22 | 783127 | 2·76 | 900240 | 1·61 | 882887 | 4·36 | 117113 | 38 |
| 23 | 783292 | 2·75 | 900144 | 1·61 | 883148 | 4·36 | 116852 | 37 |
| 24 | 783458 | 2·75 | 900047 | 1·61 | 883410 | 4·36 | 116590 | 36 |
| 25 | 783623 | 2·75 | 899951 | 1·61 | 883672 | 4·36 | 116328 | 35 |
| 26 | 783788 | 2·75 | 899854 | 1·61 | 883934 | 4·36 | 116066 | 34 |
| 27 | 783953 | 2·75 | 899757 | 1·61 | 884196 | 4·36 | 115804 | 33 |
| 28 | 784118 | 2·75 | 899660 | 1·61 | 884457 | 4·36 | 115543 | 32 |
| 29 | 784282 | 2·74 | 899564 | 1·61 | 884719 | 4·36 | 115281 | 31 |
| 30 | 784447 | 2·74 | 899467 | 1·62 | 884980 | 4·36 | 115020 | 30 |
| 31 | 9·784612 | 2·74 | 9·899370 | 1·62 | 9·885242 | 4·36 | 10·114758 | 29 |
| 32 | 784776 | 2·74 | 899273 | 1·62 | 885503 | 4·36 | 114497 | 28 |
| 33 | 784941 | 2·74 | 899176 | 1·62 | 885765 | 4·36 | 114235 | 27 |
| 34 | 785105 | 2·74 | 899078 | 1·62 | 886026 | 4·36 | 113974 | 26 |
| 35 | 785269 | 2·73 | 898981 | 1·62 | 886288 | 4·36 | 113712 | 25 |
| 36 | 785433 | 2·73 | 898884 | 1·62 | 886549 | 4·35 | 113451 | 24 |
| 37 | 785597 | 2·73 | 898787 | 1·62 | 886810 | 4·35 | 113190 | 23 |
| 38 | 785761 | 2·73 | 898689 | 1·62 | 887072 | 4·35 | 112928 | 22 |
| 39 | 785925 | 2·73 | 898592 | 1·62 | 887333 | 4·35 | 112667 | 21 |
| 40 | 786089 | 2·73 | 898494 | 1·63 | 887594 | 4·35 | 112406 | 20 |
| 41 | 9·786252 | 2·72 | 9·898397 | 1·63 | 9·887855 | 4·35 | 10·112145 | 19 |
| 42 | 786416 | 2·72 | 898299 | 1·63 | 888116 | 4·35 | 111884 | 18 |
| 43 | 786579 | 2·72 | 898202 | 1·63 | 888377 | 4·35 | 111623 | 17 |
| 44 | 786742 | 2·72 | 898104 | 1·63 | 888639 | 4·35 | 111361 | 16 |
| 45 | 786906 | 2·72 | 898006 | 1·63 | 888900 | 4·35 | 111100 | 15 |
| 46 | 787069 | 2·72 | 897908 | 1·63 | 889160 | 4·35 | 110840 | 14 |
| 47 | 787232 | 2·71 | 897810 | 1·63 | 889421 | 4·35 | 110579 | 13 |
| 48 | 787395 | 2·71 | 897712 | 1·63 | 889682 | 4·35 | 110318 | 12 |
| 49 | 787557 | 2·71 | 897614 | 1·63 | 889943 | 4·35 | 110057 | 11 |
| 50 | 787720 | 2·71 | 897516 | 1·63 | 890204 | 4·34 | 109796 | 10 |
| 51 | 9·787883 | 2·71 | 9·897418 | 1·64 | 9·890465 | 4·34 | 10·109535 | 9 |
| 52 | 788045 | 2·71 | 897320 | 1·64 | 890725 | 4·34 | 109275 | 8 |
| 53 | 788208 | 2·71 | 897222 | 1·64 | 890986 | 4·34 | 109014 | 7 |
| 54 | 788370 | 2·70 | 897123 | 1·64 | 891247 | 4·34 | 108753 | 6 |
| 55 | 788532 | 2·70 | 897025 | 1·64 | 891507 | 4·34 | 108493 | 5 |
| 56 | 788694 | 2·70 | 896926 | 1·64 | 891768 | 4·34 | 108232 | 4 |
| 57 | 788856 | 2·70 | 896828 | 1·64 | 892028 | 4·34 | 107972 | 3 |
| 58 | 789018 | 2·70 | 896729 | 1·64 | 892289 | 4·34 | 107711 | 2 |
| 59 | 789180 | 2·70 | 896631 | 1·64 | 892549 | 4·34 | 107451 | 1 |
| 60 | 789342 | 2·69 | 896532 | 1·64 | 892810 | 4·34 | 107190 | 0 |
| | Cosine | D. | Sine | 52° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·789342 | 2·69 | 9·896532 | 1·64 | 9·892810 | 4·34 | 10·107190 | 60 |
| 1 | 789504 | 2·69 | 896433 | 1·65 | 893070 | 4·34 | 106930 | 59 |
| 2 | 789665 | 2·69 | 896335 | 1·65 | 893331 | 4·34 | 106669 | 58 |
| 3 | 789827 | 2·69 | 896236 | 1·65 | 893591 | 4·34 | 106409 | 57 |
| 4 | 789988 | 2·69 | 896137 | 1·65 | 893851 | 4·34 | 106149 | 56 |
| 5 | 790149 | 2·69 | 896038 | 1·65 | 894111 | 4·34 | 105889 | 55 |
| 6 | 790310 | 2·68 | 895939 | 1·65 | 894371 | 4·34 | 105629 | 54 |
| 7 | 790471 | 2·68 | 895840 | 1·65 | 894632 | 4·33 | 105368 | 53 |
| 8 | 790632 | 2·68 | 895741 | 1·65 | 894892 | 4·33 | 105108 | 52 |
| 9 | 790793 | 2·68 | 895641 | 1·65 | 895152 | 4·33 | 104848 | 51 |
| 10 | 790954 | 2·68 | 895542 | 1·65 | 895412 | 4·33 | 104588 | 50 |
| 11 | 9·791115 | 2·68 | 9·895443 | 1·66 | 9·895672 | 4·33 | 10·104328 | 49 |
| 12 | 791275 | 2·67 | 895343 | 1·66 | 895932 | 4·33 | 104068 | 48 |
| 13 | 791436 | 2·67 | 895244 | 1·66 | 896192 | 4·33 | 103808 | 47 |
| 14 | 791596 | 2·67 | 895145 | 1·66 | 896452 | 4·33 | 103548 | 46 |
| 15 | 791757 | 2·67 | 895045 | 1·66 | 896712 | 4·33 | 103288 | 45 |
| 16 | 791917 | 2·67 | 894945 | 1·66 | 896971 | 4·33 | 103029 | 44 |
| 17 | 792077 | 2·67 | 894846 | 1·66 | 897231 | 4·33 | 102769 | 43 |
| 18 | 792237 | 2·66 | 894746 | 1·66 | 897491 | 4·33 | 102509 | 42 |
| 19 | 792397 | 2·66 | 894646 | 1·66 | 897751 | 4·33 | 102249 | 41 |
| 20 | 792557 | 2·66 | 894546 | 1·66 | 898010 | 4·33 | 101990 | 40 |
| 21 | 9·792716 | 2·66 | 9·894446 | 1·67 | 9·898270 | 4·33 | 10·101730 | 39 |
| 22 | 792876 | 2·66 | 894346 | 1·67 | 898530 | 4·33 | 101470 | 38 |
| 23 | 793035 | 2·66 | 894246 | 1·67 | 898789 | 4·33 | 101211 | 37 |
| 24 | 793195 | 2·65 | 894146 | 1·67 | 899049 | 4·32 | 100951 | 36 |
| 25 | 793354 | 2·65 | 894046 | 1·67 | 899308 | 4·32 | 100692 | 35 |
| 26 | 793514 | 2·65 | 893946 | 1·67 | 899568 | 4·32 | 100432 | 34 |
| 27 | 793673 | 2·65 | 893846 | 1·67 | 899827 | 4·32 | 100173 | 33 |
| 28 | 793832 | 2·65 | 893745 | 1·67 | 900086 | 4·32 | 099914 | 32 |
| 29 | 793991 | 2·65 | 893645 | 1·67 | 900346 | 4·32 | 099654 | 31 |
| 30 | 794150 | 2·64 | 893544 | 1·67 | 900605 | 4·32 | 099395 | 30 |
| 31 | 9·794308 | 2·64 | 9·893444 | 1·68 | 9·900864 | 4·32 | 10·099136 | 29 |
| 32 | 794467 | 2·64 | 893343 | 1·68 | 901124 | 4·32 | 098876 | 28 |
| 33 | 794626 | 2·64 | 893243 | 1·68 | 901383 | 4·32 | 098617 | 27 |
| 34 | 794784 | 2·64 | 893142 | 1·68 | 901642 | 4·32 | 098358 | 26 |
| 35 | 794942 | 2·64 | 893041 | 1·68 | 901901 | 4·32 | 098099 | 25 |
| 36 | 795101 | 2·64 | 892940 | 1·68 | 902160 | 4·32 | 097840 | 24 |
| 37 | 795259 | 2·63 | 892839 | 1·68 | 902419 | 4·32 | 097581 | 23 |
| 38 | 795417 | 2·63 | 892739 | 1·68 | 902679 | 4·32 | 097321 | 22 |
| 39 | 795575 | 2·63 | 892638 | 1·68 | 902938 | 4·32 | 097062 | 21 |
| 40 | 795733 | 2·63 | 892536 | 1·68 | 903197 | 4·31 | 096803 | 20 |
| 41 | 9·795891 | 2·63 | 9·892435 | 1·69 | 9·903455 | 4·31 | 10·096545 | 19 |
| 42 | 796049 | 2·63 | 892334 | 1·69 | 903714 | 4·31 | 096286 | 18 |
| 43 | 796206 | 2·63 | 892233 | 1·69 | 903973 | 4·31 | 096027 | 17 |
| 44 | 796364 | 2·62 | 892132 | 1·69 | 904232 | 4·31 | 095768 | 16 |
| 45 | 796521 | 2·62 | 892030 | 1·69 | 904491 | 4·31 | 095509 | 15 |
| 46 | 796679 | 2·62 | 891929 | 1·69 | 904750 | 4·31 | 095250 | 14 |
| 47 | 796836 | 2·62 | 891827 | 1·69 | 905008 | 4·31 | 094992 | 13 |
| 48 | 796993 | 2·62 | 891726 | 1·69 | 905267 | 4·31 | 094733 | 12 |
| 49 | 797150 | 2·61 | 891624 | 1·69 | 905526 | 4·31 | 094474 | 11 |
| 50 | 797307 | 2·61 | 891523 | 1·70 | 905784 | 4·31 | 094216 | 10 |
| 51 | 9·797464 | 2·61 | 9·891421 | 1·70 | 9·906043 | 4·31 | 10·093957 | 9 |
| 52 | 797621 | 2·61 | 891319 | 1·70 | 906302 | 4·31 | 093698 | 8 |
| 53 | 797777 | 2·61 | 891217 | 1·70 | 906560 | 4·31 | 093440 | 7 |
| 54 | 797934 | 2·61 | 891115 | 1·70 | 906819 | 4·31 | 093181 | 6 |
| 55 | 798091 | 2·61 | 891013 | 1·70 | 907077 | 4·31 | 092923 | 5 |
| 56 | 798247 | 2·61 | 890911 | 1·70 | 907336 | 4·31 | 092664 | 4 |
| 57 | 798403 | 2·60 | 890809 | 1·70 | 907594 | 4·31 | 092406 | 3 |
| 58 | 798560 | 2·60 | 890707 | 1·70 | 907852 | 4·31 | 092148 | 2 |
| 59 | 798716 | 2·60 | 890605 | 1·70 | 908111 | 4·30 | 091889 | 1 |
| 60 | 798872 | 2·60 | 890503 | 1·70 | 908369 | 4·30 | 091631 | 0 |
| | Cosine | D. | Sine | 51° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·798872 | 2·60 | 9·890503 | 1·70 | 9·908369 | 4·30 | 10·091631 | 60 |
| 1 | 799028 | 2·60 | 890400 | 1·71 | 908628 | 4·30 | 091372 | 59 |
| 2 | 799184 | 2·60 | 890298 | 1·71 | 908886 | 4·30 | 091114 | 58 |
| 3 | 799339 | 2·59 | 890195 | 1·71 | 909144 | 4·30 | 090856 | 57 |
| 4 | 799495 | 2·59 | 890093 | 1·71 | 909402 | 4·30 | 090598 | 56 |
| 5 | 799651 | 2·59 | 889990 | 1·71 | 909660 | 4·30 | 090340 | 55 |
| 6 | 799806 | 2·59 | 889888 | 1·71 | 909918 | 4·30 | 090082 | 54 |
| 7 | 799962 | 2·59 | 889785 | 1·71 | 910177 | 4·30 | 089823 | 53 |
| 8 | 800117 | 2·59 | 889682 | 1·71 | 910435 | 4·30 | 089565 | 52 |
| 9 | 800272 | 2·58 | 889579 | 1·71 | 910693 | 4·30 | 089307 | 51 |
| 10 | 800427 | 2·58 | 889477 | 1·71 | 910951 | 4·30 | 089049 | 50 |
| 11 | 9·800582 | 2·58 | 9·889374 | 1·72 | 9·911209 | 4·30 | 10·088791 | 49 |
| 12 | 800737 | 2·58 | 889271 | 1·72 | 911467 | 4·30 | 088533 | 48 |
| 13 | 800892 | 2·58 | 889168 | 1·72 | 911724 | 4·30 | 088276 | 47 |
| 14 | 801047 | 2·58 | 889064 | 1·72 | 911982 | 4·30 | 088018 | 46 |
| 15 | 801201 | 2·58 | 888961 | 1·72 | 912240 | 4·30 | 087760 | 45 |
| 16 | 801356 | 2·57 | 888858 | 1·72 | 912498 | 4·30 | 087502 | 44 |
| 17 | 801511 | 2·57 | 888755 | 1·72 | 912756 | 4·30 | 087244 | 43 |
| 18 | 801665 | 2·57 | 888651 | 1·72 | 913014 | 4·29 | 086986 | 42 |
| 19 | 801819 | 2·57 | 888548 | 1·72 | 913271 | 4·29 | 086729 | 41 |
| 20 | 801973 | 2·57 | 888444 | 1·73 | 913529 | 4·29 | 086471 | 40 |
| 21 | 9·802128 | 2·57 | 9·888341 | 1·73 | 9·913787 | 4·29 | 10·086213 | 39 |
| 22 | 802282 | 2·56 | 888237 | 1·73 | 914044 | 4·29 | 085956 | 38 |
| 23 | 802436 | 2·56 | 888134 | 1·73 | 914302 | 4·29 | 085698 | 37 |
| 24 | 802589 | 2·56 | 888030 | 1·73 | 914560 | 4·29 | 085440 | 36 |
| 25 | 802743 | 2·56 | 887926 | 1·73 | 914817 | 4·29 | 085183 | 35 |
| 26 | 802897 | 2·56 | 887822 | 1·73 | 915075 | 4·29 | 084925 | 34 |
| 27 | 803050 | 2·56 | 887718 | 1·73 | 915332 | 4·29 | 084668 | 33 |
| 28 | 803204 | 2·56 | 887614 | 1·73 | 915590 | 4·29 | 084410 | 32 |
| 29 | 803357 | 2·55 | 887510 | 1·73 | 915847 | 4·29 | 084153 | 31 |
| 30 | 803511 | 2·55 | 887406 | 1·74 | 916104 | 4·29 | 083896 | 30 |
| 31 | 9·803664 | 2·55 | 9·887302 | 1·74 | 9·916362 | 4·29 | 10·083638 | 29 |
| 32 | 803817 | 2·55 | 887198 | 1·74 | 916619 | 4·29 | 083381 | 28 |
| 33 | 803970 | 2·55 | 887093 | 1·74 | 916877 | 4·29 | 083123 | 27 |
| 34 | 804123 | 2·55 | 886989 | 1·74 | 917134 | 4·29 | 082866 | 26 |
| 35 | 804276 | 2·54 | 886885 | 1·74 | 917391 | 4·29 | 082609 | 25 |
| 36 | 804428 | 2·54 | 886780 | 1·74 | 917648 | 4·29 | 082352 | 24 |
| 37 | 804581 | 2·54 | 886676 | 1·74 | 917905 | 4·29 | 082095 | 23 |
| 38 | 804734 | 2·54 | 886571 | 1·74 | 918163 | 4·28 | 081837 | 22 |
| 39 | 804886 | 2·54 | 886466 | 1·74 | 918420 | 4·28 | 081580 | 21 |
| 40 | 805039 | 2·54 | 886362 | 1·75 | 918677 | 4·28 | 081323 | 20 |
| 41 | 9·805191 | 2·54 | 9·886257 | 1·75 | 9·918934 | 4·28 | 10·081066 | 19 |
| 42 | 805343 | 2·53 | 886152 | 1·75 | 919191 | 4·28 | 080809 | 18 |
| 43 | 805495 | 2·53 | 886047 | 1·75 | 919448 | 4·28 | 080552 | 17 |
| 44 | 805647 | 2·53 | 885942 | 1·75 | 919705 | 4·28 | 080295 | 16 |
| 45 | 805799 | 2·53 | 885837 | 1·75 | 919962 | 4·28 | 080038 | 15 |
| 46 | 805951 | 2·53 | 885732 | 1·75 | 920219 | 4·28 | 079781 | 14 |
| 47 | 806103 | 2·53 | 885627 | 1·75 | 920476 | 4·28 | 079524 | 13 |
| 48 | 806254 | 2·53 | 885522 | 1·75 | 920733 | 4·28 | 079267 | 12 |
| 49 | 806406 | 2·52 | 885416 | 1·75 | 920990 | 4·28 | 079010 | 11 |
| 50 | 806557 | 2·52 | 885311 | 1·76 | 921247 | 4·28 | 078753 | 10 |
| 51 | 9·806709 | 2·52 | 9·885205 | 1·76 | 9·921503 | 4·28 | 10·078497 | 9 |
| 52 | 806860 | 2·52 | 885100 | 1·76 | 921760 | 4·28 | 078240 | 8 |
| 53 | 807011 | 2·52 | 884994 | 1·76 | 922017 | 4·28 | 077983 | 7 |
| 54 | 807163 | 2·52 | 884889 | 1·76 | 922274 | 4·28 | 077726 | 6 |
| 55 | 807314 | 2·52 | 884783 | 1·76 | 922530 | 4·28 | 077470 | 5 |
| 56 | 807465 | 2·51 | 884677 | 1·76 | 922787 | 4·28 | 077213 | 4 |
| 57 | 807615 | 2·51 | 884572 | 1·76 | 923044 | 4·28 | 076956 | 3 |
| 58 | 807766 | 2·51 | 884466 | 1·76 | 923300 | 4·28 | 076700 | 2 |
| 59 | 807917 | 2·51 | 884360 | 1·76 | 923557 | 4·27 | 076443 | 1 |
| 60 | 808067 | 2·51 | 884254 | 1·77 | 923813 | 4·27 | 076187 | 0 |
| | Cosine | D. | Sine | 50° | Cotang. | D. | Tang. | M |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·808067 | 2·51 | 9·884254 | 1·77 | 9·923813 | 4·27 | 10·076187 | 60 |
| 1 | 808218 | 2·51 | 884148 | 1·77 | 924070 | 4·27 | 075930 | 59 |
| 2 | 808368 | 2·51 | 884042 | 1·77 | 924327 | 4·27 | 075673 | 58 |
| 3 | 808519 | 2·50 | 883936 | 1·77 | 924583 | 4·27 | 075417 | 57 |
| 4 | 808669 | 2·50 | 883829 | 1·77 | 924840 | 4·27 | 075160 | 56 |
| 5 | 808819 | 2·50 | 883723 | 1·77 | 925096 | 4·27 | 074904 | 55 |
| 6 | 808969 | 2·50 | 883617 | 1·77 | 925352 | 4·27 | 074648 | 54 |
| 7 | 809119 | 2·50 | 883510 | 1·77 | 925609 | 4·27 | 074391 | 53 |
| 8 | 809269 | 2·50 | 883404 | 1·77 | 925865 | 4·27 | 074135 | 52 |
| 9 | 809419 | 2·49 | 883297 | 1·78 | 926122 | 4·27 | 073878 | 51 |
| 10 | 809569 | 2·49 | 883191 | 1·78 | 926378 | 4·27 | 073622 | 50 |
| 11 | 9·809718 | 2·49 | 9·883084 | 1·78 | 9·926634 | 4·27 | 10·073366 | 49 |
| 12 | 809868 | 2·49 | 882977 | 1·78 | 926890 | 4·27 | 073110 | 48 |
| 13 | 810017 | 2·49 | 882871 | 1·78 | 927147 | 4·27 | 072853 | 47 |
| 14 | 810167 | 2·49 | 882764 | 1·78 | 927403 | 4·27 | 072597 | 46 |
| 15 | 810316 | 2·48 | 882657 | 1·78 | 927659 | 4·27 | 072341 | 45 |
| 16 | 810465 | 2·48 | 882550 | 1·78 | 927915 | 4·27 | 072085 | 44 |
| 17 | 810614 | 2·48 | 882443 | 1·78 | 928171 | 4·27 | 071829 | 43 |
| 18 | 810763 | 2·48 | 882336 | 1·79 | 928427 | 4·27 | 071573 | 42 |
| 19 | 810912 | 2·48 | 882229 | 1·79 | 928683 | 4·27 | 071317 | 41 |
| 20 | 811061 | 2·48 | 882121 | 1·79 | 928940 | 4·27 | 071060 | 40 |
| 21 | 9·811210 | 2·48 | 9·882014 | 1·79 | 9·929196 | 4·27 | 10 070804 | 39 |
| 22 | 811358 | 2·47 | 881907 | 1·79 | 929452 | 4·27 | 070548 | 38 |
| 23 | 811507 | 2·47 | 881799 | 1·79 | 929708 | 4·27 | 070292 | 37 |
| 24 | 811655 | 2·47 | 881692 | 1·79 | 929964 | 4·26 | 070036 | 36 |
| 25 | 811804 | 2·47 | 881584 | 1·79 | 930220 | 4·26 | 069780 | 35 |
| 26 | 811952 | 2·47 | 881477 | 1·79 | 930475 | 4·26 | 069525 | 34 |
| 27 | 812100 | 2·47 | 881369 | 1·79 | 930731 | 4·26 | 069269 | 33 |
| 28 | 812248 | 2·47 | 881261 | 1·80 | 930987 | 4·26 | 069013 | 32 |
| 29 | 812396 | 2·46 | 881153 | 1·80 | 931243 | 4·26 | 068757 | 31 |
| 30 | 812544 | 2·46 | 881046 | 1·80 | 931499 | 4·26 | 068501 | 30 |
| 31 | 9·812692 | 2·46 | 9·880938 | 1·80 | 9·931755 | 4·26 | 10·068245 | 29 |
| 32 | 812840 | 2·46 | 880830 | 1·80 | 932010 | 4·26 | 067990 | 28 |
| 33 | 812988 | 2·46 | 880722 | 1·80 | 932266 | 4·26 | 067734 | 27 |
| 34 | 813135 | 2·46 | 880613 | 1·80 | 932522 | 4·26 | 067478 | 26 |
| 35 | 813283 | 2·46 | 880505 | 1·80 | 932778 | 4·26 | 067222 | 25 |
| 36 | 813430 | 2·45 | 880397 | 1·80 | 933033 | 4·26 | 066967 | 24 |
| 37 | 813578 | 2·45 | 880289 | 1·81 | 933289 | 4·26 | 066711 | 23 |
| 38 | 813725 | 2·45 | 880180 | 1·81 | 933545 | 4·26 | 066455 | 22 |
| 39 | 813872 | 2·45 | 880072 | 1·81 | 933800 | 4·26 | 066200 | 21 |
| 40 | 814019 | 2·45 | 879963 | 1·81 | 934056 | 4·26 | 065944 | 20 |
| 41 | 9·814166 | 2·45 | 9·879855 | 1·81 | 9·934311 | 4·26 | 10·065689 | 19 |
| 42 | 814313 | 2·45 | 879746 | 1·81 | 934567 | 4·26 | 065433 | 18 |
| 43 | 814460 | 2·44 | 879637 | 1·81 | 934823 | 4·26 | 065177 | 17 |
| 44 | 814607 | 2·44 | 879529 | 1·81 | 935078 | 4·26 | 064922 | 16 |
| 45 | 814753 | 2·44 | 879420 | 1·81 | 935333 | 4·26 | 064667 | 15 |
| 46 | 814900 | 2·44 | 879311 | 1·81 | 935589 | 4·26 | 064411 | 14 |
| 47 | 815046 | 2·44 | 879202 | 1·82 | 935844 | 4·26 | 064156 | 13 |
| 48 | 815193 | 2·44 | 879093 | 1·82 | 936100 | 4·26 | 063900 | 12 |
| 49 | 815339 | 2·44 | 878984 | 1·82 | 936355 | 4·26 | 063645 | 11 |
| 50 | 815485 | 2·43 | 878875 | 1·82 | 936610 | 4·26 | 063390 | 10 |
| 51 | 9·815631 | 2·43 | 9·878766 | 1·82 | 9·936866 | 4·25 | 10·063134 | 9 |
| 52 | 815778 | 2·43 | 878656 | 1·82 | 937121 | 4·25 | 062879 | 8 |
| 53 | 815924 | 2·43 | 878547 | 1·82 | 937376 | 4·25 | 062624 | 7 |
| 54 | 816069 | 2·43 | 878438 | 1·82 | 937632 | 4·25 | 062368 | 6 |
| 55 | 816215 | 2·43 | 878328 | 1·82 | 937887 | 4·25 | 062113 | 5 |
| 56 | 816361 | 2·43 | 878219 | 1·83 | 938142 | 4·25 | 061858 | 4 |
| 57 | 816507 | 2·42 | 878109 | 1·83 | 938398 | 4·25 | 061602 | 3 |
| 58 | 816652 | 2·42 | 877999 | 1·83 | 938653 | 4·25 | 061347 | 2 |
| 59 | 816798 | 2·42 | 877890 | 1·83 | 938908 | 4·25 | 061092 | 1 |
| 60 | 816943 | 2·42 | 877780 | 1·83 | 939163 | 4·25 | 060837 | 0 |
| | Cosine | D. | Sine | 49° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·816943 | 2·42 | 9·877780 | 1·83 | 9·939163 | 4·25 | 10·060837 | 60 |
| 1 | 817088 | 2·42 | 877670 | 1·83 | 939418 | 4·25 | 060582 | 59 |
| 2 | 817233 | 2·42 | 877560 | 1·83 | 939673 | 4·25 | 060327 | 58 |
| 3 | 817379 | 2·42 | 877450 | 1·83 | 939928 | 4·25 | 060072 | 57 |
| 4 | 817524 | 2·41 | 877340 | 1·83 | 940183 | 4·25 | 059817 | 56 |
| 5 | 817668 | 2·41 | 877230 | 1·84 | 940438 | 4·25 | 059562 | 55 |
| 6 | 817813 | 2·41 | 877120 | 1·84 | 940694 | 4·25 | 059306 | 54 |
| 7 | 817958 | 2·41 | 877010 | 1·84 | 940949 | 4·25 | 059051 | 53 |
| 8 | 818103 | 2·41 | 876899 | 1·84 | 941204 | 4·25 | 058796 | 52 |
| 9 | 818247 | 2·41 | 876789 | 1·84 | 941458 | 4·25 | 058542 | 51 |
| 10 | 818392 | 2·41 | 876678 | 1·84 | 941714 | 4·25 | 058286 | 50 |
| 11 | 9·818536 | 2·40 | 9·876568 | 1·84 | 9·941968 | 4·25 | 10·058032 | 49 |
| 12 | 818681 | 2·40 | 876457 | 1·84 | 942223 | 4·25 | 057777 | 48 |
| 13 | 818825 | 2·40 | 876347 | 1·84 | 942478 | 4·25 | 057522 | 47 |
| 14 | 818969 | 2·40 | 876236 | 1·85 | 942733 | 4·25 | 057267 | 46 |
| 15 | 819113 | 2·40 | 876125 | 1·85 | 942988 | 4·25 | 057012 | 45 |
| 16 | 819257 | 2·40 | 876014 | 1·85 | 943243 | 4·25 | 056757 | 44 |
| 17 | 819401 | 2·40 | 875904 | 1·85 | 943498 | 4·25 | 056502 | 43 |
| 18 | 819545 | 2·39 | 875793 | 1·85 | 943752 | 4·25 | 056248 | 42 |
| 19 | 819689 | 2·39 | 875682 | 1·85 | 944007 | 4·25 | 055993 | 41 |
| 20 | 819832 | 2·39 | 875571 | 1·85 | 944262 | 4·25 | 055738 | 40 |
| 21 | 9·819976 | 2·39 | 9·875459 | 1·85 | 9·944517 | 4·25 | 10·055483 | 39 |
| 22 | 820120 | 2·39 | 875348 | 1·85 | 944771 | 4·24 | 055229 | 38 |
| 23 | 820263 | 2·39 | 875237 | 1·85 | 945026 | 4·24 | 054974 | 37 |
| 24 | 820406 | 2·39 | 875126 | 1·86 | 945281 | 4·24 | 054719 | 36 |
| 25 | 820550 | 2·38 | 875014 | 1·86 | 945535 | 4·24 | 054465 | 35 |
| 26 | 820693 | 2·38 | 874903 | 1·86 | 945790 | 4·24 | 054210 | 34 |
| 27 | 820836 | 2·38 | 874791 | 1·86 | 946045 | 4·24 | 053955 | 33 |
| 28 | 820979 | 2·38 | 874680 | 1·86 | 946299 | 4·24 | 053701 | 32 |
| 29 | 821122 | 2·38 | 874568 | 1·86 | 946554 | 4·24 | 053446 | 31 |
| 30 | 821265 | 2·38 | 874456 | 1·86 | 946808 | 4·24 | 053192 | 30 |
| 31 | 9·821407 | 2·38 | 9·874344 | 1·86 | 9·947063 | 4·24 | 10·052937 | 29 |
| 32 | 821550 | 2·38 | 874232 | 1·87 | 947318 | 4·24 | 052682 | 28 |
| 33 | 821693 | 2·37 | 874121 | 1·87 | 947572 | 4·24 | 052428 | 27 |
| 34 | 821835 | 2·37 | 874009 | 1·87 | 947826 | 4·24 | 052174 | 26 |
| 35 | 821977 | 2·37 | 873896 | 1·87 | 948081 | 4·24 | 051919 | 25 |
| 36 | 822120 | 2·37 | 873784 | 1·87 | 948336 | 4·24 | 051664 | 24 |
| 37 | 822262 | 2·37 | 873672 | 1·87 | 948590 | 4·24 | 051410 | 23 |
| 38 | 822404 | 2·37 | 873560 | 1·87 | 948844 | 4·24 | 051156 | 22 |
| 39 | 822546 | 2·37 | 873448 | 1·87 | 949099 | 4·24 | 050901 | 21 |
| 40 | 822688 | 2·36 | 873335 | 1·87 | 949353 | 4·24 | 050647 | 20 |
| 41 | 9·822830 | 2·36 | 9·873223 | 1·87 | 9·949607 | 4·24 | 10·050393 | 19 |
| 42 | 822972 | 2·36 | 873110 | 1·88 | 949862 | 4·24 | 050138 | 18 |
| 43 | 823114 | 2·36 | 872998 | 1·88 | 950116 | 4·24 | 049884 | 17 |
| 44 | 823255 | 2·36 | 872885 | 1·88 | 950370 | 4·24 | 049630 | 16 |
| 45 | 823397 | 2·36 | 872772 | 1·88 | 950625 | 4·24 | 049375 | 15 |
| 46 | 823539 | 2·36 | 872659 | 1·88 | 950879 | 4·24 | 049121 | 14 |
| 47 | 823680 | 2·35 | 872547 | 1·88 | 951133 | 4·24 | 048867 | 13 |
| 48 | 823821 | 2·35 | 872434 | 1·88 | 951388 | 4·24 | 048612 | 12 |
| 49 | 823963 | 2·35 | 872321 | 1·88 | 951642 | 4·24 | 048358 | 11 |
| 50 | 824104 | 2·35 | 872208 | 1·88 | 951896 | 4·24 | 048104 | 10 |
| 51 | 9·824245 | 2·35 | 9·872095 | 1·89 | 9·952150 | 4·24 | 10·047850 | 9 |
| 52 | 824386 | 2·35 | 871981 | 1·89 | 952405 | 4·24 | 047595 | 8 |
| 53 | 824527 | 2·35 | 871868 | 1·89 | 952659 | 4·24 | 047341 | 7 |
| 54 | 824668 | 2·34 | 871755 | 1·89 | 952913 | 4·24 | 047087 | 6 |
| 55 | 824808 | 2·34 | 871641 | 1·89 | 953167 | 4·23 | 046833 | 5 |
| 56 | 824949 | 2·34 | 871528 | 1·89 | 953421 | 4·23 | 046579 | 4 |
| 57 | 825090 | 2·34 | 871414 | 1·89 | 953675 | 4·23 | 046325 | 3 |
| 58 | 825230 | 2·34 | 871301 | 1·89 | 953929 | 4·23 | 046071 | 2 |
| 59 | 825371 | 2·34 | 871187 | 1·89 | 954183 | 4·23 | 045817 | 1 |
| 60 | 825511 | 2·34 | 871073 | 1·90 | 954437 | 4·23 | 045563 | 0 |
| | Cosine. | D. | Sine | 48° | Cotang. | D. | Tang. | |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·825511 | 2·34 | 9·871073 | 1·90 | 9·954437 | 4·23 | 10·045563 | 60 |
| 1 | 825651 | 2·33 | 870960 | 1·90 | 954691 | 4·23 | 045309 | 59 |
| 2 | 825791 | 2·33 | 870846 | 1·90 | 954945 | 4·23 | 045055 | 58 |
| 3 | 825931 | 2·33 | 870732 | 1·90 | 955200 | 4·23 | 044800 | 57 |
| 4 | 826071 | 2·33 | 870618 | 1·90 | 955454 | 4·23 | 044546 | 56 |
| 5 | 826211 | 2·33 | 870504 | 1·90 | 955707 | 4·23 | 044293 | 55 |
| 6 | 826351 | 2·33 | 870390 | 1·90 | 955961 | 4·23 | 044039 | 54 |
| 7 | 826491 | 2·33 | 870276 | 1·90 | 956215 | 4·23 | 043785 | 53 |
| 8 | 826631 | 2·33 | 870161 | 1·90 | 956469 | 4·23 | 043531 | 52 |
| 9 | 826770 | 2·32 | 870047 | 1·91 | 956723 | 4·23 | 043277 | 51 |
| 10 | 826910 | 2·32 | 869933 | 1·91 | 956977 | 4·23 | 043023 | 50 |
| 11 | 9·827049 | 2·32 | 9·869818 | 1·91 | 9·957231 | 4·23 | 10·042769 | 49 |
| 12 | 827189 | 2·32 | 869704 | 1·91 | 957485 | 4·23 | 042515 | 48 |
| 13 | 827328 | 2·32 | 869589 | 1·91 | 957739 | 4·23 | 042261 | 47 |
| 14 | 827467 | 2·32 | 869474 | 1·91 | 957993 | 4·23 | 042007 | 46 |
| 15 | 827606 | 2·32 | 869360 | 1·91 | 958246 | 4·23 | 041754 | 45 |
| 16 | 827745 | 2·32 | 869245 | 1·91 | 958500 | 4·23 | 041500 | 44 |
| 17 | 827884 | 2·31 | 869130 | 1·91 | 958754 | 4·23 | 041246 | 43 |
| 18 | 828023 | 2·31 | 869015 | 1·92 | 959008 | 4·23 | 040992 | 42 |
| 19 | 828162 | 2·31 | 868900 | 1·92 | 959262 | 4·23 | 040738 | 41 |
| 20 | 828301 | 2·31 | 868785 | 1·92 | 959516 | 4·23 | 040484 | 40 |
| 21 | 9·828439 | 2·31 | 9·868670 | 1·92 | 9·959769 | 4·23 | 10·040231 | 39 |
| 22 | 828578 | 2·31 | 868555 | 1·92 | 960023 | 4·23 | 039977 | 38 |
| 23 | 828716 | 2·31 | 868440 | 1·92 | 960277 | 4·23 | 039723 | 37 |
| 24 | 828855 | 2·30 | 868324 | 1·92 | 960531 | 4·23 | 039469 | 36 |
| 25 | 828993 | 2·30 | 868209 | 1·92 | 960784 | 4·23 | 039216 | 35 |
| 26 | 829131 | 2·30 | 868093 | 1·92 | 961038 | 4·23 | 038962 | 34 |
| 27 | 829269 | 2·30 | 867978 | 1·93 | 961291 | 4·23 | 038709 | 33 |
| 28 | 829407 | 2·30 | 867862 | 1·93 | 961545 | 4·23 | 038455 | 32 |
| 29 | 829545 | 2·30 | 867747 | 1·93 | 961799 | 4·23 | 038201 | 31 |
| 30 | 829683 | 2·30 | 867631 | 1·93 | 962052 | 4·23 | 037948 | 30 |
| 31 | 9·829821 | 2·29 | 9·867515 | 1·93 | 9·962306 | 4·23 | 10·037694 | 29 |
| 32 | 829959 | 2·29 | 867399 | 1·93 | 962560 | 4·23 | 037440 | 28 |
| 33 | 830097 | 2·29 | 867283 | 1·93 | 962813 | 4·23 | 037187 | 27 |
| 34 | 830234 | 2·29 | 867167 | 1·93 | 963067 | 4·23 | 036933 | 26 |
| 35 | 830372 | 2·29 | 867051 | 1·93 | 963320 | 4·23 | 036680 | 25 |
| 36 | 830509 | 2·29 | 866935 | 1·94 | 963574 | 4·23 | 036426 | 24 |
| 37 | 830646 | 2·29 | 866819 | 1·94 | 963827 | 4·23 | 036173 | 23 |
| 38 | 830784 | 2·29 | 866703 | 1·94 | 964081 | 4·23 | 035919 | 22 |
| 39 | 830921 | 2·28 | 866586 | 1·94 | 964335 | 4·23 | 035665 | 21 |
| 40 | 831058 | 2·28 | 866470 | 1·94 | 964588 | 4·22 | 035412 | 20 |
| 41 | 9·831195 | 2·28 | 9·866353 | 1·94 | 9·964842 | 4·22 | 10·035158 | 19 |
| 42 | 831332 | 2·28 | 866237 | 1·94 | 965095 | 4·22 | 034905 | 18 |
| 43 | 831469 | 2·28 | 866120 | 1·94 | 965349 | 4·22 | 034651 | 17 |
| 44 | 831606 | 2·28 | 866004 | 1·95 | 965602 | 4·22 | 034398 | 16 |
| 45 | 831742 | 2·28 | 865887 | 1·95 | 965855 | 4·22 | 034145 | 15 |
| 46 | 831879 | 2·28 | 865770 | 1·95 | 966109 | 4·22 | 033891 | 14 |
| 47 | 832015 | 2·27 | 865653 | 1·95 | 966362 | 4·22 | 033638 | 13 |
| 48 | 832152 | 2·27 | 865536 | 1·95 | 966616 | 4·22 | 033384 | 12 |
| 49 | 832288 | 2·27 | 865419 | 1·95 | 966869 | 4·22 | 033131 | 11 |
| 50 | 832425 | 2·27 | 865302 | 1·95 | 967123 | 4·22 | 032877 | 10 |
| 51 | 9·832561 | 2·27 | 9·865185 | 1·95 | 9·967376 | 4·22 | 10·032624 | 9 |
| 52 | 832697 | 2·27 | 865068 | 1·95 | 967629 | 4·22 | 032371 | 8 |
| 53 | 832833 | 2·27 | 864950 | 1·95 | 967883 | 4·22 | 032117 | 7 |
| 54 | 832969 | 2·26 | 864833 | 1·96 | 968136 | 4·22 | 031864 | 6 |
| 55 | 833105 | 2·26 | 864716 | 1·96 | 968389 | 4·22 | 031611 | 5 |
| 56 | 833241 | 2·26 | 864598 | 1·96 | 968643 | 4·22 | 031357 | 4 |
| 57 | 833377 | 2·26 | 864481 | 1·96 | 968896 | 4·22 | 031104 | 3 |
| 58 | 833512 | 2·26 | 864363 | 1·96 | 969149 | 4·22 | 030851 | 2 |
| 59 | 833648 | 2·26 | 864245 | 1·96 | 969403 | 4·22 | 030597 | 1 |
| 60 | 833783 | 2·26 | 864127 | 1·96 | 969656 | 4·22 | 030344 | 0 |
| | Cosine | D. | Sine | 47° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·833783 | 2·26 | 9·864127 | 1·96 | 9·969656 | 4·22 | 10·030344 | 60 |
| 1 | 833919 | 2·25 | 864010 | 1·96 | 969909 | 4·22 | 030091 | 59 |
| 2 | 834054 | 2·25 | 863892 | 1·97 | 970162 | 4·22 | 029838 | 58 |
| 3 | 834189 | 2·25 | 863774 | 1·97 | 970416 | 4·22 | 029584 | 57 |
| 4 | 834325 | 2·25 | 863656 | 1·97 | 970669 | 4·22 | 029331 | 56 |
| 5 | 834460 | 2·25 | 863538 | 1·97 | 970922 | 4·22 | 029078 | 55 |
| 6 | 834595 | 2·25 | 863419 | 1·97 | 971175 | 4·22 | 028825 | 54 |
| 7 | 834730 | 2·25 | 863301 | 1·97 | 971429 | 4·22 | 028571 | 53 |
| 8 | 834865 | 2·25 | 863183 | 1·97 | 971682 | 4·22 | 028318 | 52 |
| 9 | 834999 | 2·24 | 863064 | 1·97 | 971935 | 4·22 | 028065 | 51 |
| 10 | 835134 | 2·24 | 862946 | 1·98 | 972188 | 4·22 | 027812 | 50 |
| 11 | 9·835269 | 2·24 | 9·862827 | 1·98 | 9·972441 | 4·22 | 10·027559 | 49 |
| 12 | 835403 | 2·24 | 862709 | 1·98 | 972694 | 4·22 | 027306 | 48 |
| 13 | 835538 | 2·24 | 862590 | 1·98 | 972948 | 4·22 | 027052 | 47 |
| 14 | 835672 | 2·24 | 862471 | 1·98 | 973201 | 4·22 | 026799 | 46 |
| 15 | 835807 | 2·24 | 862353 | 1·98 | 973454 | 4·22 | 026546 | 45 |
| 16 | 835941 | 2·24 | 862234 | 1·98 | 973707 | 4·22 | 026293 | 44 |
| 17 | 836075 | 2·23 | 862115 | 1·98 | 973960 | 4·22 | 026040 | 43 |
| 18 | 836209 | 2·23 | 861996 | 1·98 | 974213 | 4·22 | 025787 | 42 |
| 19 | 836343 | 2·23 | 861877 | 1·98 | 974466 | 4·22 | 025534 | 41 |
| 20 | 836477 | 2·23 | 861758 | 1·99 | 974719 | 4·22 | 025281 | 40 |
| 21 | 9·836611 | 2·23 | 9·861638 | 1·99 | 9·974973 | 4·22 | 10·025027 | 39 |
| 22 | 836745 | 2·23 | 861519 | 1·99 | 975226 | 4·22 | 024774 | 38 |
| 23 | 836878 | 2·23 | 861400 | 1·99 | 975479 | 4·22 | 024521 | 37 |
| 24 | 837012 | 2·22 | 861280 | 1·99 | 975732 | 4·22 | 024268 | 36 |
| 25 | 837146 | 2·22 | 861161 | 1·99 | 975985 | 4·22 | 024015 | 35 |
| 26 | 837279 | 2·22 | 861041 | 1·99 | 976238 | 4·22 | 023762 | 34 |
| 27 | 837412 | 2·22 | 860922 | 1·99 | 976491 | 4·22 | 023509 | 33 |
| 28 | 837546 | 2·22 | 860802 | 1·99 | 976744 | 4·22 | 023256 | 32 |
| 29 | 837679 | 2·22 | 860682 | 2·00 | 976997 | 4·22 | 023003 | 31 |
| 30 | 837812 | 2·22 | 860562 | 2·00 | 977250 | 4·22 | 022750 | 30 |
| 31 | 9·837945 | 2·22 | 9·860442 | 2·00 | 9·977503 | 4·22 | 10·022497 | 29 |
| 32 | 838078 | 2·21 | 860322 | 2·00 | 977756 | 4·22 | 022244 | 28 |
| 33 | 838211 | 2·21 | 860202 | 2·00 | 978009 | 4·22 | 021991 | 27 |
| 34 | 838344 | 2·21 | 860082 | 2·00 | 978262 | 4·22 | 021738 | 26 |
| 35 | 838477 | 2·21 | 859962 | 2·00 | 978515 | 4·22 | 021485 | 25 |
| 36 | 838610 | 2·21 | 859842 | 2·00 | 978768 | 4·22 | 021232 | 24 |
| 37 | 838742 | 2·21 | 859721 | 2·01 | 979021 | 4·22 | 020979 | 23 |
| 38 | 838875 | 2·21 | 859601 | 2·01 | 979274 | 4·22 | 020726 | 22 |
| 39 | 839007 | 2·21 | 859480 | 2·01 | 979527 | 4·22 | 020473 | 21 |
| 40 | 839140 | 2·20 | 859360 | 2·01 | 979780 | 4·22 | 020220 | 20 |
| 41 | 9·839272 | 2·20 | 9·859239 | 2·01 | 9·980033 | 4·22 | 10·019967 | 19 |
| 42 | 839404 | 2·20 | 859119 | 2·01 | 980286 | 4·22 | 019714 | 18 |
| 43 | 839536 | 2·20 | 858998 | 2·01 | 980538 | 4·22 | 019462 | 17 |
| 44 | 839668 | 2·20 | 858877 | 2·01 | 980791 | 4·21 | 019209 | 16 |
| 45 | 839800 | 2·20 | 858756 | 2·02 | 981044 | 4·21 | 018956 | 15 |
| 46 | 839932 | 2·20 | 858635 | 2·02 | 981297 | 4·21 | 018703 | 14 |
| 47 | 840064 | 2·19 | 858514 | 2·02 | 981550 | 4·21 | 018450 | 13 |
| 48 | 840196 | 2·19 | 858393 | 2·02 | 981803 | 4·21 | 018197 | 12 |
| 49 | 840328 | 2·19 | 858272 | 2·02 | 982056 | 4·21 | 017944 | 11 |
| 50 | 840459 | 2·19 | 858151 | 2·02 | 982309 | 4·21 | 017691 | 10 |
| 51 | 9·840591 | 2·19 | 9·858029 | 2·02 | 9·982562 | 4·21 | 10·017438 | 9 |
| 52 | 840722 | 2·19 | 857908 | 2·02 | 982814 | 4·21 | 017186 | 8 |
| 53 | 840854 | 2·19 | 857786 | 2·02 | 983067 | 4·21 | 016933 | 7 |
| 54 | 840985 | 2·19 | 857665 | 2·03 | 983320 | 4·21 | 016680 | 6 |
| 55 | 841116 | 2·18 | 857543 | 2·03 | 983573 | 4·21 | 016427 | 5 |
| 56 | 841247 | 2·18 | 857422 | 2·03 | 983826 | 4·21 | 016174 | 4 |
| 57 | 841378 | 2·18 | 857300 | 2·03 | 984079 | 4·21 | 015921 | 3 |
| 58 | 841509 | 2·18 | 857178 | 2·03 | 984331 | 4·21 | 015669 | 2 |
| 59 | 841640 | 2·18 | 857056 | 2·03 | 984584 | 4·21 | 015416 | 1 |
| 60 | 841771 | 2·18 | 856934 | 2·03 | 984837 | 4·21 | 015163 | 0 |
| | Cosine | D. | Sine | 46° | Cotang. | D. | Tang. | M. |

| M. | Sine | D. | Cosine | D. | Tang. | D. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9·841771 | 2·18 | 9·856934 | 2·03 | 9·984837 | 4·21 | 10·015163 | 60 |
| 1 | 841902 | 2·18 | 856812 | 2·03 | 985090 | 4·21 | 014910 | 59 |
| 2 | 842033 | 2·18 | 856690 | 2·04 | 985343 | 4·21 | 014657 | 58 |
| 3 | 842163 | 2·17 | 856568 | 2·04 | 985596 | 4·21 | 014404 | 57 |
| 4 | 842294 | 2·17 | 856446 | 2·04 | 985848 | 4·21 | 014152 | 56 |
| 5 | 842424 | 2·17 | 856323 | 2·04 | 986101 | 4·21 | 013899 | 55 |
| 6 | 842555 | 2·17 | 856201 | 2·04 | 986354 | 4·21 | 013646 | 54 |
| 7 | 842685 | 2·17 | 856078 | 2·04 | 986607 | 4·21 | 013393 | 53 |
| 8 | 842815 | 2·17 | 855956 | 2·04 | 986860 | 4·21 | 013140 | 52 |
| 9 | 842946 | 2·17 | 855833 | 2·04 | 987112 | 4·21 | 012888 | 51 |
| 10 | 843076 | 2·17 | 855711 | 2·05 | 987365 | 4·21 | 012635 | 50 |
| 11 | 9·843206 | 2·16 | 9·855588 | 2·05 | 9·987618 | 4·21 | 10·012382 | 49 |
| 12 | 843336 | 2·16 | 855465 | 2·05 | 987871 | 4·21 | 012129 | 48 |
| 13 | 843466 | 2·16 | 855342 | 2·05 | 988123 | 4·21 | 011877 | 47 |
| 14 | 843595 | 2·16 | 855219 | 2·05 | 988376 | 4·21 | 011624 | 46 |
| 15 | 843725 | 2·16 | 855096 | 2·05 | 988629 | 4·21 | 011371 | 45 |
| 16 | 843855 | 2·16 | 854973 | 2·05 | 988882 | 4·21 | 011118 | 44 |
| 17 | 843984 | 2·16 | 854850 | 2·05 | 989134 | 4·21 | 010866 | 43 |
| 18 | 844114 | 2·15 | 854727 | 2·06 | 989387 | 4·21 | 010613 | 42 |
| 19 | 844243 | 2·15 | 854603 | 2·06 | 989640 | 4·21 | 010360 | 41 |
| 20 | 844372 | 2·15 | 854480 | 2·06 | 989893 | 4·21 | 010107 | 40 |
| 21 | 9·844502 | 2·15 | 9·854356 | 2·06 | 9·990145 | 4·21 | 10·009855 | 39 |
| 22 | 844631 | 2·15 | 854233 | 2·06 | 990398 | 4·21 | 009602 | 38 |
| 23 | 844760 | 2·15 | 854109 | 2·06 | 990651 | 4·21 | 009349 | 37 |
| 24 | 844889 | 2·15 | 853986 | 2·06 | 990903 | 4·21 | 009097 | 36 |
| 25 | 845018 | 2·15 | 853862 | 2·06 | 991156 | 4·21 | 008844 | 35 |
| 26 | 845147 | 2·15 | 853738 | 2·06 | 991409 | 4·21 | 008591 | 34 |
| 27 | 845276 | 2·14 | 853614 | 2·07 | 991662 | 4·21 | 008338 | 33 |
| 28 | 845405 | 2·14 | 853490 | 2·07 | 991914 | 4·21 | 008086 | 32 |
| 29 | 845533 | 2·14 | 853366 | 2·07 | 992167 | 4·21 | 007833 | 31 |
| 30 | 845662 | 2·14 | 853242 | 2·07 | 992420 | 4·21 | 007580 | 30 |
| 31 | 9·845790 | 2·14 | 9·853118 | 2·07 | 9·992672 | 4·21 | 10·007328 | 29 |
| 32 | 845919 | 2·14 | 852994 | 2·07 | 992925 | 4·21 | 007075 | 28 |
| 33 | 846047 | 2·14 | 852869 | 2·07 | 993178 | 4·21 | 006822 | 27 |
| 34 | 846175 | 2·14 | 852745 | 2·07 | 993430 | 4·21 | 006570 | 26 |
| 35 | 846304 | 2·14 | 852620 | 2·07 | 993683 | 4·21 | 006317 | 25 |
| 36 | 846432 | 2·13 | 852496 | 2·08 | 993936 | 4·21 | 006064 | 24 |
| 37 | 846560 | 2·13 | 852371 | 2·08 | 994189 | 4·21 | 005811 | 23 |
| 38 | 846688 | 2·13 | 852247 | 2·08 | 994441 | 4·21 | 005559 | 22 |
| 39 | 846816 | 2·13 | 852122 | 2·08 | 994694 | 4·21 | 005306 | 21 |
| 40 | 846944 | 2·13 | 851997 | 2·08 | 994947 | 4·21 | 005053 | 20 |
| 41 | 9·847071 | 2·13 | 9·851872 | 2·08 | 9·995199 | 4·21 | 10·004801 | 19 |
| 42 | 847199 | 2·13 | 851747 | 2·08 | 995452 | 4·21 | 004548 | 18 |
| 43 | 847327 | 2·13 | 851622 | 2·08 | 995705 | 4·21 | 004295 | 17 |
| 44 | 847454 | 2·12 | 851497 | 2·09 | 995957 | 4·21 | 004043 | 16 |
| 45 | 847582 | 2·12 | 851372 | 2·09 | 996210 | 4·21 | 003790 | 15 |
| 46 | 847709 | 2·12 | 851246 | 2·09 | 996463 | 4·21 | 003537 | 14 |
| 47 | 847836 | 2·12 | 851121 | 2·09 | 996715 | 4·21 | 003285 | 13 |
| 48 | 847964 | 2·12 | 850996 | 2·09 | 996968 | 4·21 | 003032 | 12 |
| 49 | 848091 | 2·12 | 850870 | 2·09 | 997221 | 4·21 | 002779 | 11 |
| 50 | 848218 | 2·12 | 850745 | 2·09 | 997473 | 4·21 | 002527 | 10 |
| 51 | 9·848345 | 2·12 | 9·850619 | 2·09 | 9·997726 | 4·21 | 10·002274 | 9 |
| 52 | 848472 | 2·11 | 850493 | 2·10 | 997979 | 4·21 | 002021 | 8 |
| 53 | 848599 | 2·11 | 850368 | 2·10 | 998231 | 4·21 | 001769 | 7 |
| 54 | 848726 | 2·11 | 850242 | 2·10 | 998484 | 4·21 | 001516 | 6 |
| 55 | 848852 | 2·11 | 850116 | 2·10 | 998737 | 4·21 | 001263 | 5 |
| 56 | 848979 | 2·11 | 849990 | 2·10 | 998989 | 4·21 | 001011 | 4 |
| 57 | 849106 | 2·11 | 849864 | 2·10 | 999242 | 4·21 | 000758 | 3 |
| 58 | 849232 | 2·11 | 849738 | 2·10 | 999495 | 4·21 | 000505 | 2 |
| 59 | 849359 | 2·11 | 849611 | 2·10 | 959748 | 4·21 | 000253 | 1 |
| 60 | 849485 | 2·11 | 849485 | 2·10 | 10·000000 | 4·21 | 10·000000 | 0 |
| | Cosine | D. | Sine | 45° | Cotang. | D. | Tang. | M. |

# RECOMMENDATIONS OF DAVIES' MATHEMATICS.

**Davies' Course of Mathematics** *are the prominent Text-Books in most of the Colleges of the United States*, **and also in the various Schools and Academies throughout the Union.**

York, Pa., *Aug.* 28, 1858.

*Davies' Series of Mathematics* I deem the very best I ever saw. From a number of authors I selected it, after a careful perusal, as a course of study to be pursued by the Teachers attending the sessions of the York Co. Normal School—believing it also to be well adapted to the wants of the schools throughout our country. Already two hundred schools are supplied with Davies' valuable *Series of Arithmetics;* and I fully believe that in a very short time the Teachers of our country *en masse* will be engaged in imparting instruction through the medium of this new and easy method of analysis of numbers. A. R. BLAIR,

*Principal of York Co. Normal School.*

Jackson Union School, Michigan, *Sept.* 25, 1858.

Messrs. A. S. Barnes & Co.:—I take pleasure in adding my testimony in favor of *Davies' Series of Mathematics*, as published by you. We have used these works in this school for more than four years; and so well satisfied are we of their superiority over any other Series, that we neither contemplate making, nor desire to make, any change in that direction. Yours truly, E. L. RIPLEY.

New Britain, *June 12th*, 1858.

Messrs. A. S. Barnes & Co.:—I have examined *Davies' Series of Arithmetics* with some care. They appear well adapted for the different grades of schools for which they are designed. The language is clear and precise; each principle is thoroughly analyzed, and the whole so arranged as to facilitate the work of instruction. Having observed the satisfaction and success with which the different books have been used by eminent teachers, it gives me pleasure to commend them to others.

DAVID N. CAMP, *Principal of Conn. State Normal School.*

I have long regarded *Davies' Series of Mathematical Text-Books* as far superior to any now before the public. We find them in every way adapted to the wants of the Normal School, and we use no other. A unity of system and method runs throughout the series, and constitutes one of its great excellences. Especially in the Arithmetics the author has earnestly endeavored to supply the wants of our Common and Union Schools: and his success is complete and undeniable. I know of no Arithmetics which exhibit so clearly the philosophy of numbers, and at the same time lead the pupil surely on to readiness and practice. A. S. WELCH.

*From* Prof. G. W. Plympton, *late of the State Normal School, N. Y.*

Out of a great number of Arithmetics that I have examined during the past year, I find none that will compare with *Davies' Intellectual* and *Davies' Analytical and Practical Arithmetics*, in clearness of demonstration or philosophical arrangement. I shall with pleasure recommend the use of these two excellent works to those who go from our institution to teach.

*From* C. May, Jr., *School Commissioner, Keene, N. H.*

I have carefully examined *Davies' Series of Arithmetics*, and *Higher Mathematics*, and am prepared to say that I consider them far superior to any with which I am acquainted.

*From* John L. Campbell, *Professor of Mathematics, Natural Philosophy, and Astronomy, in Wabash College, Indiana.*

Wabash College, *June* 22, 1858.

Messrs. A. S. Barnes & Co.:—Gentlemen: Every text-book on Science properly consists of two parts—the *philosophical* and the *illustrative.* A proper combination of abstract reasoning and practical illustration is the chief excellence in Prof. Davies' Mathematical Works. I prefer his Arithmetics, Algebras, Geometry, and Trigonometry, to all others now in use, and cordially recommend them to all who desire the advancement of sound learning, Yours, very truly, JOHN L. CAMPBELL.

Professors Mahan, Bartlett, and Church, of the United States Military Academy, West Point, say of *Davies' University Arithmetic:—*

"In the distinctness with which the various definitions are given, the clear and strictly mathematical demonstration of the rules, the convenient form and well-chosen matter of the tables, as well as in the complete and much-desired application of all to the business of the country, the *University Arithmetic* of Prof. Davies is superior to any other work of the kind with which we are acquainted."

# RECOMMENDATIONS

OF

# PARKER & WATSON'S READERS

*From* PROF. FREDERICK S. JEWELL, *of the New York State Normal School*

It gives me pleasure to find in the National Series of School Readers ample room for commendation. From a brief examination of them, I am led to believe that we have none equal to them. I hope they will prove as popular as they are excellent.

*From* HON. THEODORE FRELINGHUYSEN, *President of Rutgers' College, N. J.*

A cursory examination leads me to the conclusion that the system contained in these volumes deserves the patronage of our schools, and I have no doubt that it will become extensively used in the education of children and youth.

*From* N. A. HAMILTON, *President of Teachers' Union, Whitewater, Wis.*

The National Readers and Speller I have examined, and carefully compared with others, and must pronounce them decidedly superior, in respect to literary merit, style, and price. The gradation is more complete, and the series much more desirable for use in our schools than Sanders' or McGuffey's.

*From* PROF. T. F. THICKSTUN, *Principal of Academy and Normal School, Meadville, Pa.*

I am much pleased with the National Series of Readers after having canvassed their merits pretty thoroughly. The first of the series especially pleases me, because it affords the means of teaching the "*word-method*" in an appropriate and natural manner. They all are progressive, the rules of elocution are stated with clearness, and the selection of pieces is such as to please at the same time that they instruct.

*From* J. W. SCHERMERHORN, A. B., *Principal Coll. Institute, Middletown, N. J.*

I consider them emphatically *the* Readers of the present day, and I believe that their intrinsic merits will insure for them a full measure of popularity.

*From* PETER ROUGET, *Principal Public School No. 10, Brooklyn.*

It gives me great pleasure to be able to bear my unqualified testimony to the excellence of the National Series of Readers, by PARKER and WATSON. The gradation of the books of the series is very fine; we have reading in its elements and in its highest style. The fine taste displayed in the selections and in the collocation of the pieces deserves much praise. A distinguishing feature of the series is the variety of the subject-matter and of the style. The practical teacher knows the value of this characteristic for the development of the voice. The authors seem to have kept constantly in view the fact that a reading-book is designed for children, and therefore they have succeeded in forming a very interesting and improving collection of reading-matter, highly adapted to the wants and purposes of the school-room. In short, I look upon the National Series of Readers as a great success.

*From* A. P. HARRINGTON, *Principal of Union School, Marathon, N. Y.*

These Readers, in my opinion, are the best I have ever examined. The rhetorical exercises, in particular, are superior to any thing of the kind I have ever seen. I have had better success with my reading classes since I commenced training them on these than I ever met with before. The marked vowels in the reading exercises convey to the reader's mind at once the astonishing fact that he has been accustomed to mispronounce more than one-third of the words of the English language.

*From* CHARLES S. HALSEY, *Principal Collegiate Institute, Newton, N. J.*

In the simplicity and clearness with which the principles are stated, in the appropriateness of the selections for reading, and in the happy adaptation of the different parts of the series to each other, these works are superior to any other text-books on this subject which I have examined.

*From* WILLIAM TRAVIS, *Principal of Union School, Flint, Mich.*

I have examined the National Series of Readers, and am delighted to find it so far in advance of most other series now in use, and so well adapted to the wants of the Public Schools. It is unequaled in the skillful arrangement of the material used, beautiful typography, and the general neat and inviting appearance of its several books. I predict for it a cordial welcome and a general introduction by many of our most enterprising teachers.

# RECOMMENDATIONS
OF
# CLARK'S ENGLISH GRAMMAR.

We cannot better set forth the merits of this work than by quoting a part of a communication from Prof. F. S. Jewell, of the New York State Normal School, in which school this Grammar is now used as the text book on this subject:—

"Clark's System of Grammar is worthy of the marked attention of the friends of education. Its points of excellence are of the most decided character, and will not soon be surpassed. Among them are—

1st. "The justness of its ground principle of classification. There is no simple, philosophical, and practical classification of the elements of language, other than that built on their use or office. Our tendencies hitherto to follow the analogies of the classical languages, and classify extensively according to forms, have been mischievous and absurd. It is time we corrected them.

2d. "Its thorough and yet simple and transparent analysis of the elements of the language according to its ground principle. Without such an analysis, no broad and comprehensive view of the structure and power of the language can be attained. The absence of this analysis has hitherto precipitated the study of Grammar upon a surface of dry details and bare authorities, and useless technicalities.

3d. "Its happy method of illustrating the relations of elements by diagrams. These, however uncouth they may appear to the novice, are really simple and philosophical. Of their utility there can be no question. It is supported by the usage of other sciences, and has been demonstrated by experience in this.

4th. "The tendency of the system, when rightly taught and faithfully carried out, to cultivate habits of nice discrimination and close reasoning, together with skill in illustrating truth. In this it is not excelled by any, unless it be the mathematical sciences, and even there it has this advantage, that it deals with elements more within the present grasp of the intellect. On this point I speak advisedly.

5th. "The system is thoroughly progressive and practical, and as such, American in its character. It does not adhere to old usages, merely because they are venerably musty; and yet it does not discard things merely because they are old, or are in unimportant minutiæ not prudishly perfect. It does not overlook details and technicalities, nor does it allow them to interfere with plain philosophy or practical utility.

"Let any clear-headed, independent-minded teacher master the system, and then give it a fair trial, and there will be no doubt as to his testimony."

*A Testimonial from the Principals of the Public Schools of Rochester, N. Y.*

We regard Clark's Grammar as the clearest in its analysis, the most natural and logical in its arrangement, the most concise and accurate in its definitions, the most systematic in design, and the best adapted to the use of schools of any Grammar with which we are acquainted.

| | |
|---|---|
| C. C. MESERVE, | WM. C. FEGLES, |
| M. D. ROWLEY, | JOHN ATWATER, |
| C. R. BURRICK, | EDWARD WEBSTER, |
| J. R. VOSBURG, | S. W. STARKWEATHER, |
| E. R. ARMSTRONG, | PHILIP CURTISS. |

Lawrence Institute, *Brooklyn, Jan.* 15, 1859.

Messrs. A. S. Barnes & Co:—Having used Clark's New Grammar since its publication, I do most unhesitatingly recommend it as a work of superior merit. By the use of no other work, and I have used several, have I been enabled to advance my pupils so rapidly and thoroughly.

The author has, by an Etymological Chart and a system of Diagrams, made Grammar the study that it ought to be, interesting as well as useful.

MARGARET S. LAWRENCE, *Principal.*

# WELCH'S ENGLISH SENTENCE.

*From* Prof. J. R. Boise, A. M., *Professor of the Latin and Greek Languages and Literature in the University of Michigan.*

This work belongs to a new era in the grammatical study of our own language. We hazard nothing, in expressing the opinion, that for severe, searching, and exhaustive analysis, the work of Professor Welch is second to none. His book is not intended for beginners, but only for advanced students, and by such only it will be understood and appreciated.

# MONTEITH AND McNALLY'S GEOGRAPHIES.

## THE MOST SUCCESSFUL SERIES EVER ISSUED.

## RECOMMENDATIONS.

**A. B. Clark**, Principal of one of the largest Public Schools in Brooklyn, says:—"I have used over a thousand copies of Monteith's Manual of Geography since its adoption by the Board of Education, and am prepared to say it is the best work for junior and intermediate classes in our schools I have ever seen."

*The Series, in whole or in part, has been adopted in the*

New York State Normal School.
New York City Normal School.
New Jersey State Normal School.
Kentucky State Normal School.
Indiana State Normal School.
Ohio State Normal School.
Michigan State Normal School.
York County (Pa.) Normal School.
Brooklyn Polytechnic Institute.
Cleveland Female Seminary.
Public Schools of Milwaukie.
Public Schools of Pittsburgh.
Public Schools of Lancaster, Pa.
Public Schools of New Orleans.
Public Schools of New York.
Public Schools of Brooklyn, L. I.
Public Schools of New Haven.
Public Schools of Toledo, Ohio.
Public Schools of Norwalk, Conn.
Public Schools of Richmond, Va.
Public Schools of Madison, Wis.
Public Schools of Indianapolis.
Public Schools of Springfield, Mass.
Public Schools of Columbus, Ohio.
Public Schools of Hartford, Conn.
Public Schools of Cleveland, Ohio.
And other places too numerous to mention.

They have also been recommended by the State Superintendents of Illinois, Indiana, Wisconsin, Missouri, North Carolina, Alabama, and by numerous Teachers' Associations and Institutes throughout the country, and are in successful use in a multitude of Public and Private Schools throughout the United States.

*From* Prof. Wm. F. Phelps, A. M., *Principal of the New Jersey State Normal School.*

Trenton, *June* 17, 1858.

Messrs. A. S. Barnes & Co.:—Gentlemen: It gives me much pleasure to state that McNally's Geography has been used in this Institution from its organization in 1855, with great acceptance. The author of this work has avoided on one hand the extreme of being too meager, and on the other of going too much into detail, while he has presented, in a clear and concise manner, all those leading facts of Descriptive Geography which it is important for the young to know. The maps are accurate and well executed, the type clear, and indeed the entire work is a decided success. I most cheerfully commend it to the profession throughout the country.

Very truly yours, WM. F. PHELPS.

*From* W. V. Davis, *Principal of High School, Lancaster, Pa.*

Lancaster, Pa., *June* 26, 1858.

Dear Sirs:—I have examined your *National Geographical Series* with much care, and find them most excellent works of their kind. They have been used in the various Public Schools of this city, ever since their publication, with great success and satisfaction to both pupil and teacher. All the Geographies embraced in your series are well adapted to school purposes, and admirably calculated to impart to the pupil, in a very attractive manner, a complete knowledge of a science, annually becoming more useful and important. Their maps, illustrations, and typography, are unsurpassed. One peculiar feature of McNally's Geography—and which will recommend it at once to every practical teacher—is the arrangement of its maps and lessons; each map fronts the particular lesson which it is designed to illustrate—thus enabling the scholar to prepare his task without that constant turning over of leaves, or reference to a separate book, as is necessary with most other Geographies. Yours, &c.

Messrs. A. S. Barnes & Co., New York. V. W. DAVIS.

*From* Charles Barnes, *late President State Teachers' Association, and Superintendent of the Public Schools at New Albany, Indiana.*

Messrs. A. S. Barnes & Co.:—Dear Sirs: I have examined with considerable care the Series of Geographies published by you, and have no hesitation in saying that it is altogether the best with which I am acquainted. A trial of more than a year in the Public Schools of this city has demonstrated that *Cornell* is utterly unfit for the school-room. Yours, &c. C. BARNES.

# RECOMMENDATIONS

OF

# MONTEITH'S HISTORY OF THE UNITED STATES.

This volume is designed for youth, and we think the author has been unusually successful in its arrangement and entire preparation. Books of the same design are too often beyond the full understanding of the scholar. As history is so much neglected in all our schools, the publication of such a work as this should be hailed with pleasure; for if scholars find their first studies of history pleasant, it will become a pleasure rather than a task. This is a book of 88 pages, and finely illustrated. It is in every way worthy of a place in every Public School in the State.—*Maine Teacher.*

This is a most capital work: just the thing for children. Our boy commenced the study of it the day it came to hand. It is arranged in the catechetical form, and is finely illustrated with maps, with special reference to the matter discussed in the text. It begins with the first discoveries of America, and comes down to the laying of the Atlantic Telegraph Cable. Many spirited engravings are given to illustrate the work. It also contains brief Biographies of all prominent men who have identified themselves with the history of this country. It is the best work of the kind we have seen.—*Chester County Times.*

# WILLARD'S HISTORIES.

*From* REV. HOWARD MALCOLM, D.D., *President of the University of Lewisburg.*

I have examined, during the thirteen years that I have had charge of a College, many School Histories of the United States, and have found none, on the whole, so proper for a text-book as that of Mrs. Willard. It is neither too short nor too long, all the space given to periods, events, and persons, is happily proportioned to their importance. The style is attractive and lucid, and the narrative so woven, as both to sustain the interest and aid the memory of the student. Candor, impartiality, and accuracy, are conspicuous throughout. I think no teacher intending to commence a history class will be disappointed in adopting this book.

MRS. L. H. SIGOURNEY, *the distinguished Authoress*, writes:

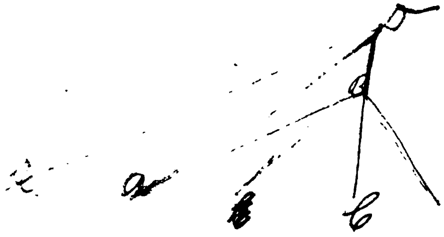
Mrs. Willard should be considered as a benefactress not only by her own sex, of whom she became in early years a prominent and permanent educator, but by the country at large, to whose good she has dedicated the gathered learning and faithful labor of life's later periods. The truths that she has recorded, and the principles that she has impressed, will win, from a future race, gratitude that cannot grow old, and a garland that will never fade.

DANIEL WEBSTER *wrote, in a letter to the Author:*

I cannot better express my sense of the value of your *History of the United States*, than by saying I keep it near me as a book of reference, accurate in facts and dates.

# DWIGHT'S MYTHOLOGY.

The mythology of the Grecians and Romans is so closely interlinked with the history and literature of the world, that some knowledge of it is indispensable to any scholarly familiarity with either that history or literature. We have seen no book so convenient in size that contains so full and elegant an exposition of mythology as the one before us. It will be found at once a most interesting and a most useful book to any one who wishes an acquaintance with the splendid myths and fables with which the great masters of ancient learning amused their leisure and cheated their faith -*Michigan Journal of Education.*



~~abc~~

BAC = 41°

DAC = 51°

DBC = 33° 45'

CA = 180

C = 90°

51° − 41° = 10°

DAB = 10°

BAD = 180 − 51 = 129°

BAD = 129°

[illegible] = 17°13' = 0.52 [illegible]

= [illegible] = 9.74 [illegible]

[illegible]

AB = [illegible] 321

[illegible]

SEP 26 1888
FEB 2 1889
OCT 2 1890
[illegible] 1892
[illegible]
JAN 25 1894
[illegible]